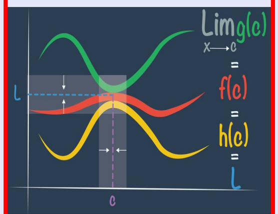
Math 261

Fall 2023

Lecture 29



Feb 19-8:47 AM

Use linear approximation to estimate $\tan 65^\circ$ $\to f(x) \approx f(a) + f'(a)(x-a)$ near $x = a$.

$\tan 65^\circ \approx \tan 60^\circ = \sqrt{3}$

$f(x) = \tan x$ $\qquad f(60^\circ) = \tan 60^\circ = \sqrt{3}$

$a = 60^\circ$ $\qquad f'(60^\circ) = \sec^2 60^\circ$

$f'(x) = \sec^2 x$ $\qquad = [\sec 60^\circ]^2 = 2^2 = 4$

$\tan x \approx \sqrt{3} + 4(x - 60^\circ)$

near 60°

$\tan 65^\circ \approx \sqrt{3} + 4(65^\circ - 60^\circ)$

$\approx \sqrt{3} + 4 \cdot 5^\circ$

$= \sqrt{3} + 4 \cdot 5 \cdot \frac{\pi}{180} = \sqrt{3} + \frac{\pi}{9}$

$= \boxed{2.081}$

≈ 2.1

Now use your calc to find $\tan 65^\circ \approx \boxed{2.145}$

≈ 2.1

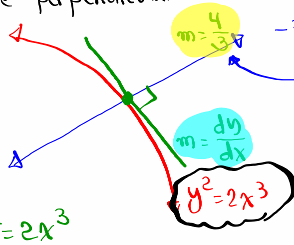
Oct 18-10:26 AM

SG 11, # 6

At what points are tangent to the curve $y^2=2x^3$ are perpendicular to $4x-3y+1=0$?

$-3y=-4x-1$

$y=\boxed{\frac{4}{3}}x+\frac{1}{3}$



Increasing line

Product of slopes must be -1

$y^2=2x^3$

$\cancel{2}y\frac{dy}{dx}=\cancel{2}\cdot 3x^2$

$\frac{dy}{dx}=\frac{3x^2}{y}$

$\frac{\cancel{3}x^2}{y}\cdot\frac{4}{\cancel{3}}=-1$

$\frac{4x^2}{y}=-1 \Rightarrow y=-4x^2$

$y^2=2x^3$

$(-4x^2)^2=2x^3$

$16x^4-2x^3=0$

$\to 8x^4-x^3=0$

Solve for x,
Plug them in into equation of the curve, and find Y-value.

Points $\to$ (,)

Oct 18-10:34 AM

Given $x\sin y - y\cos x = x^2+y^2$

Find $\frac{dy}{dx}$

$\frac{d}{dx}\left[x\sin y\right]-\frac{d}{dx}\left[y\cos x\right]=\frac{d}{dx}\left[x^2\right]+\frac{d}{dx}\left[y^2\right]$

1. $\sin y + x\cdot\cos y\cdot\frac{dy}{dx}-\left[\frac{dy}{dx}\cdot\cos x + y\cdot -\sin x\right]=2x+2y\cdot\frac{dy}{dx}$

$\sin y + x\cos y\frac{dy}{dx}-\cos x\frac{dy}{dx}+y\sin x=2x+2y\frac{dy}{dx}$

$\left[x\cos y-\cos x-2y\right]\frac{dy}{dx}=2x-\sin y-y\sin x$

$$\boxed{\frac{dy}{dx}=\frac{2x-\sin y-y\sin x}{x\cos y-\cos x-2y}}$$

Oct 18-10:44 AM

Suppose $x = f(t)$, $y = g(t)$

$$x^2 + y^2 = 100$$

Take derivative with respect to t.

$$\frac{d}{dt}[x^2] + \frac{d}{dt}[y^2] = \frac{d}{dt}[100]$$

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

If $x = 6$, $y = 8$, and $\frac{dx}{dt} = -3$, find $\frac{dy}{dt}$

$$2\cdot 6\cdot(-3) + 2\cdot 8\cdot\frac{dy}{dt} = 0$$

$$-36 + 16\frac{dy}{dt} = 0 \rightarrow \frac{dy}{dt} = \frac{36}{16} = \boxed{\frac{9}{4}}$$

Oct 18-10:53 AM

Suppose $x = x(t)$, $y = y(t)$, and $z = z(t)$

we have $x^2 + y^2 = z^2$

$3^2 + 4^2 = z^2 \rightarrow z^2 = 25$

$z = \pm 5$

At $x = 3$, and $y = 4$, $\frac{dx}{dt} = 5$, $\frac{dy}{dt} = -2$,

Find $\frac{dz}{dt}$.

Take derivative of both sides of $x^2 + y^2 = z^2$ with respect to t.

$$\frac{d}{dt}[x^2] + \frac{d}{dt}[y^2] = \frac{d}{dt}[z^2]$$

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2z\frac{dz}{dt}$$

$$x\frac{dx}{dt} + y\frac{dy}{dt} = z\frac{dz}{dt}$$

$$3\cdot 5 + 4\cdot(-2) = \pm 5\frac{dz}{dt}$$

$$15 - 8 = \pm 5\frac{dz}{dt}$$

$$7 = \pm 5\frac{dz}{dt}$$

$$\frac{dz}{dt} = \pm\frac{7}{5}$$

Oct 18-10:59 AM

Suppose $A = A(t)$, $r = r(t)$ and $A = \pi r^2$

Find $\frac{dA}{dt}$ when $\boxed{r = 5}$ and $\frac{dr}{dt} = \boxed{4}$

Area of Circle

Exact Answer

$$\frac{d}{dt}[A] = \frac{d}{dt}[\pi r^2]$$

$$\frac{dA}{dt} = \pi \cdot \frac{d}{dt}[r^2]$$

$$\frac{dA}{dt} = \pi \cdot 2r \cdot \frac{dr}{dt} = 2\pi r \frac{dr}{dt}$$

$$\frac{dA}{dt} = 2\pi(5)(4) = \boxed{40\pi}$$

Oct 18-11:08 AM

Given $V = \frac{4\pi r^3}{3}$, $r = r(t)$, $V = V(t)$

Volume of Sphere

Find $\frac{dV}{dt}$ when $r = 5$, and $\frac{dr}{dt} = -\frac{1}{\pi}$

Exact Ans. Only.

$$\frac{d}{dt}[V] = \frac{d}{dt}\left[\frac{4\pi r^3}{3}\right]$$

$$\frac{dV}{dt} = \frac{4\pi}{3} \cdot \frac{d}{dt}[r^3] = \frac{4\pi}{\cancel{3}} \cdot \cancel{3}r^2 \cdot \frac{dr}{dt}$$

$$\frac{dV}{dt} = 4\cancel{\pi}(5)^2 \cdot \left(\frac{-1}{\cancel{\pi}}\right)$$

$$= \boxed{-100}$$

Oct 18-11:12 AM

Given $V = \pi r^2 h$, $V = V(t)$, $r = r(t)$, and $h = h(t)$

Find $\frac{dV}{dt}$ when $r = 4$, $h = 8$, $\frac{dr}{dt} = \frac{5}{\pi}$, $\frac{dh}{dt} = \frac{-1}{\pi}$

$$\frac{dV}{dt} = \frac{d}{dt}\left[\pi r^2 h\right]$$

$$= \pi \frac{d}{dt}\left[r^2 h\right] = \pi\left[2r\frac{dr}{dt}\cdot h + r^2 \cdot \frac{dh}{dt}\right]$$

$$\frac{dV}{dt} = \pi\left[2\cdot 4\cdot \frac{5}{\cancel{\pi}}\cdot 8 + 4^2\cdot \frac{-1}{\cancel{\pi}}\right]$$

$$= 2\cdot 4\cdot 5\cdot 8 + 4^2\cdot(-1)$$

$$= 320 - 16 = \boxed{304}$$

Oct 18-11:22 AM