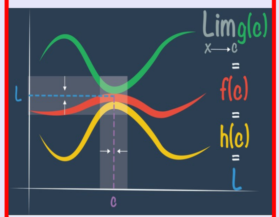
Math 261

Fall 2023

Lecture 31



Feb 19-8:47 AM

Given $z^2 = x^2 y^3$, $\frac{dx}{dt} = 4$, $\frac{dy}{dy} = -10$,

Find $\frac{dz}{dt}$ when $x=8$ & $y=4$. , Assume $z \geq 0$

$z^2 = x^2 y^3$ $\quad$ $z^2 = 8^2 \cdot 4^3 = 64 \cdot 64$

for $x=8$, $y=4$ $\quad$ $z^2 = 64^2$ $\quad$ $\boxed{z = 64}$

$$\frac{d}{dt}\left[z^2\right] = \frac{d}{dt}\left[x^2 y^3\right]$$

$$2z\frac{dz}{dt} = 2x\frac{dx}{dt} \cdot y^3 + x^2 \cdot 3y^2 \cdot \frac{dy}{dt}$$

$$2 \cdot 64 \frac{dz}{dt} = 2 \cdot 8 \cdot 4 \cdot 4^3 + 8^2 \cdot 3 \cdot 4^2 \cdot (-10)$$

$$128 \frac{dz}{dt} = -26624$$

$$\boxed{\frac{dz}{dt} = -208}$$

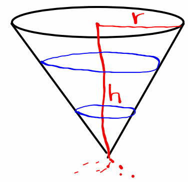
z is decreasing

Oct 24-10:26 AM

A right-Circular cone has a height twice the radius of its base.

It is leaking water at the bottom at rate of 3 ft^3/min. Assume the Cone is upside down.

How fast is the height of water changing when it is $\boxed{5 \text{ ft high}}$



$h = 2r \rightarrow r = \frac{h}{2}$

$\frac{dh}{dt} = -$

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi \left(\frac{h}{2}\right)^2 \cdot h$

$V = \frac{\pi h^3}{12}$

$\frac{dV}{dt} = \frac{\pi}{12} \cdot \frac{d}{dt}\left[h^3\right]$

$\frac{dV}{dt} = \frac{\pi}{12} \cdot 3h^2 \cdot \frac{dh}{dt}$

$-3 = \frac{\pi}{4} \cdot 5^2 \cdot \frac{dh}{dt}$

$\rightarrow -12 = 25\pi \frac{dh}{dt}$

$\frac{dh}{dt} = \frac{-12}{25\pi}$ ft/min.

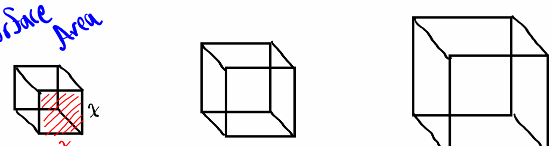
Oct 24-10:35 AM

At a certain moment, edge of cube is 8 in. and its volume increasing at 4 in^3/min. $\frac{dV}{dt} = 4$

How fast its surface area changing?

Increasing or decreasing? why?

Surface Area



$V = x^3$

$S = 6x^2$

$\frac{dS}{dt} = 6 \cdot 2x \cdot \frac{dx}{dt}$

$= \cancel{6} \cdot 2 \cdot \cancel{8} \cdot \frac{1}{\cancel{48}}$

$\frac{dV}{dt} = 3x^2 \frac{dx}{dt}$

$4 = 3 \cdot 8^2 \frac{dx}{dt}$

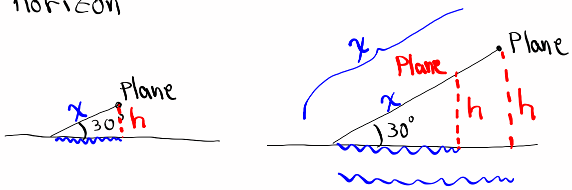
$\frac{dx}{dt} = \frac{\cancel{4}^{\,1}}{3 \cdot 8 \cdot \cancel{8}_{\,2}}$

$= \frac{1}{48}$ in/min

$\frac{dS}{dt} = 2$ in^2/min $\rightarrow$ Increasing because $\frac{dS}{dt} > 0$

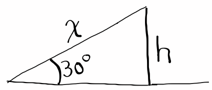
Oct 24-10:45 AM

A plane is climbing at 30° angle to the horizon



$\frac{dh}{dt} = ?$

How fast is the plane gaining altitude if its $\boxed{\text{speed is 500 mi/hr.}}$ $\frac{dx}{dt} = 500$ mi/hr.



$\sin 30° = \frac{h}{x}$

$\frac{1}{2} = \frac{h}{x} \rightarrow x = 2h$

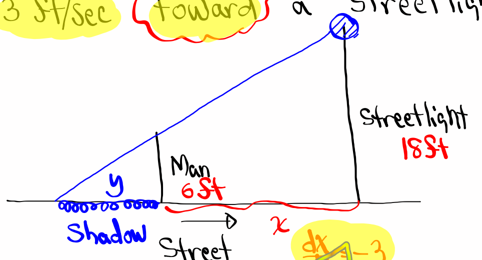
$\frac{dx}{dt} = 2 \cdot \frac{dh}{dt}$

$500 = 2\frac{dh}{dt}$

$\frac{dh}{dt} = 250$ mi/hr.

Oct 24-10:53 AM

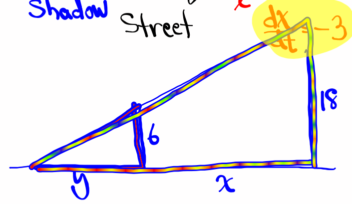
A man 6 ft tall is walking at the rate of 3 ft/sec toward a streetlight 18 ft tall.



How fast is the length of shadow changing?

$\frac{dy}{dt} = ?$

$\frac{dx}{dt} = -3$



$\frac{y}{\cancel{6}_1} = \frac{y+x}{\cancel{18}_3}$

$3y = y + x$

$2y = x$

$2\frac{dy}{dt} = \frac{dx}{dt}$

$2\frac{dy}{dt} = -3$

$\frac{dy}{dt} = -1.5$ ft/sec.

Oct 24-11:01 AM

$$f(x) = x^3 - 3x + 1$$

$$f'(x) = 3x^2 - 3$$

Solve $f'(x) = 0$

$$3x^2 - 3 = 0$$

$$x = \pm 1$$

$$f''(x) = 6x$$

Solve $f''(x) = 0$

$$6x = 0$$

$$x = 0$$

x		-1		0		1	
$f'(x)$	$+$		$-$		$-$		$+$
$f''(x)$	$-$		$-$		$+$		$+$

Oct 24-11:10 AM

$$f(x) = \frac{x^2}{x^2-1}$$

Domain: All Reals except ± 1

$$f'(x) = \frac{2x(x^2-1) - x^2 \cdot 2x}{(x^2-1)^2} = \frac{-2x}{(x^2-1)^2}$$

$f'(x) = 0 \rightarrow -2x = 0 \rightarrow x = 0$

$f'(x)$ undefined $\rightarrow x^2 - 1 = 0 \rightarrow x = \pm 1$

$$f'(x) = -2x(x^2-1)^{-2}$$

$$f''(x) = -2\left[1 \cdot (x^2-1)^{-2} + x \cdot -2(x^2-1)^{-3} \cdot 2x\right]$$

$$= -2\left[(x^2-1)^{-2} - 4x^2(x^2-1)^{-3}\right]$$

$$= -2(x^2-1)^{-3}\left[(x^2-1)^{1} - 4x^2\right]$$

$$= -2(x^2-1)^{-3} \cdot (x^2 - 1 - 4x^2)$$

$$= -2(x^2-1)^{-3}(-3x^2 - 1) = \frac{-2 \cdot -(3x^2+1)}{(x^2-1)^3}$$

$$= \frac{2(3x^2+1)}{(x^2-1)^3}$$

$f''(x) = 0 \rightarrow 2(3x^2+1) = 0 \rightarrow$ No real Soln.

$f''(x)$ undefined $\rightarrow x^2 - 1 = 0 \rightarrow x = \pm 1$

Oct 24-11:17 AM

x		-1		0		1	
$f'(x)$	$+$		$+$		$-$		$-$
$f''(x)$	$+$		$-$		$-$		$+$

$$f'(x) = \frac{-2x}{(x^2-1)^2} =$$

$$f''(x) = \frac{2(3x^2+1)}{(x^2-1)^3} = \frac{+}{+}$$

Do Same thing for $f(x) = \dfrac{x^2-1}{x^3}$

Oct 24-11:27 AM