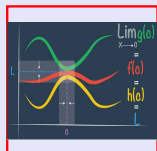


Math 261
Fall 2023
Lecture 38



Feb 19-8:47 AM

$$f(x) = \sqrt[3]{x}(x+4)$$

Defined everywhere

$f(x)$ has a domain
 $(-\infty, \infty)$



x -Int. $\rightarrow y=0 \rightarrow f(x)=0 \rightarrow \sqrt[3]{x}(x+4)=0$
 $(0,0), (-4,0)$ $x=0, x=-4$

y -Int $\rightarrow x=0 \rightarrow f(0)=\sqrt[3]{0}(0+4)=0$
 $(0,0)$

$f(x) = x^{1/3}(x+4)$  $f(x) = x^{4/3} + 4x^{1/3}$

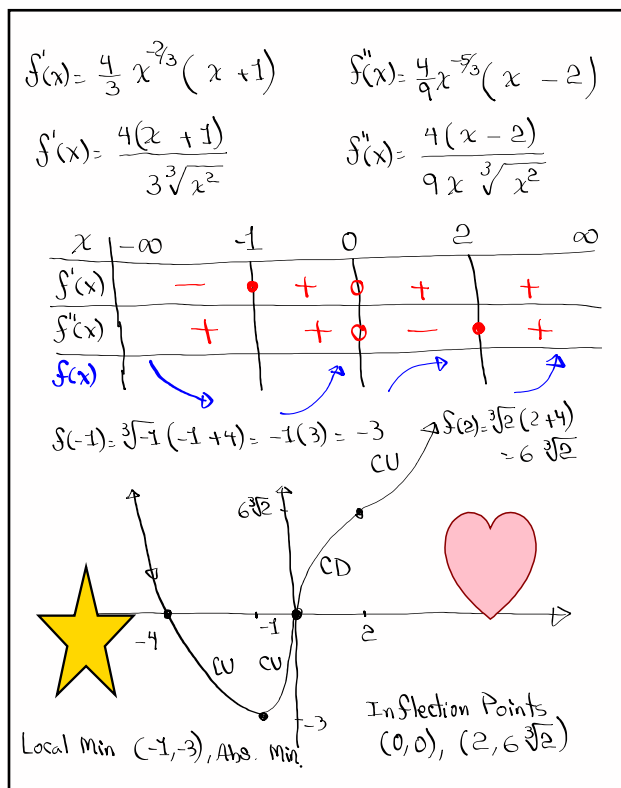
$f'(x) = \frac{4}{3}x^{1/3} + \frac{4}{3}x^{-2/3}$ $f''(x) = \frac{4}{9}x^{-2/3} - \frac{8}{9}x^{-5/3}$

$f'(x) = \frac{4}{3}x^{-2/3}(x+1)$ $f''(x) = \frac{4}{9}x^{-5/3}(x-2)$

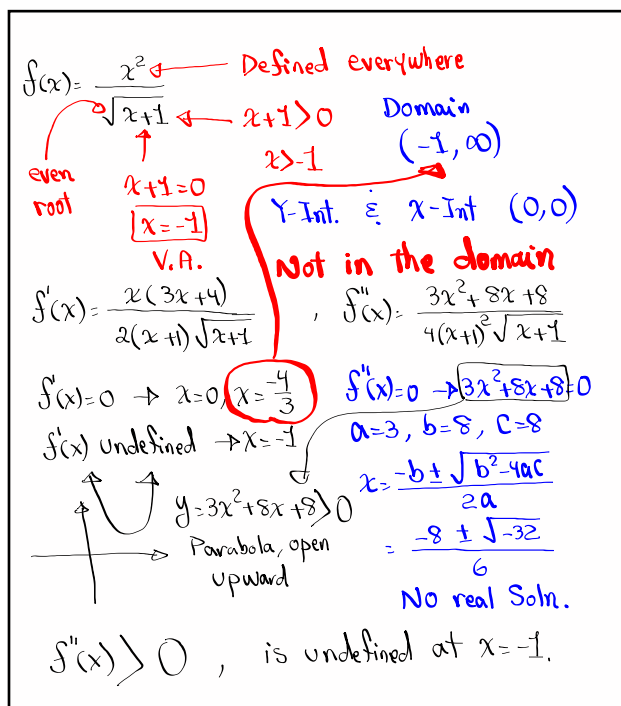
$f'(x)=0 \rightarrow x=-1$ $f''(x)=0 \rightarrow x=2$

$f'(x)$ undefined $\rightarrow x=0$ $f''(x)$ undefined $\rightarrow x=0$

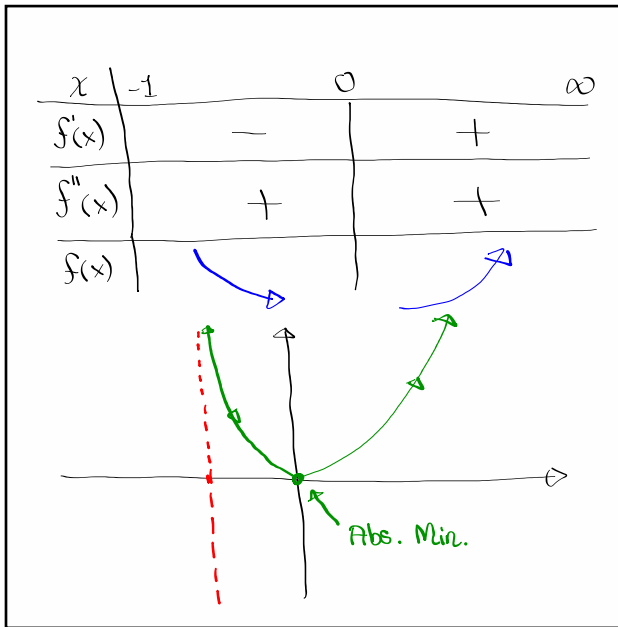
Nov 6-10:26 AM



Nov 6-10:33 AM



Nov 6-10:45 AM



Nov 6-10:55 AM

Find two positive numbers whose product is 100

$x \neq y$
 $x > 0, y > 0$
 $xy = 100$
 $y = \frac{100}{x}$

whose Sum is minimum.

$x + y$ is Min.

$x + \frac{100}{x}$ to be minimum.

$f(x) = x + 100x^{-1}$
 $f'(x) = 1 - 100x^{-2}$
 $f''(x) = 200x^{-3} = \frac{200}{x^3} > 0 \rightarrow \text{C.U.}$

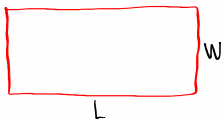
$f'(x) = 0 \rightarrow 1 - \frac{100}{x^2} = 0$
 $\frac{x^2 - 100}{x^2} = 0 \rightarrow x^2 - 100 = 0 \rightarrow x = \pm 10$
 $x = 10$

$x = 10 \rightarrow y = \frac{100}{x} = 10$
 $xy = 100$
 $x + y$ is minimum.

$(10, \text{Min.})$

Nov 6-10:58 AM

A farmer has 100 m of fencing and he wishes to have a rectangular enclosed shape using fence with largest area possible. Find the dimensions.



$$2L + 2W = 100$$

$$L + W = 50$$

Maximize LW

$$L(50 - L)$$

$$A(L) = 50L - L^2$$

$$A'(L) = 50 - 2L$$

$$A''(L) = -2 < 0$$

$$50 - 2L = 0$$

$$L = 25$$

Max. Pt.

$$W = 25$$

Dimensions are 25 m by 25 m.

Nov 6-11:05 AM

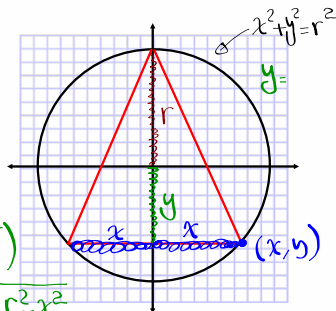
Find the dimensions of isosceles triangle with max. area inscribed in a circle with radius r .

$$A = \frac{bh}{2}$$

$$= \frac{2x \cdot (r+y)}{2}$$

$$A = x(r + \sqrt{r^2 - x^2})$$

$$A = xr + x\sqrt{r^2 - x^2}$$



Nov 6-11:11 AM

$$A = x r + x \sqrt{r^2 - x^2}$$

$$A' = r + 1(\sqrt{r^2 - x^2}) + x \cdot \frac{1}{2}(r^2 - x^2)^{-\frac{1}{2}} \cdot (-2x)$$

$$A' = r + \sqrt{r^2 - x^2} - \frac{x^2}{\sqrt{r^2 - x^2}}$$

$$A' = 0 \quad r\sqrt{r^2 - x^2} + r^2 - x^2 - x^2 = 0$$

$$(r\sqrt{r^2 - x^2})^2 = (2x^2 - r^2)^2$$

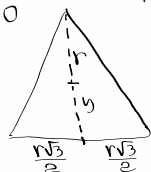
$$r^2(r^2 - x^2) = 4x^4 - 4x^2r^2 + r^4$$

$$r^4 - r^2x^2 = 4x^4 - 4x^2r^2 + r^4$$

$$4x^4 - 3x^2r^2 = 0$$

$$x^2(4x^2 - 3r^2) = 0$$

$$x = 0$$



$$\text{base} = r\sqrt{3}$$

$$\text{height} = r + \frac{r}{2} = \frac{3r}{2}$$

$$x^2 + y^2 = r^2$$

$$\frac{3r^2}{4} + y^2 = r^2$$

$$3r^2 + 4y^2 = 4r^2$$

$$y^2 = \frac{r^2}{4}$$

$$y = \frac{r}{2}$$

Nov 6-11:19 AM

$$A' = r + \sqrt{r^2 - x^2} - \frac{x^2}{\sqrt{r^2 - x^2}}$$

$$A' < 0$$

Max.

$$A' = 0$$

$$x = \frac{r\sqrt{3}}{2}$$

$$x = 0$$

$$A' = r + r - 0 = 2r > 0$$

$$x = \frac{r\sqrt{4}}{2} = \frac{2r}{2} = r$$

$$A' =$$

$$0 \quad r\frac{\sqrt{3}}{2}$$

Max

Explore First & Second derivative Test
in Calc. 1.

Nov 6-11:29 AM