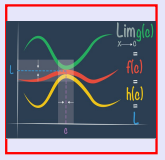
Math 261

Fall 2023

Lecture 44



Feb 19-8:47 AM

Use Newton's method to estimate $\sqrt[5]{35}$, Round to 3-decimal places.

Let $x = \sqrt[5]{35} \rightarrow x^5 = 35 \rightarrow \underbrace{x^5 - 35}_{f(x)} = 0$

$f(x) = x^5 - 35$

$f'(x) = 5x^4$

Newton's Eqn $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

$x_{n+1} = x_n - \frac{x_n^5 - 35}{5x_n^4} = \frac{5x_n^5 - x_n^5 + 35}{5x_n^4}$

$x_{n+1} = \frac{4x_n^5 + 35}{5x_n^4}$

Now $\sqrt[5]{35} \approx \sqrt[5]{32} = 2$

Let's start with $x_1 = 2$

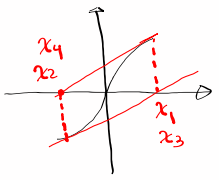
$x_2 = \frac{4(2)^5 + 35}{5(2)^4} = 2.038$, $x_3 = \frac{4(2.038)^5 + 35}{5(2.038)^4} = 2.036$

$x_4 = \frac{4(2.036)^5 + 35}{5(2.036)^4} = \boxed{2.036}$ Soln ≈ 2.036

check $(2.036)^5 \approx 34.986 \approx 35$

check $\sqrt[5]{35} = 35^{\frac{1}{5}} = 35^{.2} \approx 2.036$

Nov 15-10:25 AM



Make Sure to review Newton's Method for exam 3.

Nov 15-10:37 AM

$f(x) = \frac{x^2}{x-1}$

1) Domain $(-\infty, 1) \cup (1, \infty)$

2) V. A. $x = 1$

3) y-Int $(0,0)$ 4) x-Int $(0,0)$

4) $\lim_{x \to \infty} f(x) = \lim_{x \to \infty} \frac{x^2}{x-1} = \infty$

5) $\lim_{x \to -\infty} f(x) = \lim_{x \to -\infty} \frac{x^2}{x-1} = -\infty$

Nov 15-10:40 AM

$f(x) = \frac{x^2}{x-1}$ $\quad f'(x) = \frac{x(x-2)}{(x-1)^2}$ $\quad f''(x) = \frac{2}{(x-1)^3}$

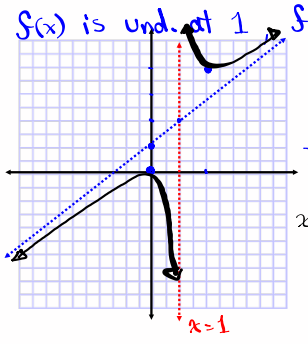
$f'(x) = 0 \rightarrow x = 0, x = 2$ $\quad f''(x) \neq 0$

$f'(x)$ und. $\rightarrow x = 1$ $\quad f''(x)$ und. $\rightarrow x = 1$

x	$-\infty$		0		1		2		∞
$f'(x)$		+	0	−		−	0	+	
$f''(x)$		−		−		+	0	+	
$f(x)$		↗		↘		↘		↗	

$f(0) = 0$, $f(x)$ is und. at 1 $\quad f(2) = \frac{2^2}{2-1} = 4$

$y = x + 1$ slant Asym.



$\frac{x^2}{x-1} = (x+1) + \frac{1}{x-1}$

$x-1 \overline{)\, x^2+0x+0}$

$x+1$

$-(x^2 - x)$

$x + 0$

$-(x - 1)$

1

Nov 15-10:45 AM

$f(x) = \frac{x^2-1}{x^3}$ $\quad$ Domain: $(-\infty, 0) \cup (0, \infty)$

V.A. $x = 0$

H.A. $y = 0$

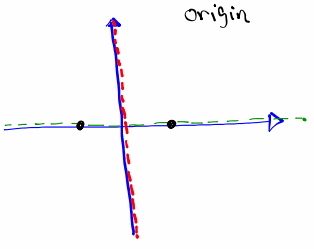
odd function $\rightarrow$ Symmetry w/t origin

No Y-Int, $x \neq 0$

x-Ints $(\pm 1, 0)$

$f'(x) = \frac{3 - x^2}{x^4}$

$f''(x) = \frac{2(x^2 - 6)}{x^5}$



Nov 15-10:56 AM

$f(x) = \frac{x^2-1}{x^3}$

$f'(x) = \frac{3-x^2}{x^4}$

$f''(x) = \frac{2(x^2-6)}{x^5}$

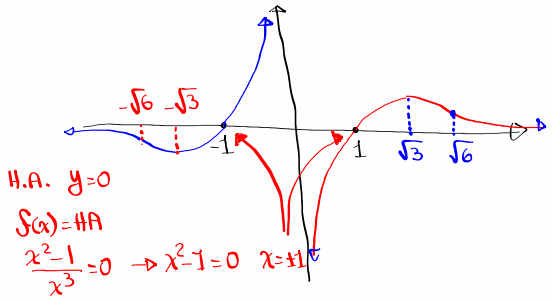
$f'(x) = 0 \rightarrow x = \pm\sqrt{3}$

$f'(x)$ undefined $\rightarrow x = 0$

$f''(x) = 0 \rightarrow x = \pm\sqrt{6}$

$f''(x)$ undefined $\rightarrow x = 0$

x	$-\infty$		$-\sqrt{6}$		$-\sqrt{3}$		0		$\sqrt{3}$		$\sqrt{6}$		∞
$f'(x)$		$-$		$-$	0	$+$	$\not\exists$	$+$	0	$-$		$-$	
$f''(x)$		$-$	0	$+$		$+$	$\not\exists$	$-$		$-$	0	$+$	



H.A. $y = 0$

$f(x) = HA$

$\frac{x^2-1}{x^3} = 0 \rightarrow x^2 - 1 = 0 \quad x = \pm 1$

Nov 15-11:01 AM

$f'(x) = \boxed{5x^4} - \boxed{3x^2} + \boxed{4}$, $f(-1) = 2$

find $f(x)$

$f(x) = x^5 - x^3 + 4x + C$ General Solution

$f(-1) = (-1)^5 - (-1)^3 + 4(-1) + C = 2$

$-1 + 1 - 4 + C = 2$

$\boxed{C = 6}$

$\boxed{f(x) = x^5 - x^3 + 4x + 6}$

Nov 15-11:11 AM

For $-\frac{\pi}{2} < x < \frac{\pi}{2}$, $f'(x) = 2\cos x + \sec^2 x$

$f\left(\frac{\pi}{3}\right) = 4$

find $f(x)$

$f'(x) = 2\cos x + \sec^2 x$

$f(x) = 2\sin x + \tan x + C$

$f\left(\frac{\pi}{3}\right) = 2\sin\frac{\pi}{3} + \tan\frac{\pi}{3} + C = 4$ ($\frac{\pi}{3} = 60°$)

$2 \cdot \frac{\sqrt{3}}{2} + \sqrt{3} + C = 4$

$2\sqrt{3} + C = 4$

$C = 4 - 2\sqrt{3}$

$f(x) = 2\sin x + \tan x + 4 - 2\sqrt{3}$

Nov 15-11:16 AM

$f''(x) = \sin x + \cos x$ $\quad f'(0) = 4$, $f(0) = 3$

find $f(x)$.

$f'(x) = -\cos x + \sin x + C$

$f'(0) = -\cos 0 + \sin 0 + C = 4$ ($\cos 0 = 1$, $\sin 0 = 0$)

$C = 5$

$f'(x) = -\cos x + \sin x + 5$

$f(x) = -\sin x + -\cos x + 5x + C$

$f(x) = -\sin x - \cos x + 5x + C$

$f(0) = -\sin 0 - \cos 0 + 5(0) + C = 3$ ($\sin 0 = 0$, $\cos 0 = 1$)

$C = 4$

$f(x) = -\sin x - \cos x + 5x + 4$

Nov 15-11:22 AM

$f''(x) = 20x^3 + 12x^2 + 4$ $\quad f(0)=8,\ f(1)=5$

$f''(x) = 5 \cdot 4x^3 + 4 \cdot 3x^2 + 4$

$f'(x) = 5x^4 + 4x^3 + 4x + C$

$f(x) = x^5 + x^4 + 2x^2 + Cx + D$

$f(0) = 0^5 + 0^4 + 2(0)^2 + C(0) + D = 8$ $\quad D = 8$

$f(1) = 1^5 + 1^4 + 2(1)^2 + C(1) + 8 = 5$

$4 + C + 8 = 5$

$C = 5 - 12$ $\quad C = -7$

$f(x) = x^5 + x^4 + 2x^2 - 7x + 8$

Review Related Rates & Optimization

Nov 15-11:28 AM