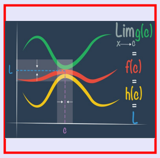
Math 261

Fall 2023

Lecture 33



Feb 19-8:47 AM

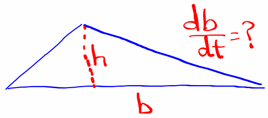
The height of a triangle increases @ 1cm/min.

The area increases @ $2\,cm^2/min$.

At what $\boxed{\text{rate its base}}$ changing when $h = 10cm$,

and $A = 100\,cm^2$?

$A = \frac{bh}{2}$, $\frac{dh}{dt} = 1$, $\frac{dA}{dt} = 2$



$\frac{db}{dt} = ?$

$2A = bh$

$\frac{d}{dt}[2A] = \frac{d}{dt}[bh]$

$2 \cdot \frac{dA}{dt} = \frac{db}{dt} \cdot h + b \cdot \frac{dh}{dt}$

$2 \cdot 2 = \frac{db}{dt} \cdot 10 + 20 \cdot 1$

$A = \frac{bh}{2}$

$100 = \frac{b \cdot 10}{2}$

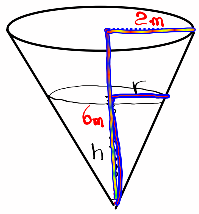
$\boxed{b = 20}$

$\frac{db}{dt} = \boxed{}$

Oct 26-10:27 AM

An inverted Conical tank is leaking water at rate of $10000\ cm^3/min$.

$\frac{dV}{dt} = -10{,}000\ cm^3/min$



height is 6m & diameter on top is 4m.

At what rate the water level decreasing when water level is 3m?

$\frac{h}{r} = \frac{6}{2}$ $\quad \frac{h}{r} = 3 \rightarrow h = 3r \rightarrow r = \frac{h}{3}$ $\quad$ ↳300 cm

$V = \frac{1}{3}\pi r^2 h \rightarrow V = \frac{1}{3}\pi\left(\frac{h}{3}\right)^2 \cdot h$

$V = \frac{\pi}{27} h^3$ $\quad \frac{dV}{dt} = \frac{\pi}{27} \cdot 3h^2 \cdot \frac{dh}{dt}$

$-10000 = \frac{\pi}{27} \cdot 3 \cdot (300)^2 \cdot \frac{dh}{dt}$

$-\cancel{10000} = \frac{\pi}{\cancel{27}} \cdot \cancel{3} \cdot \cancel{90000} \cdot \frac{dh}{dt}$

$\boxed{\frac{dh}{dt} = \frac{-1}{\pi}\ cm/min}$

Oct 26-10:34 AM

Two sides of a triangle are 4cm & 5cm. Angle between them is increasing at the rate of .06 Rad/Sec.

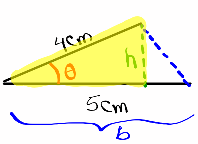
$\frac{d\theta}{dt} = .06$

At what rate is its area changing when the angle is $\frac{\pi}{3}$.

$\frac{dA}{dt}$ $\quad A = \frac{bh}{2}$

$\sin\theta = \frac{h}{4}$

$h = 4\sin\theta$



$A = \frac{b \cdot 4\sin\theta}{2}$

$= \frac{5 \cdot 4 \cdot \sin\theta}{2}$

$A = 10\sin\theta$

$A = \frac{1}{2}ab\sin C$

$= \frac{1}{2}bc\sin A$

$= \frac{1}{2}ac\sin B$

$\frac{dA}{dt} = 10 \cdot \cos\theta \cdot \frac{d\theta}{dt}$

$\frac{dA}{dt} = 10 \cdot \cos\frac{\pi}{3} \cdot (.06)$

$= 10 \cdot \frac{1}{2} \cdot (.06)$

$\boxed{\frac{dA}{dt} = .3\ cm^2/s}$

Oct 26-10:47 AM

try $f(x)=\frac{x^3}{x^2+1}$ Domain: $(-\infty,\infty)$

x-Int. $\to y=0 \to f(x)=0 \to x^3=0 \to x=0$

$(0,0)$

Y-Int $\to x=0 \to f(0)=\frac{0^3}{0^2+1}=\frac{0}{1}=0$

$f(-x)=\frac{(-x)^3}{(-x)^2+1}=\frac{-x^3}{x^2+1}=-\frac{x^3}{x^2+1}=-f(x)$

odd function

Symmetric with respect to origin

Oct 25-11:08 AM

$f(x)=\frac{x^3}{x^2+1}$

$f'(x)=\frac{x^4+3x^2}{(x^2+1)^2}=\frac{x^2(x^2+3)}{(x^2+1)^2}$

$f'(x)=0 \to x=0$

$f'(x)$ defined everywhere

$f''(x)=\frac{-2x(x^2-3)}{(x^2+1)^3}$

$f''(x)=0 \to x=0$ $\quad x=0$

$\to x^2-3=0$ $\quad x=\pm\sqrt{3}$

x		$-\sqrt{3}$		0		$\sqrt{3}$	
$f'(x)$	+		+	•	+		+
$f''(x)$	+	•	−	•	+	•	−

Oct 26-11:04 AM

$y = \frac{1}{x^2-9}$ Domain All Reals except ± 3

$f(x) = \frac{1}{x^2-9}$ $f(-x) = \frac{1}{(-x)^2-9} = \frac{1}{x^2-9} = f(x)$

even

Symmetric $\rightarrow$ Y-axis

y-Int $\rightarrow x=0 \rightarrow y = \frac{-1}{9}$

x-Int $\rightarrow y=0 \rightarrow \frac{1}{x^2-9} \neq 0$ No x-Int.

$y = (x^2-9)^{-1}$ $y' = -1(x^2-9)^{-2} \cdot 2x = \frac{-2x}{(x^2-9)^2}$

$y'' = -2 \cdot \frac{1(x^2-9)^2 - x \cdot 2(x^2-9)^1 \cdot 2x}{(x^2-9)^4}$

$y' = 0 \rightarrow x = 0$

y' und. $\rightarrow x = \pm 3$

$= -2 \cdot \frac{\cancel{(x^2-9)}[x^2-9-4x^2]}{(x^2-9)^{\cancel{4}}} = \frac{-2(-3x^2-9)}{(x^2-9)^3}$

$y'' = \frac{6(x^2+3)}{(x^2-9)^3}$ $y'' \neq 0$

y'' undef. at ± 3

Oct 26-11:14 AM

$y' = \frac{-2x}{(x^2-9)^2}$ $y'' = \frac{6(x^2+3)}{(x^2-9)^3}$

x		-3		0		3	
$f'(x)$	$+$		$+$	$\bullet$	$-$		$-$
$f''(x)$	$+$		$-$		$-$		$+$

Oct 26-11:23 AM