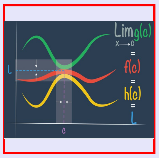
Math 261

Fall 2023

Lecture 41



Feb 19-8:47 AM

Mean Value Theorem

Let $f(x)$ be a function that satisfies the following

1) $f(x)$ is cont. on $[a,b]$

2) $f(x)$ is diff. on (a,b),

then there is a number c in (a,b) such that $f'(c) = \dfrac{f(b) - f(a)}{b-a}$

Nov 9-10:26 AM

Given $f(x)=x^3-3x+2$ over $[-2,2]$

$f(x)$ is a polynomial function, therefore it is cont. & diff. everywhere.

Conditions of MVT ✓

$f(2)=2^3-3(2)+2=4$

$f(-2)=(-2)^3-3(-2)+2=0$

$$f'(c)=\frac{f(b)-f(a)}{b-a}$$

Conclusion of MVT

$f'(x)=3x^2-3$

$$3c^2-3=\frac{4-0}{2-(-2)}$$

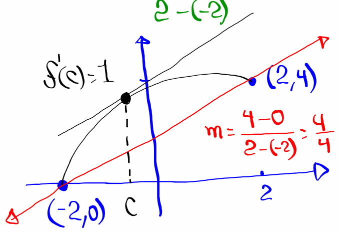
$3c^2-3=1$

$3c^2=4$

$c=\pm\frac{2}{\sqrt{3}}$

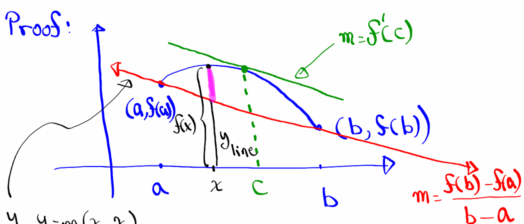
$c\approx\pm 1.2$

$(-2,2)$



Nov 9-10:29 AM

Proof:



$y-y_1=m(x-x_1)$

$$y-f(a)=\frac{f(b)-f(a)}{b-a}(x-a) \rightarrow \boxed{y=f(a)+\frac{f(b)-f(a)}{b-a}(x-a)}$$

Vertical distance between the curve & Secant line

Curve − Line

$$\boxed{f(x)-\left[f(a)+\frac{f(b)-f(a)}{b-a}(x-a)\right]}$$

$f(x)$ is cont. & diff.

Line is cont. & diff.

$$h(x)=f(x)-\left[f(a)+\frac{f(b)-f(a)}{b-a}(x-a)\right]$$

is also cont. & diff.

$$h(a)=f(a)-\left[f(a)+\frac{f(b)-f(a)}{b-a}\cancel{(a-a)}\right]=f(a)-f(a)=0$$

$$h(b)=f(b)-\left[\cancel{f(a)}+\frac{f(b)-\cancel{f(a)}}{\cancel{b-a}}\cancel{(b-a)}\right]=f(b)-f(b)=0$$

Nov 9-10:37 AM

$$h(x) = f(x) - \left[f(a) + \frac{f(b)-f(a)}{b-a}(x-a)\right]$$

$h(x)$ is cont. on $[a,b]$
$h(x)$ is diff. on (a,b)
$h(a) = h(b)$

$h(x)$ satisfies conditions of Rolle's Theorem then
$h'(c) = 0$

$$h(x) = f(x) - \left[f(a) + \frac{f(b)-f(a)}{b-a}(x-a)\right]$$

$$h'(x) = f'(x) - \left[0 + \frac{f(b)-f(a)}{b-a}\cdot 1\right]$$

By conclusion of Rolle's Thrm $h'(c) = 0$

$$f'(c) - \frac{f(b)-f(a)}{b-a} = 0 \Rightarrow \boxed{f'(c) = \frac{f(b)-f(a)}{b-a}}$$

Conclusion of MVT

Make sure to review this proof.

Nov 9-10:50 AM

Given $f(x)$ is cont. on $[0,4]$, $\boxed{f(0) = 1}$, and $2 \le f'(x) \le 5$ for all x in $(0,4)$

Find **range of values for $f(4)$**

Since $2 \le f'(x) \le 5$ by M.V.T.

$$2 \le \frac{f(b)-f(a)}{b-a} \le 5 \quad \text{over } (0,4)$$

$$2 \le \frac{f(4)-f(0)}{4-0} \le 5$$

$$2 \le \frac{f(4)-1}{4} \le 5$$

$$8 \le f(4) - 1 \le 20$$

$$\boxed{9 \le f(4) \le 21}$$

Nov 9-10:57 AM

$f(x) = x^4 - 3x^3 + 3x^2 - x$

Polynomial Func. $\to$ Cont. & diff. $(-\infty, \infty)$

y-Int $(0,0)$

Is $(1,0)$ an x-Int? $f(1) = 1 - 3 + 3 - 1 = 0$

Yes

$$\begin{array}{r|rrrrr} 1 & 1 & -3 & 3 & -1 & 0 \\ & & 1 & -2 & 1 & 0 \\ \hline & 1 & -2 & 1 & 0 & 0 \end{array}$$

$f(x) = (x-1)(x^3 - 2x^2 + x + 0)$

$= x(x-1)(x^2 - 2x + 1)$

$f(x) = x(x-1)(x-1)(x-1) \Rightarrow f(x) = x(x-1)^3$

x-Int $(0,0), (1,0)$

Nov 9-11:03 AM

$f(x) = x^4 - 3x^3 + 3x^2 - x$

$f'(x) = 4x^3 - 9x^2 + 6x - 1 = (x-1)(4x^2 - 5x + 1)$

$f'(1) = 0$

$= (x-1)(x-1)(4x-1)$

$$\begin{array}{r|rrrr} 1 & 4 & -9 & 6 & -1 \\ & & 4 & -5 & 1 \\ \hline & 4 & -5 & 1 & 0 \end{array}$$

$= (x-1)^2(4x-1)$

$\boxed{\text{C.P. at } 1, \frac{1}{4}}$

$f''(x) = 12x^2 - 18x + 6 = 6(2x^2 - 3x + 1)$

$= 6(x-1)(2x-1)$

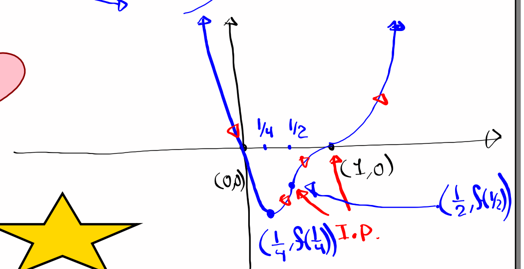
P.I.P. $x = 1$, $x = \frac{1}{2}$

Nov 9-11:09 AM

$f'(x) = (x-1)^2(4x-1)$ $\qquad$ $f''(x) = 6(x-1)(2x-1)$

C.P. $1, \frac{1}{4}$ $\qquad$ P.I.P. $1, \frac{1}{2}$

x	$-\infty$		$\frac{1}{4}$		$\frac{1}{2}$		1		∞
$f'(x)$		$-$	•	$+$		$+$	•	$+$	
$f''(x)$		$+$		$+$	•	$-$	•	$+$	
$f(x)$									



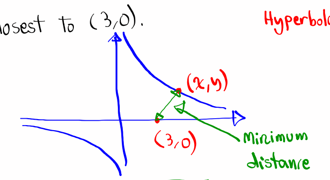
Nov 9-11:14 AM

find the point on the graph of $\underline{xy=8}$ that is closest to $(3,0)$. Hyperbola

$xy = 8$

$y = \frac{8}{x}$

Recall $y = \frac{1}{x}$



$d = \sqrt{(x-3)^2 + (y-0)^2} = \underbrace{\sqrt{(x-3)^2 + y^2}}_{\text{need to minimize this}}$

$f(x) = (x-3)^2 + \left(\frac{8}{x}\right)^2$ $\quad 64x^{-2}$

need to minimize this

$f'(x) = 2(x-3)^1 - 128x^{-3}$

$f''(x) = 2 + 128(3)x^{-4}$

$= 2 + \frac{128(3)}{x^4} > 0$

C.U.

min.

$f'(x) = 0$

$2(x-3) - \frac{128}{x^3} = 0$

$2x^3(x-3) - 128 = 0$

$x^3(x-3) - 64 = 0$

$x^4 - 3x^3 - 64 = 0$

1 |	1	-3	0	0	-64
		1	-2	-2	-2
	1	-2	-2	-2	-66

NO

2 |	1	-3	0	0	-64
		2	-2	-4	-8
	1	-1	-2	-4	-72

NO

4 |	1	-3	0	0	-64
		4	4	16	64
	1	1	4	16	0

4 is a Solution

$y = \frac{8}{x}$ $\quad y = \frac{8}{4} = 2$

$\boxed{\text{Point } (4,2)}$

Nov 9-11:25 AM