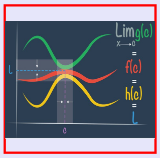
Math 261

Fall 2023

Lecture 43



Feb 19-8:47 AM

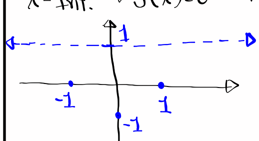
Given $f(x)=\dfrac{x^2-1}{x^2+1}$

$x^2+1\neq 0$, Domain: $(-\infty,\infty)$

$f(-x)=\dfrac{(-x)^2-1}{(-x)^2+1}=\dfrac{x^2-1}{x^2+1}=f(x) \rightarrow$ F(x) even function, Sym. w/t Y-axis

Y-Int $\rightarrow x=0 \rightarrow f(0)=-1 \rightarrow$ Y-Int $(0,-1)$

X-Int. $\rightarrow f(x)=0 \rightarrow x^2-1=0 \rightarrow$ X-Int $(\pm 1,0)$



$\lim\limits_{x\to\infty} f(x)=\lim\limits_{x\to\infty}\dfrac{x^2-1}{x^2+1}=1$

$\lim\limits_{x\to-\infty} f(x)=1$

H.A. $y=1$

Nov 14-10:27 AM

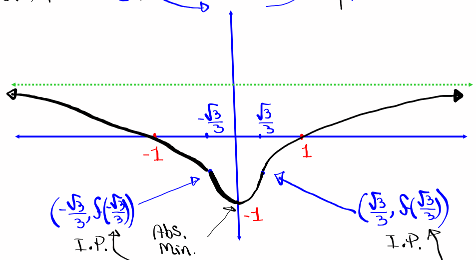
Please verify this later

$f'(x) = \dfrac{4x}{(x^2+1)^2}$, $f''(x) = \dfrac{4(1-3x^2)}{(x^2+1)^3}$

$f'(x)$ is defined $(-\infty,\infty)$ $\quad$ $f''(x)$ is defined $(-\infty,\infty)$

$f'(x)=0 \rightarrow 4x=0 \rightarrow x=0$ $\quad$ $f''(x)=0 \rightarrow 1-3x^2=0 \rightarrow x=\pm\frac{\sqrt{3}}{3}$

x	$-\infty$	$-\frac{\sqrt{3}}{3}$	0	$\frac{\sqrt{3}}{3}$	∞
$f'(x)$	$-$	$-$	$+$	$+$	
$f''(x)$	$-$	$+$	$+$	$-$	
$f(x)$	↘	↘	↗	↗	



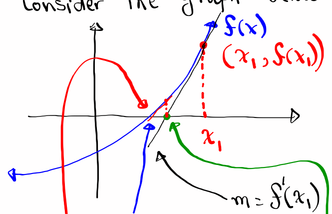
$f\left(\frac{\sqrt{3}}{3}\right) = \dfrac{\left(\frac{\sqrt{3}}{3}\right)^2-1}{\left(\frac{\sqrt{3}}{3}\right)^2+1} = \dfrac{\frac{3}{9}-1}{\frac{3}{9}+1} = \dfrac{\frac{1}{3}-1}{\frac{1}{3}+1} = \dfrac{1-3}{1+3} = \dfrac{-2}{4} = \boxed{-\frac{1}{2}}$

$f(x) = \text{H.A.}$ $\rightarrow x^2-1 = x^2+1$ $\rightarrow$ Graph will not cross H.A.

$\dfrac{x^2-1}{x^2+1} = 1$ $\qquad -1 = 1$ False

Nov 14-10:32 AM

Consider the graph below



$y - f(x_1) = f'(x_1)(x - x_1)$

If we let $y=0$

$-f(x_1) = f'(x_1)(x-x_1)$

Solve for x

$x - x_1 = -\dfrac{f(x_1)}{f'(x_1)}$

$x = x_1 - \dfrac{f(x_1)}{f'(x_1)}$

$x_2 = x_1 - \dfrac{f(x_1)}{f'(x_1)}$

$x_3 = x_2 - \dfrac{f(x_2)}{f'(x_2)}$

$x_4 = x_3 - \dfrac{f(x_3)}{f'(x_3)}$

$\vdots$

$\boxed{x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}}$ Newton's eqn or Method.

Nov 14-10:46 AM

Solve $x^6 = 2$

$x^6 - 2 = 0$

$f(x) = x^6 - 2 \qquad f'(x) = 6x^5$

Newton's equation

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} = x_n - \frac{x_n^6 - 2}{6x_n^5}$$

$$x_{n+1} = \frac{6x_n^6 - x_n^6 + 2}{6x_n^5} \Rightarrow x_{n+1} = \frac{5x_n^6 + 2}{6x_n^5}$$

Suppose our first guess is $1 \rightarrow x_1 = 1$

$$x_2 = \frac{5(1)^6 + 2}{6(1)^5} = \frac{7}{6} = 1.17$$

$$x_3 = \frac{5(1.17)^6 + 2}{6(1.17)^5} = 1.13$$

$$x_4 = \frac{5(1.13)^6 + 2}{6(1.13)^5} = 1.12$$

$$x_5 = \frac{5(1.12)^6 + 2}{6(1.12)^5} = 1.12$$

Soln $\rightarrow 1.12$

check

$1.12^6 = 1.9738\ldots$

≈ 2

Nov 14-10:54 AM

Solve $x^5 - x - 1 = 0$ using Newton's method with $x_1 = 1$.

$f(x) = x^5 - x - 1$

$f'(x) = 5x^4 - 1$

Newton's eqn

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_{n+1} = x_n - \frac{x_n^5 - x_n - 1}{5x_n^4 - 1}$$

$$x_{n+1} = \frac{x_n(5x_n^4 - 1) - (x_n^5 - x_n - 1)}{5x_n^4 - 1}$$

$$x_{n+1} = \frac{4x_n^5 + 1}{5x_n^4 - 1} \quad \text{with } x_1 = 1$$

$$x_2 = \frac{4(1)^5 + 1}{5(1)^4 - 1} = \frac{5}{4} = 1.25$$

$$x_3 = \frac{4(1.25)^5 + 1}{5(1.25)^4 - 1} = 1.18$$

$$x_4 = \frac{4(1.18)^5 + 1}{5(1.18)^4 - 1} = 1.17$$

$$x_5 = \frac{4(1.17)^5 + 1}{5(1.17)^4 - 1} = 1.17$$

$\Rightarrow$ Soln $x = 1.17$

check

$1.17^5 - 1.17 - 1 = .022\ldots$

≈ 0

Nov 14-11:04 AM

Use Newton's method to Solve $x - \cos x = 0$ with $x_1 = 1$

$f(x) = x - \cos x$

$f'(x) = 1 + \sin x$

$\Rightarrow$ Newton's eqn

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_{n+1} = x_n - \frac{x_n - \cos x_n}{1 + \sin x_n}$$

$$= \frac{x_n(1 + \sin x_n) - (x_n - \cos x_n)}{1 + \sin x_n}$$

$$x_{n+1} = \frac{x_n \sin x_n + \cos x_n}{1 + \sin x_n} \quad \text{with } x_1 = 1$$

$$x_2 = \frac{1 \cdot \sin 1 + \cos 1}{1 + \sin 1} = .75$$

$$x_3 = \frac{.75 \sin .75 + \cos .75}{1 + \sin .75} = .74$$

$\Rightarrow$ Soln $x = .74$

$$x_4 = \frac{.74 \sin .74 + \cos .74}{1 + \sin .74} = .74$$

check

$.74 - \cos .74 \approx .002$

≈ 0

Nov 14-11:14 AM