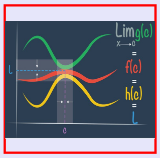
Math 261

Fall 2023

Lecture 42



Feb 19-8:47 AM

Given $f(x)=\cos 2x$ and $\left[\frac{\pi}{8},\frac{7\pi}{8}\right]$

$f(x)$ is cont. & diff every where

$f\left(\frac{\pi}{8}\right)=\cos\frac{2\pi}{8}=\cos\frac{\pi}{4}=\frac{\sqrt{2}}{2}$

$f\left(\frac{7\pi}{8}\right)=\cos 2\left(\frac{7\pi}{8}\right)=\cos\frac{7\pi}{4}=\cos\frac{\pi}{4}=\frac{\sqrt{2}}{2}$

All three conditions of Rolle's thrm are met,

So there is a number c in $\left(\frac{\pi}{8},\frac{7\pi}{8}\right)$ where

$f'(c)=0$

$f(x)=\cos 2x$

$f'(x)=-\sin 2x\cdot 2$

$f'(x)=-2\sin 2x$

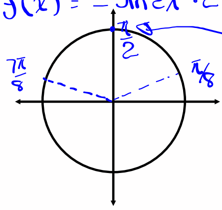
$f'(c)=0$

$-2\sin 2c=0$

$\sin 2c=0$

where $2c=\pi$

$\boxed{c=\frac{\pi}{2}}$



Nov 13-10:26 AM

$f(x) = x^3 - 2x \quad , \quad [-2, 2]$

$f(2) = 2^3 - 2(2) = 4$

$f(-2) = (-2)^3 - 2(-2) = -4$

$f'(x) = 3x^2 - 2$

$f(x)$ is a polynomial function.

It is cont. & diff. everywhere

By MVT, there is a number c in $(-2,2)$

where $f'(c) = \dfrac{f(2) - f(-2)}{2-(-2)}$

$3c^2 - 2 = \dfrac{4-(-4)}{2-(-2)}$

$3c^2 - 2 = \dfrac{8}{4}$

$3c^2 = 4$

$(-2, 2) \leftarrow \boxed{c = \pm \dfrac{2}{\sqrt{3}}}$

Nov 13-10:33 AM

Use MVT to Prove $|\sin b - \sin a| \leq |b-a|$

for all a and b.

Consider $f(x) = \sin x$ on $[a,b]$ $\rightarrow f'(x) = \cos x$

$f(x)$ is diff. & cont. everywhere

by MVT, there is a number c in (a,b)

Such that $f'(c) = \dfrac{f(b) - f(a)}{b-a}$

$\cos c = \dfrac{\sin b - \sin a}{b-a}$

$|\cos c| = \dfrac{|\sin b - \sin a|}{|b-a|}$

$-1 \leq \cos c \leq 1 \rightarrow |\cos c| \leq 1$

$\dfrac{|\sin b - \sin a|}{|b-a|} \leq 1$

$\boxed{|\sin b - \sin a| \leq |b-a|}$

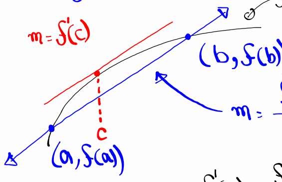
for all a & b.

Nov 13-10:38 AM

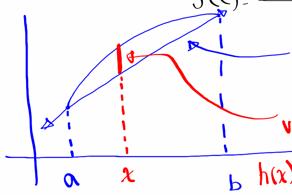
Let's go over the proof of MVT.

$f(x)$ cont on $[a,b]$

$f(x)$ diff on (a,b).



$$f'(c) = \frac{f(b) - f(a)}{b-a}$$



$h(x) = y_{curve} - y_{line}$

$$h(x) = f(x) - \left[\frac{f(b)-f(a)}{b-a}(x-a) + f(a)\right]$$

$h(x)$ is cont. on $[a,b]$

$h(x)$ is diff. on (a,b)

$$h(a) = f(a) - \left[\frac{f(b)-f(a)}{b-a}\cancel{(a-a)} + f(a)\right] = 0$$

$$h(b) = f(b) - \left[\frac{f(b)-\cancel{f(a)}}{b-\cancel{a}}\cancel{(b-a)} + \cancel{f(a)}\right] = 0$$

So we can apply the Rolle's thrm

Nov 13-10:44 AM

$$h(x) = f(x) - \left[\frac{f(b)-f(a)}{b-a}(x-a) + f(a)\right]$$

$$h'(x) = f'(x) - \frac{f(b)-f(a)}{b-a}$$

by Conclusion of Rolle's Thrm

$$h'(c) = 0$$

$$f'(c) - \frac{f(b)-f(a)}{b-a} = 0$$

$$\boxed{f'(c) = \frac{f(b)-f(a)}{b-a}}$$ Conclusion of MVT.

Nov 13-10:54 AM

$f(x) = \sqrt{x} - \sqrt[4]{x}$

Domain $[0, \infty)$

$f(0) = 0 \rightarrow$ Y-Int $(0,0)$

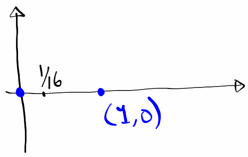
$f(x) = 0 \rightarrow$ x-Int. $\sqrt{x} - \sqrt[4]{x} = 0$

$(0,0), (1,0)$ $\sqrt{x} = \sqrt[4]{x}$ Raise to 4th Power

$(\sqrt{x})^4 = (\sqrt[4]{x})^4$

$x^2 = x$

$x^2 - x = 0$ $\rightarrow x = 0$, $x = 1$



Nov 13-10:56 AM

$\frac{-3}{4} + \boxed{?} = \frac{-1}{2}$

$f(x) = x^{1/2} - x^{1/4}$

$f'(x) = \frac{1}{2} x^{-1/2} - \frac{1}{4} x^{-3/4} = \frac{1}{4} x^{-3/4} \left[2x^{\frac{1}{4}} - 1 \right]$

$= \dfrac{2\sqrt[4]{x} - 1}{4\sqrt[4]{x^3}}$

$f'(x)$ undefined at $x = 0$

$f'(x) = 0 \rightarrow 2\sqrt[4]{x} - 1 = 0 \rightarrow x = \frac{1}{16}$

$\frac{-7}{4} + \boxed{?} = \frac{-5}{4}$

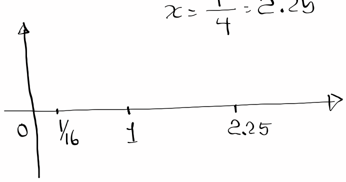
$f'(x) = \frac{1}{2} x^{-1/4} - \frac{1}{4} x^{-3/4}$

$f''(x) = \frac{-1}{8} x^{-5/4} + \frac{3}{16} x^{-7/4} = \frac{-1}{16} x^{-7/4} \left[2x^{\frac{1}{2}} - 3 \right]$

$f''(x)$ undefined at 0. $= \dfrac{-(2\sqrt{x} - 3)}{16 \sqrt[4]{x^7}}$

$f''(x) = 0 \rightarrow 2\sqrt{x} - 3 = 0$

$x = \frac{9}{4} = 2.25$



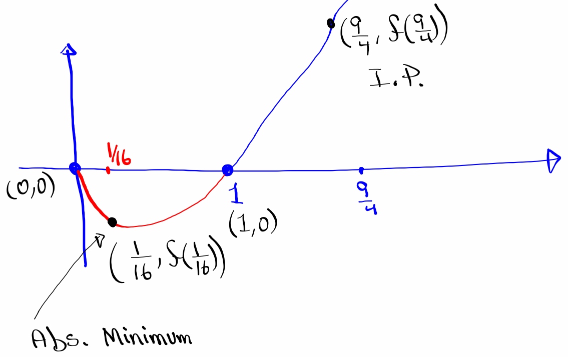
Nov 13-11:00 AM

x	0		$\frac{1}{16}$		1		$\frac{9}{4}$		∞
$f'(x)$		$-$	•		$+$			$+$	
$f''(x)$		$+$			$+$		•	$-$	
$f(x)$									

$f'(x) = \dfrac{2\sqrt[4]{x} - 1}{4\sqrt[4]{x^3}}$

$f''(x) = \dfrac{-(2\sqrt{x} - 3)}{16\sqrt[4]{x^7}}$

I.P. at $x = \frac{9}{4}$



Nov 13-11:10 AM

Find two positive numbers with sum of 16 and smallest value of sum of their squares.

$1, 15 \longrightarrow 1^2 + 15^2 = 1 + 225 = 226$

$2, 14 \longrightarrow 2^2 + 14^2 = \quad = 200$

$\vdots$

$6, 10 \longrightarrow 6^2 + 10^2 = \cdots = 136$

$8, 8 \longrightarrow 8^2 + 8^2 = \cdots = 128$

$x + y = 16$

$x^2 + y^2$ must be minimum

$x^2 + (16 - x)^2$ " " "

$f(x) = x^2 + (16 - x)^2$

$\longrightarrow f'(x) = 2x + 2(16 - x) \cdot -1$

$f'(x) = 2x + 2x - 32$

$f'(x) = 4x - 32$

$f''(x) = 4 > 0$

C.U.

$f'(x) = 0$

$4x - 32 = 0$

$\boxed{x = 8}$

Nov 13-11:19 AM

$f''(x) = \boxed{-2} + \boxed{12x} - \boxed{12x^2}$

$f(0) = 4$

$f'(0) = 12$

Find $f(x)$.

$f'(x) = -2x + 6x^2 - 4x^3 + C$

$f'(0) = \cancel{-2(0)}^{0} + \cancel{6(0)^2}^{0} - \cancel{4(0)^3}^{0} + C = 12$

$C = 12$

$f'(x) = -\boxed{2x} + \boxed{6x^2} - \boxed{4x^3} + \boxed{12}$

$f(x) = -x^2 + 2x^3 - x^4 + 12x + C$

$f(0) = 0 + 0 - 0 + 0 + C = 4$

$\boxed{C=4}$

$\boxed{f(x) = -x^2 + 2x^3 - x^4 + 12x + 4}$

Nov 13-11:26 AM