

Math 261

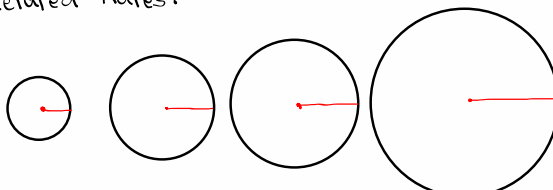
Fall 2023

Lecture 30



Feb 19-8:47 AM

Related Rates:



Suppose its radius changes at 2 cm/min .

$$\frac{dr}{dt} = 2 \text{ cm/min}$$

we want to know how fast its area changes
When $r = 5\text{ cm}$.

$$A = \pi r^2$$

$$\frac{dA}{dt} = \frac{d}{dt}[\pi r^2]$$

$$\frac{dA}{dt} = \pi \frac{d}{dt}[r^2]$$

$$\frac{dA}{dt} = \pi \cdot 2r \cdot \frac{dr}{dt}$$

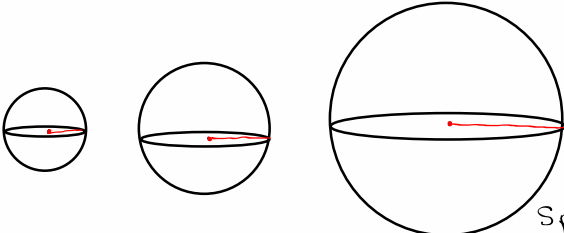
$$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$$

when $r = 5$

$$\frac{dA}{dt} = 2\pi(5) \cdot 2$$

$$= 20\pi \text{ cm}^2/\text{min}$$

Oct 23-10:28 AM



Sphere

Suppose $\frac{dr}{dt} = 4 \text{ in/min.}$ $\rightarrow V = \frac{4\pi r^3}{3}$

How fast its volume changes when $r = 5 \text{ in.}$?

$V = \frac{4\pi r^3}{3}$

$\frac{dV}{dt} = \frac{4\pi}{3} \cdot \frac{d}{dt}[r^3]$

$\frac{dV}{dt} = \frac{4\pi}{3} \cdot \cancel{3} r^2 \cdot \frac{dr}{dt}$

$= 4\pi r^2 \frac{dr}{dt}$

When $r = 5 \text{ in.}$

$\frac{dV}{dt} = 4\pi(5)^2 \cdot 4$

$= 400\pi \text{ in}^3/\text{min.}$

Oct 23-10:35 AM

Two cars leave an intersection at the same time. Assume roads are \perp to each other.

one going east at 30 miles/hr.

other one going north at 40 miles/hr.

How fast the distance between them changes after 1 hr?

using Pythagorean thm

$z^2 = x^2 + y^2$

$\cancel{2z} \frac{dz}{dt} = \cancel{2x} \frac{dx}{dt} + \cancel{2y} \frac{dy}{dt}$

$\frac{dz}{dt} = ?$

after 1 hr

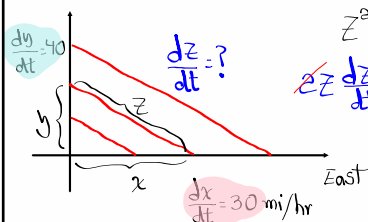
$x = 30, y = 40$

$z^2 = x^2 + y^2$

$z^2 = 30^2 + 40^2$

$z = 50$

$\frac{dz}{dt} = \frac{900 + 1600}{50} = \frac{2500}{50} = 50 \text{ mi/hr}$



Oct 23-10:41 AM

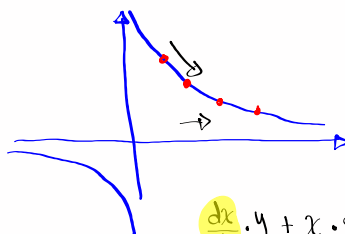
An object is moving along the curve $xy=4$

How fast is y changing when x increases

at 5 units/sec. at the point $(4,1)$?

$$xy=4 \rightarrow y=\frac{4}{x} \quad \text{Is } (4,1) \text{ on the graph? Yes}$$

$$1 = \frac{4}{4} \checkmark$$



$$xy=4$$

$$\frac{d}{dt}(xy) = \frac{d}{dt}(4)$$

$$\frac{dx}{dt} \cdot y + x \cdot \frac{dy}{dt} = 0$$

$$\text{at } (4,1) \quad 5 \cdot 1 + 4 \cdot \frac{dy}{dt} = 0$$

$$4 \frac{dy}{dt} = -5 \quad \frac{dy}{dt} = -\frac{5}{4}$$

$$= -1.25$$

as x increases, y decreases.

units/sec.

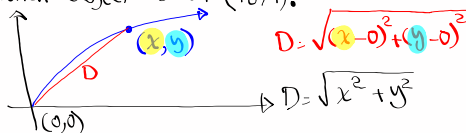
Oct 23-10:53 AM

An object is moving along the curve $y=\sqrt{x}$

suppose $\frac{dx}{dt} = 4$ units/min., $\frac{dy}{dt} = 3$ units/min.

How fast its distance from the origin changes

when object is at $(16,4)$?



$$D = \sqrt{(x-0)^2 + (y-0)^2}$$

$$D = \sqrt{x^2 + y^2}$$

$$D^2 = x^2 + y^2$$

$$2D \frac{dD}{dt} = 2x \frac{dx}{dt} + 2y \frac{dy}{dt}$$

at $(16,4)$

$$D = \sqrt{x^2 + y^2}$$

$$\sqrt{272} \frac{dD}{dt} = 16 \cdot 4 + 4 \cdot 3$$

$$= \sqrt{16^2 + 4^2}$$

$$\frac{dD}{dt} = \frac{64 + 12}{\sqrt{272}} = \frac{76}{\sqrt{272}}$$

$$= \sqrt{272}$$

$$= \frac{76}{4\sqrt{17}} = \frac{19}{\sqrt{17}}$$

$$= \sqrt{\frac{19 \cdot 19}{17}} \text{ units/min.}$$

Oct 23-11:02 AM

$$f(x) = x^3 - 3x^2 + 1$$

$$f(0) = 1$$

$$f'(x) = 3x^2 - 6x$$

$$f''(x) = 6x - 6$$

Find **points** where $f'(x) \hat{=}$ $f''(x)$ are equal to 0.

$$f'(x) = 0 \rightarrow 3x^2 - 6x = 0 \rightarrow 3x(x-2) = 0$$

$$f(0) = 0^3 - 3(0)^2 + 1 = 1 \rightarrow (0, 1) \quad \downarrow \quad \downarrow$$

$$f(2) = 2^3 - 3(2)^2 + 1 = 8 - 12 + 1 = -3 \rightarrow (2, -3)$$

$$f''(x) = 0 \rightarrow 6x - 6 = 0 \rightarrow x = 1$$

$$f(1) = 1^3 - 3(1)^2 + 1 = -1 \rightarrow (1, -1)$$

Oct 23-11:12 AM

$$f(x) = \frac{x}{x-1} \quad \text{Domain: All Reals except } 1$$

$$x-1 \neq 0 \quad x \neq 1$$

$$(-\infty, 1) \cup (1, \infty)$$

$$f(2) = \frac{2}{2-1} = \frac{2}{1} = 2 \rightarrow (2, 2)$$

$$f'(x) = \frac{1(x-1) - x(1)}{(x-1)^2} = \frac{-1}{(x-1)^2} \quad \begin{matrix} f'(x) \neq 0 \\ f'(x) \text{ is} \\ \text{undefined at} \\ x=1 \end{matrix}$$

$$f'(x) = -1(x-1)^{-2}$$

$$f''(x) = -1 \cdot 2(x-1)^{-3} \cdot 1 \quad f''(x) = \frac{2}{(x-1)^3}$$

$$f''(x) \neq 0$$

$f''(x)$ is undefined
at $x=1$.

Oct 23-11:18 AM

$$f(x) = x + \cos x \quad \text{on } \underline{\underline{[0, 2\pi]}}$$

$$f(0) = 0 + \cos 0 = 1$$

$$f'(x) = 1 - \sin x$$

$$f'(x) = 0$$

$$1 - \sin x = 0$$

$$\sin x = 1 \rightarrow x = \frac{\pi}{2}$$

$$f''(x) = -\cos x$$

$$f''(x) = 0$$

$$-\cos x = 0$$

$$\cos x = 0 \rightarrow x = \frac{\pi}{2}, \frac{3\pi}{2}$$

Oct 23-11:28 AM