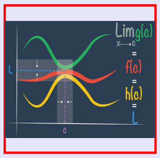
Math 261

Fall 2023

Lecture 35

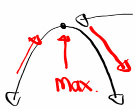


Feb 19-8:47 AM

Find $\underbrace{\text{two numbers}}_{x \,\&\, y}$ with $\underbrace{\text{sum of 10}}_{x+y=10}$, and $\underbrace{\text{their product}}_{x\,y}$ is $\boxed{\text{maximum}}$

$y = 10 - x$

$P(x) = x(10-x) = -x^2 + 10x$



Max.

at the vertex

$x = \frac{-b}{2a} = \frac{-10}{2(-1)} = \frac{-10}{-2} = 5$

$(5,5) \rightarrow$ Sum of 10

Product $= 25 \leftarrow$ Max.

$P'(x) = -2x + 10$

$P'(x) = 0 \rightarrow -2x + 10 = 0$

$\boxed{x = 5}$

x		5	
$P'(x)$	$+$		$-$

Max

Oct 31-10:27 AM

Find two numbers whose difference is 100

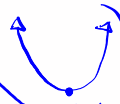
x & y $\qquad x - y = 100$

and their product is $\boxed{\text{minimum}}$

xy

$y = x - 100$

$\boxed{y = -50}$



Vertex

$P(x) = x(x-100) = x^2 - 100x$

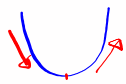
$x = \frac{-b}{2a} = \frac{-(-100)}{2(1)} = \boxed{50}$

$P'(x) = 2x - 100$

$P'(x) = 0 \rightarrow 2x - 100 = 0 \rightarrow \boxed{x = 50}$

x		50	
$P'(x)$	$-$		$+$

min.



$(50, -2500)$

$\boxed{50 \text{ \& } -50}$

$y = x - 100$

$= 50 - 100$

$= \boxed{-50}$

Oct 31-10:34 AM

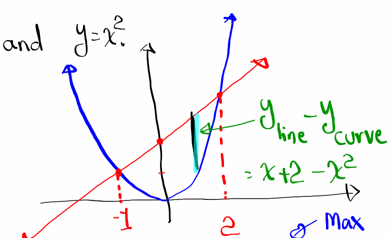
For $-1 \le x \le 2$, find a $\boxed{\text{maximum}}$ vertical distance between $y = x+2$ and $y = x^2$.

$x^2 = x + 2$

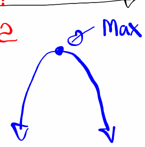
$x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

$x = 2$, $x = -1$



$f(x) = x + 2 - x^2$



$f'(x) = 1 - 2x$ $\qquad f'(x) = 0 \rightarrow x = \frac{1}{2}$

x		1/2	
$f'(x)$	$+$		$-$

Vertical distance

$f(\frac{1}{2}) = \frac{1}{2} + 2 - (\frac{1}{2})^2$

$= \frac{1}{2} + 2 - \frac{1}{4} = 2 + \frac{1}{4} = \boxed{\frac{9}{4}}$

Oct 31-10:41 AM

Find dimensions of a rectangle with area 1000 m^2 whose perimeter is as small as possible.

$A = 1000$

$LW = 1000$

W

L

$P = 2L + 2W$

$P(L) = 2L + 2\left(\frac{1000}{L}\right)$ $\qquad P(L) = 2L + \frac{2000}{L}$ $\qquad 2000L^{-1}$

$P'(L) = 2 - 2000L^{-2}$

$P'(L) = 0 \qquad 2 - \frac{2000}{L^2} = 0 \qquad 2L^2 - 2000 = 0$

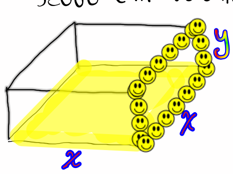
$L^2 - 1000 = 0 \longrightarrow \boxed{L = \sqrt{1000}} \quad \boxed{L = 10\sqrt{10}}$

$W = \frac{1000}{L} = \frac{1000}{10\sqrt{10}} = \frac{100}{\sqrt{10}} = \frac{100\sqrt{10}}{10} = 10\sqrt{10} \quad \boxed{W = 10\sqrt{10}}$

$\boxed{10\sqrt{10} \text{ by } 10\sqrt{10} \text{ meters}}$

Oct 31-10:50 AM

A box has a square base and open top with 32000 cm^3 volume. $V = LWH$



$x \cdot x \cdot y = 32000$

$x^2 y = 32000$

$y = \frac{32000}{x^2}$

Find dimensions of the box with least amount of materials.

$m(x,y) = x^2 + 4xy$

$f(x) = x^2 + 4x \cdot \frac{32000}{x^2} \qquad f(x) = x^2 + \frac{128000}{x}$

$f'(x) = 2x - \frac{128000}{x^2}$

$f''(x) = 2 + \frac{256000}{x^3} > 0$

Concave Up $\quad$ Min.

Oct 31-10:57 AM

$f'(x) = 2x - \frac{128000}{x^2}$ $\quad f'(x) = 0$

$2x - \frac{128000}{x^2} = 0$

$2(41) - \frac{128000}{41^2} = +$

$2x^3 - 128000 = 0$

$x^3 = \frac{128000}{2}$ $\quad x^3 = 64000$ $\quad \boxed{x = 40}$

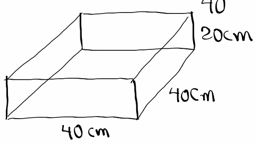
x		40	
$f'(x)$	$-$		$+$
$f''(x)$	$+$		$+$

Min.

$x^2 y = 32000$

$40^2 y = 32000$

$y = \frac{32000}{40^2}$ $\quad \boxed{y = 20}$



Oct 31-11:07 AM

$f(x) = \frac{x}{x^2+9}$

1) Domain: $(-\infty, \infty)$

2) x-Int: $(0, 0)$

3) Y-Int: $(0, 0)$

4) Discuss Symmetry

$f(-x) = \frac{-x}{(-x)^2+9} = \frac{-x}{x^2+9} = -f(x)$

odd function → Origin

5) $f'(x) = \frac{1(x^2+9) - x(2x)}{(x^2+9)^2} = \frac{9-x^2}{(x^2+9)^2}$

$f'(x) = 0 \rightarrow 9 - x^2 = 0 \rightarrow x = \pm 3$

$f'(x)$ is defined everywhere.

Oct 31-11:14 AM

6) Find $f''(x)$

$f'(x) = \dfrac{9-x^2}{(x^2+9)^2}$ $\quad f''(x) = \dfrac{-2x(x^2+9)^2-(9-x^2)\cdot 2(x^2+9)\cdot 2x}{[(x^2+9)^2]^2}$

$f''(x) = \dfrac{-2x(x^2+9)^2-4x(9-x^2)(x^2+9)}{(x^2+9)^4}$

$= \dfrac{\cancel{(x^2+9)}[-2x(x^2+9)-4x(9-x^2)]}{(x^2+9)^{\cancel{4}}} = \dfrac{-2x[x^2+9+2(9-x^2)]}{(x^2+9)^3}$

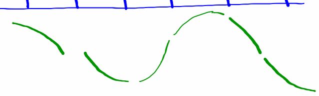
$f''(x) = \dfrac{-2x(27-x^2)}{(x^2+9)^3}$ $\quad f'(x) = \dfrac{9-x^2}{(x^2+9)^2}$

$f''(x)=0 \longrightarrow x=0,\ x=\pm 3\sqrt{3}$

$f''(x)$ is defined everywhere

8) Sign chart

x		$-3\sqrt{3}$		-3		0		3		$3\sqrt{3}$	
$f'(x)$	−		−	•	+		+	•	−		−
$f''(x)$	−	•	+		+	•	−		−	•	+



Oct 31-11:23 AM