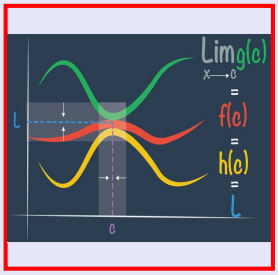
Math 261

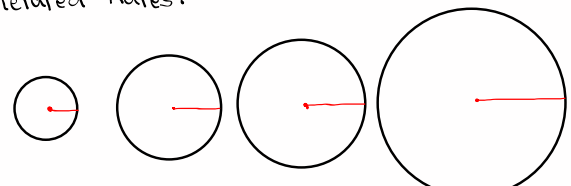
Fall 2023

Lecture 30



Feb 19-8:47 AM

Related Rates:



Suppose its radius changes at 2cm/min.

$\frac{dr}{dt} = 2$ cm/min.

we want to know how fast its area changes when $r = 5$cm?

$A = \pi r^2$

$\frac{dA}{dt} = \frac{d}{dt}[\pi r^2]$

$\frac{dA}{dt} = \pi \frac{d}{dt}[r^2]$

$\frac{dA}{dt} = \pi \cdot 2r \cdot \frac{dr}{dt}$

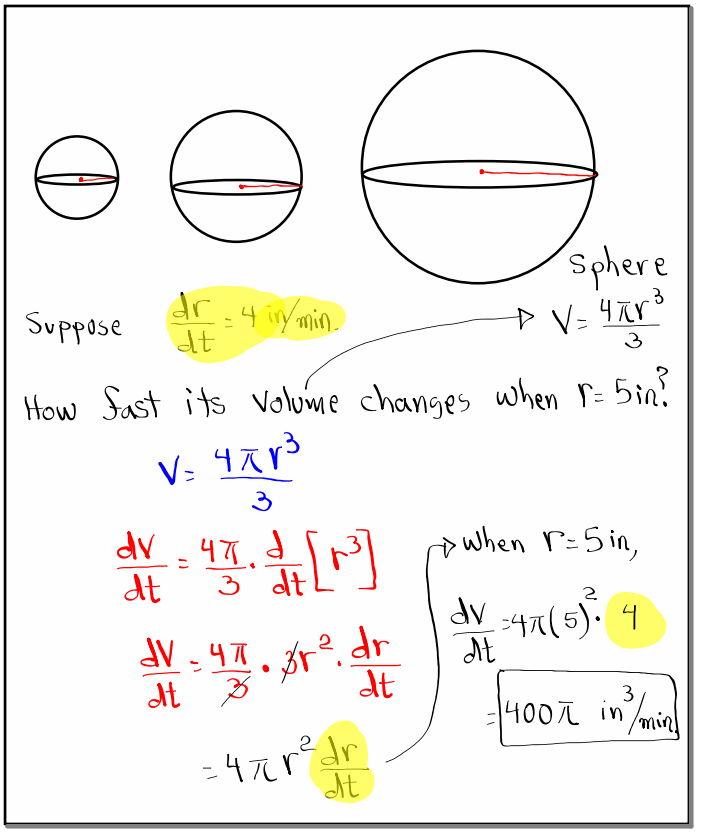
$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

when $r = 5$

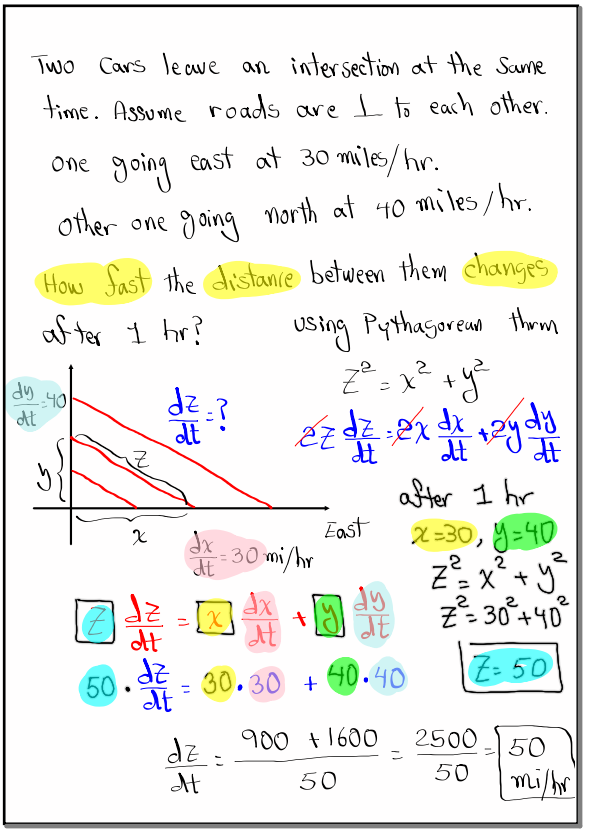
$\frac{dA}{dt} = 2\pi(5) \cdot 2$

$= 20\pi$ cm^2/min

Oct 23-10:28 AM



Oct 23-10:35 AM



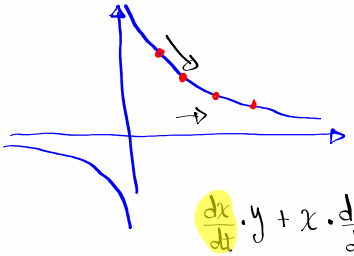
Oct 23-10:41 AM

An object is moving along the curve $xy=4$

How fast is y changing when x increases at 5 units/sec. at the point $(4,1)$?

$xy=4 \rightarrow y=\frac{4}{x}$

Is $(4,1)$ on the graph? Yes $1=\frac{4}{4}$ ✓



$xy=4$

$\frac{d}{dt}(xy)=\frac{d}{dt}(4)$

$\frac{dx}{dt}\cdot y + x\cdot\frac{dy}{dt}=0$

at $(4,1)$ $\quad 5\cdot 1+4\cdot\frac{dy}{dt}=0$

$4\frac{dy}{dt}=-5 \qquad \frac{dy}{dt}=-\frac{5}{4}$

$=-1.25$ units/sec.

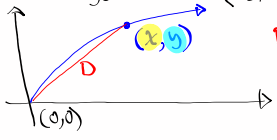
as x increases, y decreases.

Oct 23-10:53 AM

An object is moving along the curve $y=\sqrt{x}$

Suppose $\frac{dx}{dt}=4$ units/min., $\frac{dy}{dt}=3$ units/min.

How fast its distance from the origin changes when object is at $(16,4)$?



$D=\sqrt{(x-0)^2+(y-0)^2}$

$D=\sqrt{x^2+y^2}$

$D^2=x^2+y^2$

$2D\frac{dD}{dt}=2x\frac{dx}{dt}+2y\frac{dy}{dt}$

at $(16,4)$

$D=\sqrt{x^2+y^2}$

$=\sqrt{16^2+4^2}$

$=\sqrt{272}$

$\sqrt{272}\frac{dD}{dt}=16\cdot 4+4\cdot 3$

$\frac{dD}{dt}=\frac{64+12}{\sqrt{272}}=\frac{76}{\sqrt{272}}$

$=\frac{76}{4\sqrt{17}}=\frac{19}{\sqrt{17}}$

$=\frac{19\sqrt{17}}{17}$ units/min.

Oct 23-11:02 AM

$f(x) = x^3 - 3x^2 + 1$

$f(0) = 1$ $\quad f'(x) = 3x^2 - 6x$

$f''(x) = 6x - 6$

Find points where $f'(x)$ & $f''(x)$ are equal to 0.

$F'(x) = 0 \rightarrow 3x^2 - 6x = 0 \rightarrow 3x(x-2) = 0$

$x = 0$ $\quad x = 2$

$f(0) = 0^3 - 3(0)^2 + 1 = 1 \rightarrow (0,1)$

$f(2) = 2^3 - 3(2)^2 + 1 = 8 - 12 + 1 = -3 \rightarrow (2,-3)$

$F''(x) = 0 \rightarrow 6x - 6 = 0 \rightarrow x = 1$

$f(1) = 1^3 - 3(1)^2 + 1 = -1 \rightarrow (1,-1)$

Oct 23-11:12 AM

$f(x) = \frac{x}{x-1}$ Domain: All Reals except 1

$x - 1 \neq 0 \quad x \neq 1$

$(-\infty, 1) \cup (1, \infty)$

$f(2) = \frac{2}{2-1} = \frac{2}{1} = 2 \rightarrow (2,2)$

$f'(x) = \frac{1(x-1) - x(1)}{(x-1)^2} = \frac{-1}{(x-1)^2}$

$f'(x) \neq 0$

$f'(x)$ is undefined at $x = 1$

$f'(x) = -1(x-1)^{-2}$

$f''(x) = -1 \cdot -2(x-1)^{-3} \cdot 1$ $\quad f''(x) = \frac{2}{(x-1)^3}$

$f''(x) \neq 0$

$f''(x)$ is undefined at $x = 1$.

Oct 23-11:18 AM

$f(x) = x + \cos x$ on $[0, 2\pi]$

$f(0) = 0 + \cos 0 = 1$

$f'(x) = 1 - \sin x$

$f'(x) = 0$

$1 - \sin x = 0$

$\sin x = 1 \rightarrow x = \frac{\pi}{2}$

$f''(x) = -\cos x$

$f''(x) = 0$

$-\cos x = 0$

$\cos x = 0 \rightarrow x = \frac{\pi}{2}, \frac{3\pi}{2}$

Oct 23-11:28 AM