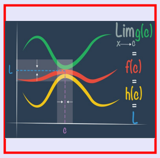
Math 261

Fall 2023

Lecture 46



Feb 19-8:47 AM

1) find $\int (\sec^2 x - \csc^2 x)\, dx$

$= \int \sec^2 x\, dx - \int \csc^2 x\, dx$

$= \tan x - (-\cot x) + C$

$= \boxed{\tan x + \cot x + C}$

Indifinite Integral $\rightarrow +C$

$\frac{d}{dx}[\tan x] = \sec^2 x$

$\frac{d}{dx}[\cot x] = -\csc^2 x$

2) Evaluate $\int_2^4 (3x^2 + 6x)\, dx$

Definite integral $\rightarrow$ NO C

$= \left(\cancel{3} \frac{x^3}{\cancel{3}} + \overset{3}{\cancel{6}} \frac{x^2}{\cancel{2}} \right) \Big|_2^4$

$= (x^3 + 3x^2) \Big|_2^4 = [4^3 + 3(4)^2] - [2^3 + 3(2)^2]$

$= 112 - 20 = \boxed{92}$

Nov 21-10:25 AM

Evaluate

$$\int_0^1 \underbrace{(2x+3)(x-3)}_{\text{Foil \& Simplify}}\,dx$$

$$= \int_0^1 (2x^2 - 3x - 9)\,dx$$

$$= \left[2\frac{x^3}{3} - \frac{3x^2}{2} - 9x\right]_0^1$$

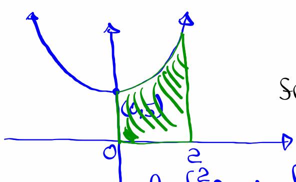
$$= \left(\frac{2\cdot 1^3}{3} - \frac{3\cdot 1^2}{2} - 9(1)\right) - \cancel{\left(\frac{2\cdot 0^3}{3} - \frac{3\cdot 0^2}{2} - 9(0)\right)} \to 0$$

$$= \frac{2}{3} - \frac{3}{2} - 9 = \boxed{-\frac{59}{6}}$$

Nov 21-10:33 AM

Find the area below $\underline{f(x) = 3x^2+5}$, above x-axis from $x=0$ to $x=2$.

Parabola
opens upward
Vertex $(0,5)$



$f(x) \geq 0$, $f(x)$ is cont. & diff.

$$A = \int_0^2 f(x)\,dx = \int_0^2 (3x^2+5)\,dx$$

$$= \left(\frac{3x^3}{3} + 5x\right)\Big|_0^2$$

$$= (x^3 + 5x)\Big|_0^2$$

$$= 2^3 + 5(2) - 0$$

$$= \boxed{18}$$

Nov 21-10:37 AM

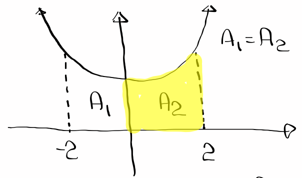
Find the area below $f(x)=3x^2+5$, above x-axis from $x=-2$ to $x=2$.

$f(-x)=3(-x)^2+5=3x^2+5=f(x)$

Even Function

Symmetric w/r Y-axis

$A_1=A_2$



$$A=\int_{-2}^{2}(3x^2+5)\,dx = 2\int_{0}^{2}(3x^2+5)\,dx = 2\cdot 18 = \boxed{36}$$

$$= (x^3+5x)\Big|_{-2}^{2} = (2^3+5(2)) - ((-2)^3+5(-2))$$

$$= 18 - (-18) = 18+18 = \boxed{36}$$

Nov 21-10:42 AM

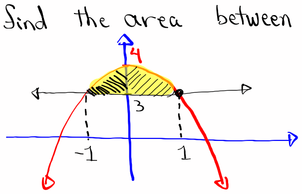
If $f(x)\geq g(x)$ over $[a,b]$, both cont. & diff.

then area between them is

$$A=\int_a^b [f(x)-g(x)]\,dx$$

Find the area between $f(x)=4-x^2$ and $y=3$. Horizontal line

Parabola open downward vertex $(0,4)$



$4-x^2=3$

$x^2=1 \quad x=\pm 1$

$$A=\int_{-1}^{1}(\text{Top}-\text{Bottom})\,dx = \int_{-1}^{1}((4-x^2)-3)\,dx$$

$$=\int_{-1}^{1}(1-x^2)\,dx = 2\int_{0}^{1}(1-x^2)\,dx$$

even function

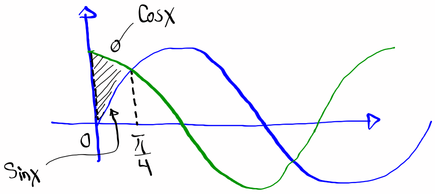
$$=2\left[x-\frac{x^3}{3}\right]\Bigg|_0^1$$

$$=2\left[\left(1-\cancel{\frac{1^3}{3}}\right)-\left(0-\cancel{\frac{0^3}{3}}\right)\right] \quad \frac{2}{3} \quad 0$$

$$=2\cdot\frac{2}{3}=\boxed{\frac{4}{3}}$$

Nov 21-10:48 AM

Find the area enclosed by $f(x) = \sin x$ and $g(x) = \cos x$ from $x = 0$ to $x = \frac{\pi}{4}$.



$f(x) = g(x)$

$\sin x = \cos x$

$\frac{\sin x}{\cos x} = 1$

$\tan x = 1$

$x = \pi/4$

$\frac{d}{dx}[\sin x] = \cos x$

$\frac{d}{dx}[\cos x] = -\sin x$

$$A = \int_0^{\pi/4} [\text{Top} - \text{Bottom}]\,dx =$$

$$= \int_0^{\pi/4} [\cos x - \sin x]\,dx =$$

$$= (\sin x + \cos x)\Big|_0^{\pi/4}$$

$$= \left(\cancel{\sin \tfrac{\pi}{4}}^{\frac{\sqrt{2}}{2}} + \cancel{\cos \tfrac{\pi}{4}}^{\frac{\sqrt{2}}{2}}\right) - \left(\cancel{\sin 0}^{0} + \cancel{\cos 0}^{1}\right)$$

$$= \boxed{\sqrt{2} - 1}$$

Nov 21-10:56 AM

Integration by Substitution Method

$$\int (2x+3)^4\,dx$$

Let $u = 2x + 3$

$\frac{du}{dx} = 2 \rightarrow du = 2\,dx$

$\frac{du}{2} = dx$

$$= \int u^4 \cdot \frac{du}{2}$$

$$= \int \frac{1}{2} u^4\,du = \frac{1}{2} \cdot \frac{u^5}{5} + C$$

$$= \boxed{\frac{1}{10}(2x+3)^5 + C}$$

Check:

$$\frac{d}{dx}\left[\frac{1}{10}(2x+3)^5 + C\right] = \frac{1}{\cancel{10}} \cdot \cancel{5}(2x+3)^4 \cdot \cancel{2} + 0$$

$$= (2x+3)^4$$

Nov 21-11:04 AM

$\int 3x^2 \cos x^3 \, dx$

Let $u = x^3$

$\frac{du}{dx} = 3x^2$

$du = 3x^2 \, dx$

$= \int \cos u \, du$

$= \sin u + C$

$= \boxed{\sin x^3 + C}$

$\frac{d}{dx}[\sin x] = \cos x$

$\int \cos x \, dx = \sin x$

check:

$\frac{d}{dx}[\sin x^3 + C] = \cos x^3 \cdot 3x^2 + 0 = \boxed{3x^2 \cos x^3}$

Nov 21-11:10 AM

$\int (2x - 1)(x^2 - x + 10)^5 \, dx$

Let $u = x^2 - x + 10$

$du = (2x - 1) \, dx$

$= \int u^5 \, du = \frac{u^6}{6} + C$

$= \boxed{\frac{1}{6}(x^2 - x + 10)^6 + C}$

check:

$\frac{d}{dx}\left[\frac{1}{6}(x^2 - x + 10)^6 + C\right] = \frac{1}{\cancel{6}} \cdot \cancel{6}(x^2 - x + 10)^5 \cdot (2x - 1) + 0$

$= \boxed{(2x - 1)(x^2 - x + 10)^5}$

Nov 21-11:15 AM

$$\int x^2 \sqrt{x^3+1}\, dx$$

Hint : Let $u = x^3+1$

$du = 3x^2 dx$

$\frac{du}{3} = x^2 dx$

$$= \int \sqrt{u}\, \frac{du}{3}$$

$$= \frac{1}{3}\int u^{1/2}\, du = \frac{1}{3} \cdot \frac{u^{1/2+1}}{\frac{1}{2}+1} + C$$

$$= \frac{1}{3} \cdot \frac{u^{3/2}}{3/2} + C$$

$$= \frac{1}{3} \cdot \frac{2}{3} \cdot u^{3/2} + C$$

$$= \frac{2}{9} \sqrt{u^3} + C$$

$$= \boxed{\frac{2}{9} \sqrt{(x^3+1)^3} + C}$$

$$= \boxed{\frac{2}{9} (x^3+1)\sqrt{x^3+1} + C}$$

Assume $x \geq 0$

Nov 21-11:21 AM