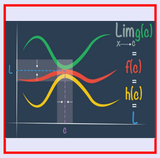
Math 261

Fall 2023

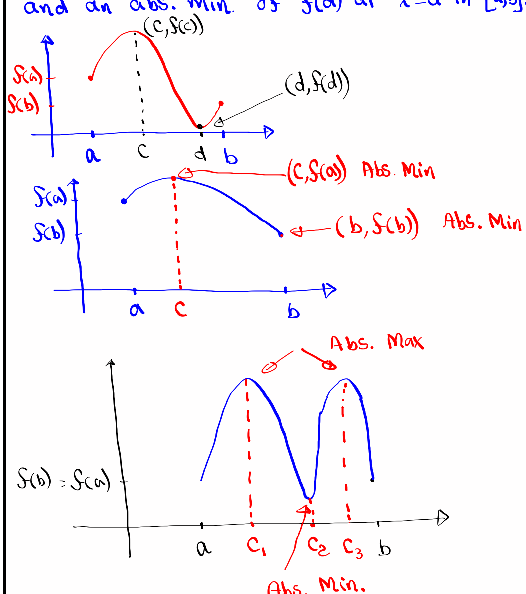
Lecture 39



Feb 19-8:47 AM

The Extreme Value Theorem:

If $f(x)$ is continuous on $[a,b]$, then $f(x)$ attains an abs. max of $f(c)$ at $x=c$ and an abs. min. of $f(d)$ at $x=d$ in $[a,b]$.

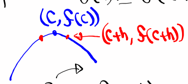


Nov 7-10:28 AM

Fermat's Theorem:

If $f(x)$ has a local Max or Min at c, and if $f'(c)$ exists, then $f'(c)=0$

Suppose $f(c) \geq f(c+h)$ for very small h.



$f(c) \geq f(c+h)$ $\qquad$ $0 \geq f(c+h) - f(c)$

$f(c+h) - f(c) \leq 0$

for $h>0$ $\quad \dfrac{f(c+h) - f(c)}{h} \leq \dfrac{0}{h}$

$$\frac{f(c+h) - f(c)}{h} \leq 0$$

$$\lim_{h \to 0} \frac{f(c+h) - f(c)}{h} \leq \lim_{h \to 0} 0 = 0$$

$h \to 0^+$, $h \to 0^-$ $\qquad \displaystyle\lim_{h \to 0^+} \frac{f(c+h) - f(c)}{h} = 0 = \lim_{h \to 0^-} \frac{f(c+h) - f(c)}{h}$

$$\lim_{h \to 0} \frac{f(c+h) - f(c)}{h} = 0$$

$$\Downarrow$$

$$f'(c) = 0$$

Max or Min happens where $f'(c)=0$.

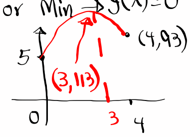
Nov 7-10:36 AM

Given $f(x) = 5 + 54x - 2x^3$ on $[0,4]$

$f(x)$ is a polynomial function $\to$ Cont. everywhere

By Extreme Value Theorem, $f(x)$ attains abs. Max and abs. Min. on $[0,4]$

By Fermat's theorem, Max or Min $\to f'(x)=0$



$f(0) = 5 + 54(0) - 2(0)^3 = 5$

$f(4) = 5 + 54(4) - 2(4)^3 = 93$

$f'(x) = 54 - 6x^2 \qquad f'(x) = 0 \qquad 54 - 6x^2 = 0$

$9 - x^2 = 0$

$x = 3$, $\cancel{x = -3}$

$3 \in [0,4] \qquad [0,4]$

$f(3) = 5 + 54(3) - 2(3)^3 = 113$

Min at $x=0 \to 5$

Max at $x=3 \to 113$

Nov 7-10:46 AM

$f(x) = x + \frac{1}{x}$, $[\frac{1}{5}, 4]$

Not cont. at $x=0$, It is cont. for $x \neq 0$.

$f(x)$ is cont. on $[\frac{1}{5}, 4]$

By E.V.T., $f(x)$ has abs. Max and abs. Min on $[\frac{1}{5}, 4]$.

Fermat's Thrm → Max or Min → $f'(x) = 0$

$f(x) = x + \frac{1}{x}$, $[\frac{1}{5}, 4]$

$f(\frac{1}{5}) = \frac{1}{5} + \frac{1}{\frac{1}{5}} = .2 + 5 = \boxed{5.2}$

$f(4) = 4 + \frac{1}{4} = 4 + .25 = \boxed{4.25}$

$f(x) = x + x^{-1}$

$f'(x) = 1 - x^{-2}$

$= 1 - \frac{1}{x^2}$

$f'(x) = 0$

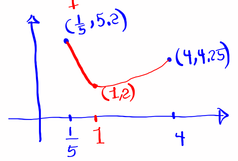
$1 - \frac{1}{x^2} = 0$

$x^2 = 1$

$x = 1$, $\cancel{x = -1}$

Not $[\frac{1}{5}, 4]$

$f(1) = 1 + \frac{1}{1} = 1 + 1 = 2$



Max at $x = \frac{1}{5}$

Min at $x = 1$

Explore Mean-Value Theorem and Rolle's Theorem

Nov 7-10:55 AM

$f(x) = 1 + \frac{1}{x} + \frac{1}{x^2}$ Domain $(-\infty, 0) \cup (0, \infty)$

$\lim_{x \to \infty} f(x) = \lim_{x \to \infty} \left(1 + \cancelto{0}{\frac{1}{x}} + \cancelto{0}{\frac{1}{x^2}}\right) = 1$

V.A. at $x = 0$

H.A. at $y = 1$

$f(x) = 1 + x^{-1} + x^{-2}$

$f'(x) = -x^{-2} - 2x^{-3}$

$= -x^{-3}(x + 2)$

$f'(x) = \frac{-(x+2)}{x^3}$

$f'(x) = 0 \rightarrow x = -2$

$f'(x)$ undefined $\rightarrow x = 0$

$f''(x) = 2x^{-3} + 6x^{-4}$

$= 2x^{-4}(x + 3)$

$f''(x) = \frac{2(x+3)}{x^4}$

$f''(x) = 0 \rightarrow x = -3$

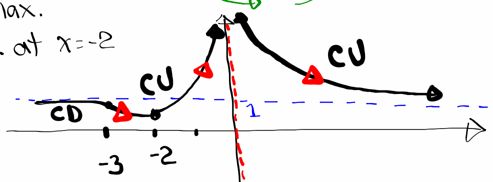
$f''(x)$ undefined $\rightarrow x = 0$

Nov 7-11:05 AM

$f'(x) = \frac{-(x+2)}{x^3}$ $\qquad$ $f''(x) = \frac{2(x+3)}{x^4}$

x	$-\infty$	-3		-2		0	∞
$f'(x)$	$-$		$-$	$\bullet$	$+$	$\circ$	$-$
$f''(x)$	$-$	$\bullet$	$+$		$+$	$\circ$	$+$
$f(x)$							

No max.
Min. at $x=-2$



No y-Int $\rightarrow x \neq 0$

x-Int $\rightarrow y=0$

$1 + \frac{1}{x} + \frac{1}{x^2} = 0$

$x^2 + x + 1 = 0 \rightarrow x = \frac{-1 \pm \sqrt{1^2 - 4(1)(1)}}{2(1)}$ No real Soln.

Use Q-formula

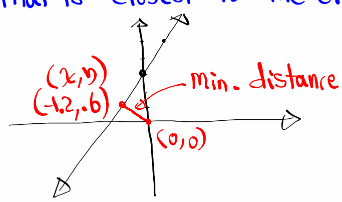
$f(-2) = 1 + \frac{1}{-2} + \frac{1}{(-2)^2} = \frac{3}{4}$
$= .75$

$f(-3) = 1 + \frac{1}{-3} + \frac{1}{(-3)^2} \approx .777$

Nov 7-11:12 AM

Find a point on the line $y = 2x+3$ that is closest to the origin.

$y = 2(-1.2)+3$
$= \boxed{.6}$



$d = \sqrt{(x_1-x_2)^2 + (y_1-y_2)^2}$

$= \sqrt{(x-0)^2 + (y-0)^2}$

$= \sqrt{x^2 + y^2}$

We need to minimize $x^2 + y^2$

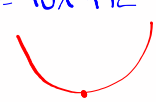
$f(x) = x^2 + (2x+3)^2$

$f'(x) = 2x + 2(2x+3)^1 \cdot 2$

$= 2x + 4(2x+3) = 2x + 8x + 12$

$= 10x + 12$

$f''(x) = 10 > 0$ C.U.



$f'(x) = 0$ $\qquad$ $10x + 12 = 0$ $\qquad$ $\boxed{x = -1.2}$ $\qquad$ Min. Value

Nov 7-11:26 AM