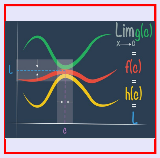
Math 261

Fall 2023

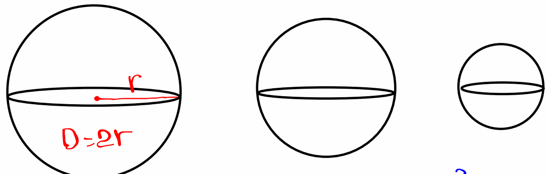
Lecture 32



Feb 19-8:47 AM

A snowball melts and its surface area decreases at the rate of 1 cm^2/min. $\frac{dS}{dt} = -1$

At what rate its diameter decreases when the diameter is 10 cm. $\frac{dD}{dt} = ?$



Surface Area of Sphere $S = 4\pi r^2$

$S = 4\pi\left(\frac{D}{2}\right)^2$ $\qquad$ $S = \pi D^2$

$$\frac{dS}{dt} = 2\pi D \frac{dD}{dt}$$

$$-1 = 2\pi(10)\frac{dD}{dt}$$

it decreases

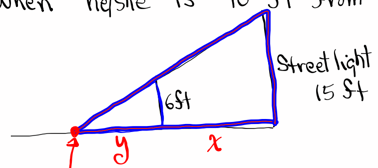
$$\frac{dD}{dt} = \frac{-1}{20\pi} \text{ cm/min}$$

Oct 25-10:26 AM

Street light is 15-ft tall.

A 6-ft tall person walks away from the Pole at the rate of $\boxed{5 \text{ ft/sec}}$

At what rate is the tip of his shadow moving when he/she is 40 ft from the Pole?



Tip of shadow

$\frac{dx}{dt} = 5$

$\frac{y}{6} = \frac{y+x}{15}$

$15y = 6y + 6x$

$9y = 6x$

$3y = 2x$

$y = \frac{2}{3}x$

$\frac{dy}{dt} = \frac{2}{3} \cdot \frac{dx}{dt}$

$\frac{dy}{dt} = \frac{2}{3} \cdot 5 = \frac{10}{3}$ ft/sec.

For tip of the shadow

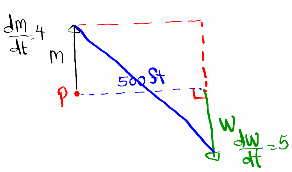
$\frac{d}{dt}[y+x] = \frac{dy}{dt} + \frac{dx}{dt} = \frac{10}{3} + 5 = \frac{25}{3}$ ft/sec

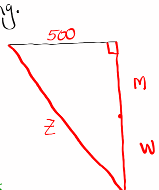
Oct 25-10:33 AM

A man starts walking north at 4 ft/sec from a fixed point P.

5 minutes later, A woman going South @ 5 ft/sec from a point 500 ft east of P.

At what rate they are moving apart 15 mins after woman starts walking.





15 mins → w
20 " → m

$z^2 = (m+w)^2 + 500^2$

$\cancel{2}z \frac{dz}{dt} = \cancel{2}(m+w)^1 \cdot \left[\frac{dm}{dt} + \frac{dw}{dt}\right] + 0$

$9313 \frac{dz}{dt} = (4800+4500) \cdot [4 + 5]$ $\frac{dz}{dt} \approx 8.987$ ≈ 9 ft/s.

In 15 mins → $w = 5 \cdot 15 \cdot 60 = 4500$
In 20 " → $m = 4 \cdot 20 \cdot 60 = 4800$

$z^2 = (m+w)^2 + 500^2$
$= (4800+4500)^2 + 500^2$
$= \sqrt{86740000}$

$z \approx 9313$

Oct 25-10:44 AM

Do Same thing for $f(x) = \frac{x^2-1}{x^3}$

$f(x) = \frac{x^2-1}{x^3}$ Domain $\rightarrow$ All Reals except 0.

x-Int. $\rightarrow f(x)=0 \rightarrow x^2-1=0 \rightarrow x=\pm 1$

$\rightarrow (1,0),(-1,0)$

No Y-Int Since $x \neq 0$

$f(x) = \frac{x^2}{x^3} - \frac{1}{x^3}$ $F(x) = \frac{1}{x} - \frac{1}{x^3}$

$F(x) = x^{-1} - x^{-3}$

$f'(x) = -x^{-2} + 3x^{-4}$

$f''(x) = 2x^{-3} - 12x^{-5}$

Oct 24-11:27 AM

$f'(x) = -x^{-2} + 3x^{-4} \rightarrow f'(x) = \frac{-1}{x^2} + \frac{3}{x^4}$

$f''(x) = 2x^{-3} - 12x^{-5}$

$f'(x) = \frac{-x^2+3}{x^4}$

$\boxed{f'(x) = \frac{3-x^2}{x^4}}$

$f''(x) = \frac{2}{x^3} - \frac{12}{x^5}$

$f''(x) = \frac{2x^2-12}{x^5}$

$\boxed{f''(x) = \frac{2(x^2-6)}{x^5}}$

Oct 25-11:05 AM

$$\boxed{f'(x) = \frac{3 - x^2}{x^4}}$$

$$\boxed{f''(x) = \frac{2(x^2 - 6)}{x^5}}$$

$f'(x) = 0 \to 3 - x^2 = 0 \to x = \pm\sqrt{3}$

$f'(x)$ undefined $\to x = 0$

$f''(x) = 0 \to x^2 - 6 = 0 \to x = \pm\sqrt{6}$

$f''(x)$ undefined $\to x = 0$

x		$-\sqrt{6}$		$-\sqrt{3}$		0		$\sqrt{3}$		$\sqrt{6}$	
$f'(x)$	$-$		$-$		$+$		$+$		$-$		$-$
$f''(x)$	$-$		$+$		$+$		$-$		$-$		$+$

try $f(x) = \frac{x^3}{x^2 + 1}$

Oct 25-11:08 AM

$f(x) = \frac{x^2}{x^2 + 3}$

Domain: All Real #

$x^2 + 3 \neq 0$

x-Int $\to f(x) = 0 \to x^2 = 0 \to x = 0 \to (0,0)$

Y-Int $\to x = 0 \to \frac{0^2}{0^2 + 3} = 0 \to (0,0)$

Find $f(-x) = \frac{(-x)^2}{(-x)^2 + 3} = \frac{x^2}{x^2 + 3} = f(x)$

even function

Symmetric $\to$ Y-axis.

Oct 25-11:15 AM

Find $f'(x)$

$$f(x) = \frac{x^2}{x^2+3} = \frac{x^2+3 \;\; -3}{x^2+3} = \frac{x^2+3}{x^2+3} - \frac{3}{x^2+3}$$

$$f(x) = 1 - \frac{3}{x^2+3} \qquad f(x) = 1 - 3(x^2+3)^{-1}$$

$$f'(x) = 0 \;\; +3(x^2+3)^{-2} \cdot 2x \qquad f'(x) = \frac{6x}{(x^2+3)^2}$$

$$f'(x) = 6x(x^2+3)^{-2}$$

$$f''(x) = 6\left[1\cdot(x^2+3)^{-2} + x \cdot -2(x^2+3)^{-3} \cdot 2x\right]$$

$$= 6\left[(x^2+3)^{-2} - 4x^2(x^2+3)^{-3}\right]$$

$$= 6(x^2+3)^{-3}\left[(x^2+3)^{1} - 4x^2\right]$$

$$= 6(x^2+3)^{-3}(3-3x^2)$$

$$\boxed{f''(x) = \frac{18(1-x^2)}{(x^2+3)^3}}$$

Oct 25-11:19 AM

$$f'(x) = \frac{6x}{(x^2+3)^2}$$

$f'(x) = 0 \to 6x = 0 \to x = 0$

$f'(x)$ is always defined

$x^2+3 \neq 0$

$$f''(x) = \frac{18(1-x^2)}{(x^2+3)^3}$$

$f''(x) = 0 \to 18(1-x^2) = 0$

$\to \boxed{x = \pm 1}$

$f''(x)$ is always defined

x		-1		0		1	
$f'(x)$	−		−		+		+
$f''(x)$	−		+		+		−

Oct 25-11:30 AM