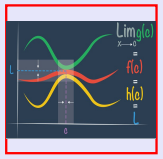
Math 261

Fall 2023

Lecture 34



Feb 19-8:47 AM

If $x^2 + y^2 + z^2 = 9$, $\frac{dx}{dt} = 5$, $\frac{dy}{dt} = 4$

find $\frac{dz}{dt}$ when $(x, y, z) = (2, 2, 1)$

ordered - Triple

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} + 2z\frac{dz}{dt} = 0$$

$$2 \cdot 5 + 2 \cdot 4 + 1 \cdot \frac{dz}{dt} = 0$$

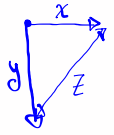
$$\frac{dz}{dt} = -18$$

z is decreasing

Oct 30-10:30 AM

Two Cars leave the Same point at Same time. One going South at 60 mph and the one going east at 25 mph. At what rate is the distance between them changing 2 hrs later?

$\frac{dx}{dt} = 25, \frac{dy}{dt} = 60$



$z^2 = x^2 + y^2$

$2z\frac{dz}{dt} = 2x\frac{dx}{dt} + 2y\frac{dy}{dt}$

In 2 hrs

$x = 50, y = 120$

$z^2 = x^2 + y^2$

$z^2 = 50^2 + 120^2$

$z = 130$

$\frac{dz}{dt} = \frac{x\frac{dx}{dt} + y\frac{dy}{dt}}{z}$

$= \frac{50 \cdot 25 + 120 \cdot 60}{130}$

$\frac{dz}{dt} = 65$ mph

Oct 30-10:34 AM

Gravel is being dumped from a belt at the rate of $30\ ft^3/min$. $\frac{dV}{dt} = 30\ ft^3/min$.

It is forming a Circular cone.

$d = h \rightarrow 2r = h$, $r = \frac{h}{2}$

$V = \frac{1}{3}\pi r^2 h$

Diameter is always equal to height.

How fast is the height of pile increasing when it is 40 ft high?

$\frac{dh}{dt} = ?$

$V = \frac{1}{3}\pi\left(\frac{h}{2}\right)^2 h$

$V = \frac{1}{12}\pi h^3$

$\frac{dV}{dt} = \frac{1}{12} \cdot \pi \cdot 3h^2 \frac{dh}{dt}$

$30 = \frac{1}{4} \cdot \pi \cdot 10^2 \cdot \frac{dh}{dt}$

$120 = 100\pi \frac{dh}{dt}$

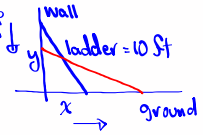
$\frac{dh}{dt} = \frac{6}{5\pi}$ ft/min

$\frac{dh}{dt} = \frac{\cancel{120}^{6}}{\cancel{100}\pi_{5}}$ ft/min

Oct 30-10:41 AM

A ladder is 10 ft long and it is resting against a wall.

If the bottom of ladder slides away at 1 ft/sec, How fast the top of the ladder slides down when the bottom of the ladder is 6 ft from the wall?



$x^2 + y^2 = 10^2$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$

$\frac{dy}{dt} = -\frac{x\frac{dx}{dt}}{y}$

$\frac{dx}{dt} = 1$ ft/s

$x^2 + y^2 = 10^2$

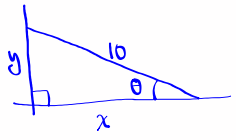
$6^2 + y^2 = 10^2$

$y = 8$

$\frac{dy}{dt} = \frac{-6 \cdot 1}{8} = -\frac{3}{4}$ ft/s

How fast the angle between the ladder and ground changing when the bottom of the ladder is 6 ft from the wall?

Find $\frac{d\theta}{dt}$



Oct 30-10:51 AM

$f(x) = x^4 - 4x^3$

$f'(x) = 4x^3 - 12x^2$

$f''(x) = 12x^2 - 24x$

$f'(x) = 0$

$4x^3 - 12x^2 = 0$

$4x^2(x-3) = 0$

$x = 0$ $\quad$ $x = 3$

$f''(x) = 0$

$12x^2 - 24x = 0$

$12x(x-2) = 0$

$x = 0$ $\quad$ $x = 2$

x		0		2		3	
$f'(x)$	−	0	−		−	0	+
$f''(x)$	+	0	−	0	+		+

Oct 30-11:01 AM

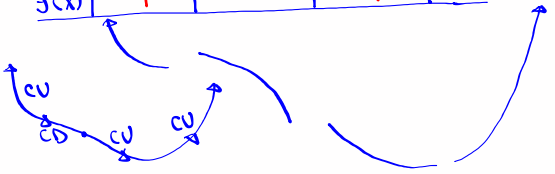
When $f'(x) > 0$ → $f(x)$ is increasing

When $f'(x) < 0$ → $f(x)$ is decreasing

When $f''(x) > 0$ → $f(x)$ is Concave up.

When $f''(x) < 0$ → $f(x)$ is Concave down

x		0		2		3	
$f'(x)$	−		−		−		+
$f''(x)$	+		−		+		+



Oct 30-11:08 AM

$f(x) = -2x^3 + 3x^2 + 36x$

$f'(x) = -6x^2 + 6x + 36$ $\quad f''(x) = -12x + 6$

$f'(x) = 0$

$-6x^2 + 6x + 36 = 0$

$-6(x^2 - x - 6) = 0$

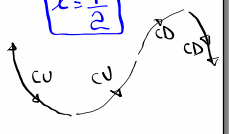
$-6(x-3)(x+2) = 0$

$x = 3$ $\quad x = -2$

$f''(x) = 0$

$-12x + 6 = 0$

$x = \frac{1}{2}$



x		-2		$\frac{1}{2}$		3	
$f'(x)$	−		+		+		−
$f''(x)$	+		+		−		−
	Dec. C.U.		Inc., CU		Inc., CD		Dec., CD

Oct 30-11:13 AM

$f(x) = \frac{x}{x-1}$ Domain : $(-\infty, 1) \cup (1, \infty)$

$f'(x) = \frac{1(x-1) - x \cdot 1}{(x-1)^2}$

$\boxed{f'(x) = \frac{-1}{(x-1)^2}}$

$f'(x) = -(x-1)^{-2}$

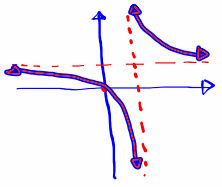
$f''(x) = -(-2)(x-1)^{-3} \cdot 1$

$\boxed{f''(x) = \frac{2}{(x-1)^3}}$

$f'(x) \neq 0$, $f''(x) \neq 0$

$f'(x)$ & $f''(x)$ undefined at $x = 1$

x		1	
$f'(x)$		—	—
$f''(x)$		—	+
		Dec. CD	Dec. CU



Oct 30-11:23 AM