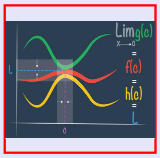
Math 261

Fall 2023

Lecture 40



Feb 19-8:47 AM

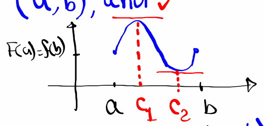
Rolle's Theorem:

If $f(x)$ satisfies the following

1) It is Cont. on $[a,b]$,

2) It is diff. on (a,b), and

3) $f(a) = f(b)$



then there is a number c in (a,b) such that $f'(c)=0$

Nov 8-10:27 AM

$f(x) = 3x^2 - 12x + 5$ on $[1,3]$

$f(x)$ is a polynomial function

1) It is cont. on $[1,3]$ ✓

2) It is diff. on $(1,3)$ ✓

3) $f(1) = f(3)$ ✓

$f(1) = 3(1)^2 - 12(1) + 5$
$= 3 - 12 + 5$
$= \boxed{-4}$

$f(3) = 3(3)^2 - 12(3) + 5$
$= 27 - 36 + 5$
$= \boxed{-4}$

by Rolle's Thrm, there is a C in $(1,3)$

Such that $f'(c) = 0$

$f'(x) = 6x - 12$

$6C - 12 = 0$

$\boxed{C = 2}$

Nov 8-10:30 AM

$f(x) = \underbrace{\sqrt{x} - \frac{x}{3}}_{\text{Domain } [0,\infty)}$, $[0,9]$

1) $f(x)$ is cont. on $[0,9]$

$f'(x) = \frac{1}{2}x^{-1/2} - \frac{1}{3}$ $\quad f'(x) = \frac{1}{2\sqrt{x}} - \frac{1}{3}$

2) $f(x)$ is diff. on $(0,9)$

$f(0) = \sqrt{0} - \frac{0}{3}$
$= \boxed{0}$

$f(9) = \sqrt{9} - \frac{9}{3}$
$= 3 - 3$
$= \boxed{0}$

3) $f(0) = f(9)$

By Rolle's Thrm, there is a number C in $(0,9)$

Such that $f'(c) = 0$

$f'(x) = \frac{1}{2\sqrt{x}} - \frac{1}{3}$

$\frac{1}{2\sqrt{c}} - \frac{1}{3} = 0$

$\frac{1}{2\sqrt{c}} = \frac{1}{3}$

$2\sqrt{c} = 3$

$(2\sqrt{c})^2 = (3)^2$

$4C = 9$

$C = \frac{9}{4}$

$\boxed{C = 2.25}$

Nov 8-10:34 AM

Mean - Value Theorem

If $f(x)$ satisfies the following conditions

1) It is cont. on $[a,b]$, and
2) It is diff. on (a,b)

then, there is a number c in (a,b) such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

Nov 8-10:41 AM

$f(x) = 2x^2 - 3x + 1$, $[0,2]$

$f(0) = 1$

$f(2) = 3$

$f'(x) = 4x - 3$

$f(x)$ is a polynomial function

1) It is cont. on $[0,2]$
2) It is diff. on $(0,2)$

By MVT, there is a number c in $(0,2)$ such that

$$f'(c) = \frac{f(2) - f(0)}{2 - 0}$$

$$4c - 3 = \frac{3 - 1}{2 - 0}$$

$$4c - 3 = \frac{2}{2} \qquad 4c - 3 = 1$$

$$4c = 4$$

$$(0,2) \leftarrow \boxed{c = 1}$$

Nov 8-10:45 AM

$f(x) = \frac{1}{x}$, $[1,3]$

$f(3) = \frac{1}{3}$

$f(1) = 1$

$f(x)$ is Cont. everywhere except at 0.

So $f(x)$ is cont. on $[1,3]$

$f'(x) = \frac{-1}{x^2}$ Cont. everywhere except at $x=0$ $\Rightarrow$ $f(x)$ is diff. on $(1,3)$

By MVT, there is a number c in $(1,3)$ Such that $f'(c) = \frac{f(3) - f(1)}{3-1}$

$\frac{-1}{c^2} = \frac{\frac{1}{3} - 1}{2}$

$\frac{-1}{c^2} = \frac{\frac{-2}{3}}{2}$

$\frac{2}{3}c^2 = 2$

$\Rightarrow 2c^2 = 6$

$c^2 = 3$

$c = \pm\sqrt{3}$

$\boxed{c = \sqrt{3}}$

Nov 8-10:50 AM

$f(x) = \sqrt[3]{x}$, $[0,1]$

$f(0) = \sqrt[3]{0} = \boxed{0}$

$f(1) = \sqrt[3]{1} = \boxed{1}$

1) Is $f(x)$ cont. on $[0,1]$? ✓Yes

2) Is $f(x)$ diff. on $(0,1)$? ✓ Yes

$f'(x) = \frac{1}{3\sqrt[3]{x^2}}$

By MVT

There is a number c in $(0,1)$ Such that $f'(c) = \frac{f(1) - f(0)}{1-0}$

$\frac{1}{3\sqrt[3]{c^2}} = \frac{1-0}{1-0}$

$\frac{1}{3\sqrt[3]{c^2}} = 1$

$\Rightarrow (3\sqrt[3]{c^2})^3 = (1)^3$

$27c^2 = 1$

$c^2 = \frac{1}{27} \rightarrow \boxed{c = \frac{1}{3\sqrt{3}}}$

Nov 8-10:56 AM

$f(x) = x - 3\sqrt[3]{x}$

Domain: $(-\infty, \infty)$

y-Int $(0,0)$

x-Int $\rightarrow y = 0$

$(0,0), (3\sqrt{3}, 0), (-3\sqrt{3}, 0)$

$x - 3\sqrt[3]{x} = 0$

$x = 3\sqrt[3]{x}$

$x^3 = 27x$

$x^3 - 27x = 0$

$x(x^2 - 27) = 0$

$x = 0$ $\quad$ $x = \pm 3\sqrt{3}$

$f(-x) = -x - 3\sqrt[3]{-x}$

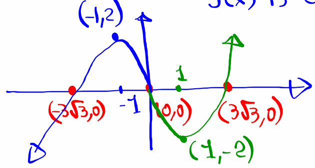
$= -x - 3 \cdot -\sqrt[3]{x}$

$= -x + 3\sqrt[3]{x}$

$= -(x - 3\sqrt[3]{x})$

$= -f(x)$ Symmetric to origin

$f(x)$ is odd



Nov 8-11:02 AM

$f(x) = x - 3x^{1/3}$

$f'(x) = 1 - 3 \cdot \frac{1}{3} x^{-2/3}$

$f'(x) = 1 - \frac{1}{\sqrt[3]{x^2}}$

$f'(x) = 1 - x^{-2/3}$

$f''(x) = - \cdot -\frac{2}{3} x^{-5/3}$

$f''(x) = \frac{2}{3\sqrt[3]{x^5}} = \frac{2}{3x\sqrt[3]{x^2}}$

$f'(x) = 0$

$1 - \frac{1}{\sqrt[3]{x^2}} = 0$

$\sqrt[3]{x^2} = 1$

$\boxed{x = \pm 1}$

$f'(x)$ is undef. $\boxed{x = 0}$

$f''(x) \neq 0$

$f''(x)$ is undef. at $\boxed{x = 0}$

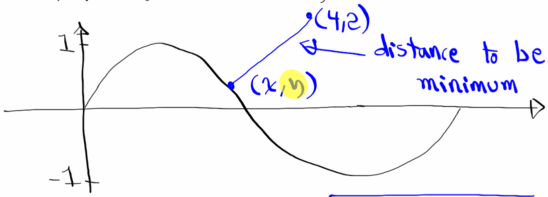
I.P. $(0,0)$

x	$-\infty$	-1		0		1		∞
$f'(x)$		•		0	$-$	•	$+$	
$f''(x)$	$-$		$-$	0	$+$		$+$	
$f(x)$								

$f(1) = -2$

Nov 8-11:08 AM

Find the point on the graph of $y = \sin x$ that is closest to $(4,2)$.



$$d = \sqrt{(x_1-x_2)^2+(y_1-y_2)^2} = \underbrace{\sqrt{(x-4)^2+(y-2)^2}}_{\text{we need to minimize this}}$$

$$f(x) = (x-4)^2 + (\sin x - 2)^2$$

$$f'(x) = 2(x-4)^1 \cdot 1 + 2(\sin x - 2)^1 \cdot \cos x$$

$$f'(x) = 2x - 8 + 2\sin x \cos x - 4\cos x$$

Nov 8-11:21 AM

$f'(x) = \cos x + 2x$, $f(0) = 4$

Find $f(x)$

$$f(x) = \sin x + x^2 + C$$

$$f(0) = \sin 0 + 0^2 + C = 4$$

$$\boxed{C=4}$$

$$\boxed{f(x) = \sin x + x^2 + 4}$$

Nov 8-11:29 AM

$f'(x) = 3x^2 - 2x + 1 \qquad f(2) = 5$

$f(x) = x^3 - x^2 + x + C$

$f(2) = 2^3 - 2^2 + 2 + C = 5$

$8 - 4 + 2 + C = 5$

$6 + C = 5 \qquad \boxed{C = -1}$

$\boxed{f(x) = x^3 - x^2 + x - 1}$

Nov 8-11:32 AM