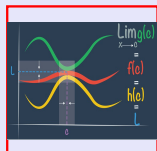


Math 261
Fall 2023
Lecture 47



Feb 19-8:47 AM

1) Evaluate $\int (x-4) dx$

$$= \int x dx - \int 4 dx = \boxed{\frac{x^2}{2} - 4x + C}$$

2) Evaluate $\int (x-4)^2 dx$

method I: $\int (x-4)^2 dx = \int (x^2 - 8x + 16) dx$

$$= \frac{x^3}{3} - \frac{8x^2}{2} + 16x + C$$

$$= \boxed{\frac{1}{3}x^3 - 4x^2 + 16x + C}$$

method II: $\int (x-4)^2 dx$

$u = x-4$
 $du = dx$

$$= \int u^2 du = \frac{u^3}{3} + C$$

$$= \boxed{\frac{1}{3}(x-4)^3 + C}$$

Nov 22-10:25 AM

3) Evaluate $\int \sqrt{x-4} \, dx$

Method I: $u = x-4$
 $du = dx$

Method II:

$$u = \sqrt{x-4}$$

$$u^2 = x-4$$

$$2u \, du = dx$$

$$\int \sqrt{x-4} \, dx = \int u \cdot 2u \, du$$

$$= \int 2u^2 \, du = 2 \cdot \frac{u^3}{3} + C$$

$$= \frac{2}{3} (\sqrt{x-4})^3 + C$$

$$= \frac{2}{3} (\sqrt{x-4})^2 (\sqrt{x-4}) + C$$

$$= \frac{2}{3} (x-4) \sqrt{x-4} + C$$

$$\int \sqrt{x-4} \, dx = \int \sqrt{u} \, du = \int u^{1/2} \, du$$

$$= \frac{u^{1/2+1}}{\frac{1}{2}+1} + C$$

$$= \frac{u^{3/2}}{3/2} + C$$

$$= \frac{2}{3} \sqrt{u^3} + C$$

$$= \frac{2}{3} u \sqrt{u} + C$$

$$= \frac{2}{3} (x-4) \sqrt{x-4} + C$$

Nov 22-10:30 AM

Evaluate $\int x \sin x^2 \, dx$

Let $u = x^2$ $du = 2x \, dx$ $\frac{du}{2} = x \, dx$

$$\int x \sin x^2 \, dx = \int \sin u \cdot \frac{du}{2} = \frac{1}{2} \int \sin u \, du$$

$$= \frac{1}{2} \cdot (-\cos u) + C$$

$$= -\frac{1}{2} \cos x^2 + C$$

$$\frac{d}{dx} [\cos x] = -\sin x$$

$$\int \sin x \, dx = -\cos x + C$$

Nov 22-10:36 AM

Find $\int \cos x \cdot \sec^2(\sin x) dx$ Hint: Let $u = \sin x$

Let $u = \sin x$
 $du = \cos x dx$

$$\frac{d}{dx} [\tan x] = \sec^2 x$$

$$\int \sec^2 x dx = \tan x + C$$

$$= \int \sec^2 u du$$

$$= \tan u + C$$

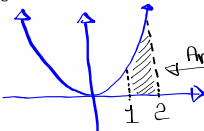
$$= \boxed{\tan(\sin x) + C}$$

Nov 22-10:40 AM

Evaluate $\int_1^2 x^2 dx = \frac{x^3}{3} \Big|_1^2 = \frac{1}{3} [2^3 - 1^3]$

What does this answer represent?

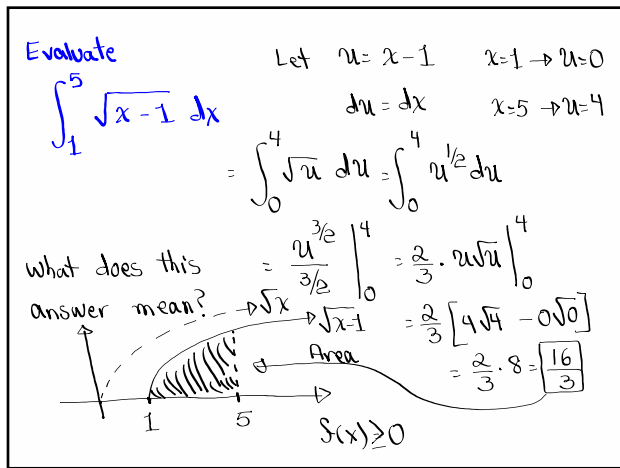
$f(x) = x^2 \geq 0$



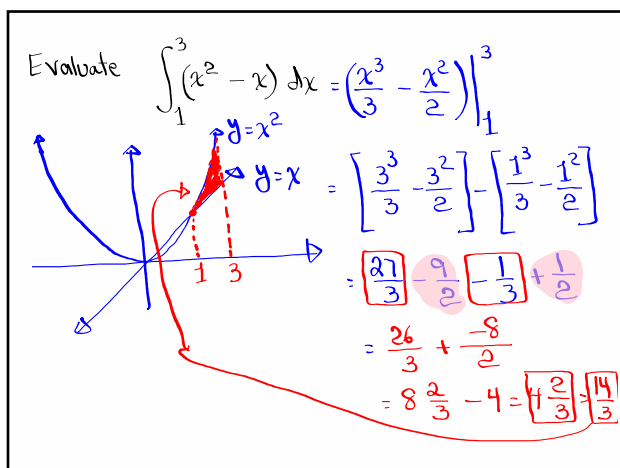
$= \frac{1}{3} [8 - 1]$

$\boxed{\frac{7}{3}}$

Nov 22-10:45 AM

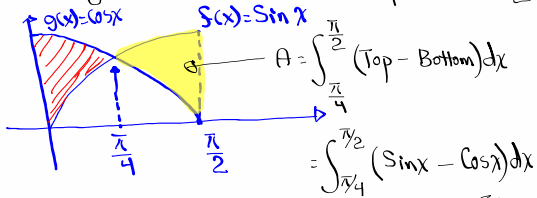


Nov 22-10:49 AM



Nov 22-10:54 AM

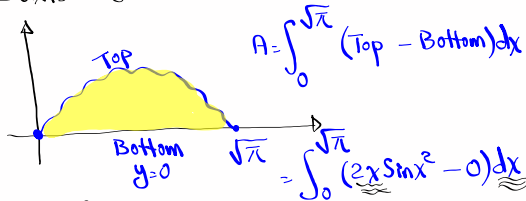
Find the area below $f(x) = \sin x$ and above $g(x) = \cos x$ from $x = \frac{\pi}{4}$ to $x = \frac{\pi}{2}$.



$$\begin{aligned}
 \int \sin x \, dx &= -\cos x + C \\
 \int \cos x \, dx &= \sin x + C \\
 A &= \int_{\pi/4}^{\pi/2} (\sin x - \cos x) \, dx \\
 &= (-\cos x - \sin x) \Big|_{\pi/4}^{\pi/2} \\
 &= \left(-\cos \frac{\pi}{2} - \sin \frac{\pi}{2} \right) - \left(-\cos \frac{\pi}{4} - \sin \frac{\pi}{4} \right) \\
 &= -1 - (-\sqrt{2}) \\
 &= \boxed{\sqrt{2} - 1}
 \end{aligned}$$

Nov 22-11:00 AM

Find the area below $f(x) = 2x \sin x^2$, above x -axis from $x=0$ and $x=\sqrt{\pi}$.

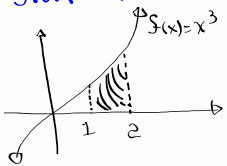


$$\begin{aligned}
 \text{Let } u &= x^2 \quad x=0 \rightarrow u=0 \\
 du &= 2x \, dx \quad x=\sqrt{\pi} \rightarrow u=\pi \\
 A &= \int_0^{\sqrt{\pi}} (2x \sin x^2 - 0) \, dx \\
 &= \int_0^{\pi} \sin u \, du \\
 &= -\cos u \Big|_0^{\pi} \\
 &= -[\cos \pi - \cos 0] \\
 &= -[-1 - 1] = \boxed{2}
 \end{aligned}$$

Nov 22-11:07 AM

class QZ 13

Find the area below $f(x)=x^3$, above x -axis
from $x=1$ to $x=2$. Exact Ans. only.



Give some drawing.

$$A = \int_1^2 x^3 dx = \frac{x^4}{4} \Big|_1^2$$

$$= \frac{1}{4} [2^4 - 1^4] = \frac{1}{4} [16 - 1] = \frac{15}{4}$$

$$= 3.75$$

Nov 22-11:14 AM