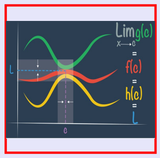
Math 261

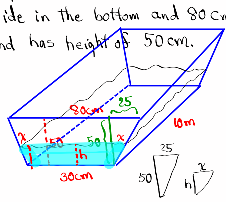
Fall 2023

Lecture 37



Feb 19-8:47 AM

A trough is 10 m long and its ends have a shape of isosceles Trapezoid that is 30cm wide in the bottom and 80 cm wide on the top and has height of 50 cm.



It is being field with water at the rate of .2 m^3/min.

$$\frac{dV}{dt} = .2$$

How fast is the water level rising when the water is 30 cm deep?

$$\frac{50}{25} = \frac{h}{x} \rightarrow \boxed{h = 2x}$$

$$x = \frac{h}{2}$$

$V = \underbrace{\text{Area of the base}} \cdot \underbrace{\text{height}}_{\text{Trough}}$

$$V = \frac{h(B+b)}{2} \cdot 10$$

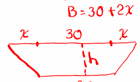
Blue Trapezoid

heigh h

$b = 30$

$B = 30 + 2x$

$$V = 5h(B+b)$$



$$V = 5h(.3 + 2x + .3)$$

$$V = 5h(.6 + h)$$

$b = .3\,m$

$B = .3 + 2x$

$$V = 3h + 5h^2$$

$$\frac{dV}{dt} = 3\frac{dh}{dt} + 10h\frac{dh}{dt}$$

$$.2 = 3\frac{dh}{dt} + 10(.3)\frac{dh}{dt}$$

$$.2 = 6\frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{.2}{6}\ m/min$$

$$\boxed{\frac{dh}{dt} = \frac{1}{30}\ m/min}$$

Book Says $\frac{dh}{dt} = \frac{10}{3}$ cm/min.

$1m = 100\,cm$

$$\frac{dh}{dt} = \frac{\cancel{1m}}{30 \text{ mins}} \cdot \frac{100cm}{\cancel{1m}}$$

$$= \frac{10}{3}\ cm/min.$$

Nov 1-10:39 AM

$f(x) = 5x^3 - 3x^5$

Polynomial → Cont. $(-\infty, \infty)$, diff. $(-\infty, \infty)$

$f(-x) = 5(-x)^3 - 3(-x)^5 = -5x^3 + 3x^5 = -(5x^3 - 3x^5) = -f(x)$

Since $f(-x) = -f(x)$

odd function → symmetric w/r origin

$f'(x) = 15x^2 - 15x^4$

$f'(x) = 0$

$15x^2(1 - x^2) = 0$

$x = 0 \quad x = \pm 1$

Critical numbers

$f''(x) = 30x - 60x^3$

$f''(x) = 0$

$30x(1 - 2x^2) = 0$

$x = 0 \quad x = \pm\frac{\sqrt{2}}{2} = \pm\frac{1}{\sqrt{2}}$

$\left.\begin{array}{l} f(0) = 0 \\ f(1) = 2 \\ f(-1) = -2 \end{array}\right\}$ Critical Points

Possible inflection numbers

$f(0) = 0$

$f\left(\frac{\sqrt{2}}{2}\right) = \frac{7}{4\sqrt{2}}$

$f\left(-\frac{\sqrt{2}}{2}\right) = \frac{-7}{4\sqrt{2}}$

Possible inflection Points

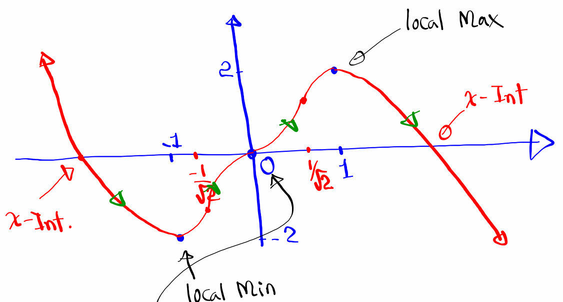
$y = 5\left(\frac{1}{\sqrt{2}}\right)^3 - 3\left(\frac{1}{\sqrt{2}}\right)^5 = \frac{5}{2\sqrt{2}} - \frac{3}{4\sqrt{2}} = \frac{7}{4\sqrt{2}}$

Nov 2-10:54 AM

x	$-\infty$ to -1	-1 to $-\frac{1}{\sqrt{2}}$	$-\frac{1}{\sqrt{2}}$ to 0	0 to $\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$ to 1	1 to ∞
$f'(x)$	− ↘	+ ↗	+ ↗	+ ↗	+ ↗	− ↘
$f''(x)$	+	+	−	+	−	−
	CU	CU	CD	CU	CD	CD

I.P.   I.P.   I.P.

$f'(x) = 15x^2(1 - x^2)$ $f''(x) = 30x(1 - 2x^2)$



Find x-Ints → $y = 0$ → $f(x) = 0$ → $5x^3 - 3x^5 = 0$

$x^3(5 - 3x^2) = 0$ → $x = \pm\sqrt{\frac{5}{3}}$

$x = 0$

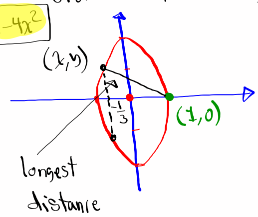
Nov 2-11:03 AM

Find all points on the graph of $4x^2+y^2=4$ that are farthest away from the point $(1,0)$.

$4x^2+y^2=4 \quad \boxed{y^2=4-4x^2}$

$\frac{x^2}{1}+\frac{y^2}{4}=1$

Center $(0,0)$



$d=\sqrt{(x-1)^2+(y-0)^2}$

$d=\sqrt{(x-1)^2+y^2}=\sqrt{(x-1)^2+4-4x^2}$

$f(x)=(x-1)^2+4-4x^2$

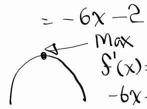
maximum

$f'(x)=2(x-1)^1\cdot 1+0-8x=2x-2-8x$

$=-6x-2$

$f''(x)=-6$

$f''(x)<0 \rightarrow$ C.D.



$f'(x)=0$

$-6x-2=0$

$x=-\frac{1}{3}$

$y^2=4-4x^2 \quad y^2=4-4\left(\frac{-1}{3}\right)^2$

$=4-\frac{4}{9} \Rightarrow y^2=\frac{32}{9}$

$y=\pm\sqrt{\frac{32}{9}}=\pm\frac{4\sqrt{2}}{3}$

Points are

$\left(\frac{-1}{3},\frac{4\sqrt{2}}{3}\right),\left(\frac{-1}{3},\frac{-4\sqrt{2}}{3}\right)$

Nov 2-11:18 AM