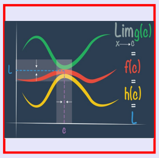
Math 261

Fall 2023

Lecture 45



Introduction to Integration

"Reverse of differentiation"

$$\int f(x)\,dx$$

Integral Integrand Respect to x.

$$\int f(x)\,dx = F(x) + C$$

where $\frac{d}{dx}[F(x)] = f(x)$

1) $\int \cos x\,dx = \boxed{\sin x + C}$

$\frac{d}{dx}[\sin x] = \cos x$

2) $\int 2x\,dx = x^2 + C$

$\frac{d}{dx}[x^2] = 2x$

$\int \sec x \tan x\,dx = \sec x + C$ $\frac{d}{dx}[\sec x] = \sec x \tan x$

$\int 4\,dx = 4x + C$, $\frac{d}{dx}[4x] = 4$

Power rule

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C \quad , \; n \neq -1$$

$$\int x^5\,dx = \frac{x^{5+1}}{5+1} + C = \boxed{\frac{1}{6}x^6 + C}$$

$$\int \frac{1}{x^5}\,dx = \int x^{-5}\,dx = \frac{x^{-5+1}}{-5+1} + C$$

$$= \frac{x^{-4}}{-4} + C$$

$$= \frac{-1}{4\,x^4} + C$$

check

$$\frac{d}{dx}\left[\frac{-1}{4x^4} + C\right]$$

$$= \frac{d}{dx}\left[\frac{-1}{4}x^{-4} + C\right] = \frac{-1}{\cancel{4}}\cdot(\cancel{-4})\cdot x^{-4-1} + 0$$

$$= x^{-5} = \boxed{\frac{1}{x^5}}$$

$$\int [f(x) \pm g(x)]\,dx = \int f(x)\,dx \pm \int g(x)\,dx$$

$$\int [\cos x + 4x^3]\,dx = \int \cos x\,dx + \int 4x^3\,dx$$

$$= \sin x + C_1 + 4\cdot\frac{x^4}{4} + C_2$$

$$= \boxed{\sin x + x^4 + C}$$

$$\int [\sqrt{x} - 3\sin x]\,dx =$$

$$\int \sqrt{x}\,dx - \int 3\sin x\,dx =$$

$$\frac{d}{dx}[\cos x] = -\sin x$$

$$\int x^{1/2}\,dx - 3\int \sin x\,dx =$$

$$\int \sin x\,dx = -\cos x$$

$$\frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} - 3\cdot(-\cos x) + C =$$

$$\frac{x^{3/2}}{3/2} + 3\cos x + C = \boxed{\frac{2}{3}x^{3/2} + 3\cos x + C}$$

check

$$\frac{d}{dx}\left[\frac{2}{3}x^{3/2} + 3\cos x + C\right] = \frac{\cancel{2}}{\cancel{3}}\cdot\frac{\cancel{3}}{\cancel{2}}\,x^{\frac{3}{2}-1} + 3\cdot(-\sin x) + 0$$

$$= x^{1/2} - 3\sin x = \boxed{\sqrt{x} - 3\sin x}$$

$$\int (6x^2 - 8x + 5)\,dx$$

$$= \int 6x^2\,dx - \int 8x\,dx + \int 5\,dx$$

$$= 6\int x^2\,dx - 8\int x\,dx + 5\int 1\,dx$$

$$= 6\cdot\frac{x^3}{3} - 8\cdot\frac{x^2}{2} + 5x + C$$

$$= \boxed{2x^3 - 4x^2 + 5x + C}$$

check $\frac{d}{dx}\left[2x^3 - 4x^2 + 5x + C\right] =$

$$2\cdot 3x^2 - 4\cdot 2x^1 + 5 + 0 =$$

$$\boxed{6x^2 - 8x + 5}$$

Find $\int \left(\sqrt[3]{x} - \frac{1}{x^3}\right) dx$

$$= \int \left(x^{1/3} - x^{-3}\right) dx$$

$$= \frac{x^{\frac{1}{3}+1}}{\frac{1}{3}+1} - \frac{x^{-3+1}}{-3+1} + C$$

$$= \frac{x^{4/3}}{4/3} - \frac{x^{-2}}{-2} + C$$

$$= \frac{3}{4}x^{4/3} + \frac{1}{2x^2} + C$$

$$= \boxed{\frac{3}{4}x\sqrt[3]{x} + \frac{1}{2x^2} + C}$$

find $\int \underbrace{(x+4)(x-4)}_{\text{foil \& Simplify}}\,dx$

$= \int (x^2 - 16)\,dx = \boxed{\dfrac{x^3}{3} - 16x + C}$

Check $\dfrac{d}{dx}\left[\dfrac{x^3}{3} - 16x + C\right] =$

$\dfrac{1}{3} \cdot 3x^2 - 16 + 0 = x^2 - 16$

$\int f'(x)\,dx = f(x) + C$ Indefinite Integral

$\int_a^b f'(x)\,dx = f(x)\Big|_a^b = f(b) - f(a)$ Definite Integral

ex: $\int_1^3 (2x+5)\,dx = (x^2+5x)\Big|_1^3 = [3^2+5(3)] - [1^2+5(1)]$

$= [9+15] - [1+5]$

$= 24 - 6 = \boxed{18}$

$$\int_0^{\pi/2} \cos x \, dx = \sin x \Big|_0^{\pi/2}$$

b, a, $f'(x)$

$$\frac{d}{dx}[\sin x] = \cos x$$

$$\int \cos x \, dx = \sin x + C$$

$$= \sin\frac{\pi}{2} - \sin 0$$

$$= 1 - 0 = \boxed{1}$$

$$\int_{-\frac{\pi}{4}}^{\pi/4} \sec^2 x \, dx = \tan x \Big|_{-\pi/4}^{\frac{\pi}{4}}$$

$$\frac{d}{dx}[\tan x] = \sec^2 x$$

$$\int \sec^2 x \, dx = \tan x + C$$

$$= \tan\left(\frac{\pi}{4}\right) - \tan\left(-\frac{\pi}{4}\right)$$

$$= 1 - (-1) = \boxed{2}$$

Evaluate $\int_4^9 \sqrt{x} \, dx = \int_4^9 x^{1/2} \, dx$

$$= \frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} \Bigg|_4^9$$

$$= \frac{x^{\frac{3}{2}}}{\frac{3}{2}} \Bigg|_4^9 = \frac{2}{3} x^{3/2} \Big|_4^9 = \frac{2}{3} x\sqrt{x} \Big|_4^9$$

$$= \frac{2}{3}\left[9\sqrt{9} - 4\sqrt{4}\right] = \frac{2}{3}[27 - 8] = \frac{2}{3} \cdot 19$$

$$= \boxed{\frac{38}{3}}$$

Evaluate $\int_{-2}^{2} x^2\,dx$

$$= \frac{x^3}{3}\Bigg|_{-2}^{2} = \frac{1}{3}\left[2^3 - (-2)^3\right]$$

$$= \frac{1}{3}\left[8 - (-8)\right] = \boxed{\frac{16}{3}}$$

Evaluate $\int_{-1}^{1} x^3\,dx = \frac{x^4}{4}\Bigg|_{-1}^{1}$

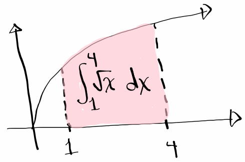
$$= \frac{1}{4}\left[1^4 - (-1)^4\right] = \frac{1}{4}\left[1 - 1\right] = \boxed{0}$$

If $f(x) \geq 0$ on $[a, b]$ and $f(x)$ is diff. on (a, b),

then the area below $f(x)$ and above x-axis

is $\int_a^b f(x)\,dx$.

$f(x) = \sqrt{x}$ over $[1, 4]$

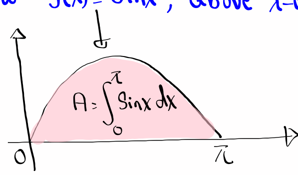


$$A = \int_1^4 \sqrt{x}\,dx = \int_1^4 x^{1/2}\,dx = \frac{x^{3/2}}{3/2}\Bigg|_1^4 = \frac{2}{3}x\sqrt{x}\Bigg|_1^4$$

$$= \frac{2}{3}\left[4\sqrt{4} - 1\sqrt{1}\right]$$

$$= \frac{2}{3}\left[8 - 1\right] = \boxed{\frac{14}{3}}$$

Find the area below $f(x) = \sin x$, above x-axis from 0 to π.



$A = \int_0^{\pi} \sin x \, dx = -\cos x \Big|_0^{\pi}$

$= -\left[\cos \pi - \cos 0\right]$

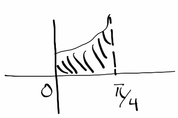
$= -\left[-1 - 1\right]$

$= -(-2) = \boxed{2}$

$\frac{d}{dx}[\cos x] = -\sin x$

$\int \sin x \, dx = -\cos x + C$

Find the area below $f(x) = \sec^2 x$, above x-axis from 0 to $\frac{\pi}{4}$.



$A = \int_0^{\pi/4} \sec^2 x \, dx$

$= \tan x \Big|_0^{\pi/4} = \tan \pi/4 - \tan 0$

$= \boxed{1}$