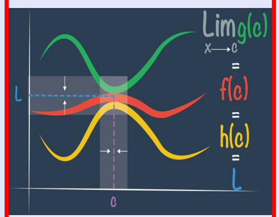
Math 261

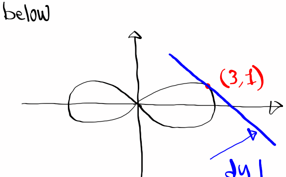
Fall 2023

Lecture 28



Feb 19-8:47 AM

Graph of $8(x^2+y^2)^2 = 100(x^2-y^2)$ is given below



$m = \left.\frac{dy}{dx}\right|_{(3,1)}$

verify that (3,1) is on the graph.

$8(3^2+1^2)^2 = 100(3^2-1^2)$

$8 \cdot 10^2 = 100 \cdot 8$

$800 = 800$

$8(x^2+y^2)^2 = 100(x^2-y^2)$

Take derivative of both sides

$8 \cdot 2(x^2+y^2)^1 \cdot \left(2x + 2y \cdot \frac{dy}{dx}\right) = 100\left(2x - 2y \cdot \frac{dy}{dx}\right)$

at $(3,1) \to \left.\frac{dy}{dx}\right|_{(3,1)} = m_{\text{tan. line}}$

$16(\underbrace{3^2+1^2}_{10}) \cdot (6 + 2m) = 100(6 - 2m)$

$\overset{8}{\cancel{160}}(6+2m) = \overset{5}{\cancel{100}}(6-2m)$

$48 + 16m = 30 - 10m$

$26m = -18 \qquad m = \frac{-18}{26} = \frac{-9}{13}$

$y - 1 = -\frac{9}{13}(x-3)$

$13y - 13 = -9(x-3)$

$13y - 13 = -9x + 27$

$\boxed{9x + 13y = 40}$

Standard form.

Oct 17-10:25 AM

Find $\frac{d^2y}{dx^2}$ Given $x^3+y^3=1$

1) Find $\frac{dy}{dx}$

$$3x^2+3y^2\cdot\frac{dy}{dx}=0$$

$$y^2\frac{dy}{dx}=-x^2$$

$$\frac{dy}{dx}=\boxed{\frac{-x^2}{y^2}}$$

2) Find $\frac{d^2y}{dx^2}$

$$\frac{d^2y}{dx^2}=\frac{d}{dx}\left[\frac{dy}{dx}\right]=\frac{d}{dx}\left[\frac{-x^2}{y^2}\right]=-\frac{d}{dx}\left[\frac{x^2}{y^2}\right]$$

$$=-\frac{2x\cdot y^2-x^2\cdot 2y\cdot\frac{dy}{dx}}{(y^2)^2}$$

$$=-\frac{2xy^2-2x^2y\cdot\frac{-x^2}{y^2}}{y^4}$$

LCD $=y^2$

$$=-\frac{2xy^4+2x^4y}{y^6}=-\frac{2xy(\overbrace{y^3+x^3}^{1})}{y^{\cancel{6}\,5}}$$

$$\left(2x^2y\cdot\frac{x^2}{\cancel{y^2}}\right)\cdot\cancel{y^2}=2x^4y$$

$$=\boxed{-\frac{2x}{y^5}}$$

Oct 17-10:37 AM

use linear approximation to estimate $\frac{1}{(2.01)^2+1}$.

$f(x)\approx f(a)+f'(a)(x-a)$

Near 2

$f(x)\approx f(2)+f'(2)(x-2)$

$$\frac{1}{(2.01)^2+1}\approx\frac{1}{2^2+1}=\frac{1}{5}=.2$$

$$\frac{1}{x^2+1}\approx\frac{1}{5}-\frac{4}{25}(x-2)$$

$f(x)=\frac{1}{x^2+1}$

$a=2$

$F(x)=(x^2+1)^{-1}$

$F'(x)=-1(x^2+1)^{-2}\cdot 2x$

$f'(x)=\frac{-2x}{(x^2+1)^2}$

$F(2)=\frac{1}{2^2+1}=\frac{1}{5}$

$F'(2)=\frac{-2(2)}{(2^2+1)^2}=\frac{-4}{5^2}=\frac{-4}{25}$

$$\frac{1}{x^2+1}\approx\frac{1}{5}-\frac{4}{25}(x-2)$$

for $x=2.01$

$$\frac{1}{(2.01)^2+1}\approx\frac{1}{5}-\frac{4}{25}(2.01-2)$$

$$=\frac{1}{5}-\frac{4}{25}(.01)$$

$$=\frac{1}{5}-\frac{\cancel{4}}{25}\cdot\frac{1}{\cancel{100}_{25}}$$

$$=\frac{1}{5}-\frac{1}{625}$$

$$=\frac{125}{625}-\frac{1}{625}$$

$$=\frac{124}{625}\approx\boxed{.1984}$$

Use your calc. to find

$$\frac{1}{2.01^2+1}\approx\boxed{.1984}08$$

Oct 17-10:48 AM

Use linear approximation to estimate $\sqrt[3]{66}$

$\to f(x) \approx f(a) + f'(a)(x-a)$

$\sqrt[3]{66} \approx \sqrt[3]{64} = 4$

$\sqrt[3]{x} \approx f(64) + f'(64)(x-64)$

$= 4 + \frac{1}{48}(x-64)$

$f(x) = \sqrt[3]{x}$ $\quad f(64) = \sqrt[3]{64} = 4$

$a = 64$ $\quad f'(x) = \frac{1}{3}x^{-2/3} = \frac{1}{3\sqrt[3]{x^2}}$

$f'(64) = \frac{1}{3\cdot\sqrt[3]{64^2}} = \frac{1}{3\cdot 16} = \frac{1}{48}$

Near $a=64$

$\sqrt[3]{x} \approx 4 + \frac{1}{48}(x-64)$

Now let $x = 66$

$\sqrt[3]{66} \approx 4 + \frac{1}{48}(66-64) = 4 + \frac{2}{48}$

$= 4 + \frac{1}{24}$

$= \frac{97}{24}$

$= \boxed{4.042}$

Now use your calc to find $\sqrt[3]{66} \approx \boxed{4.0412}$

Oct 17-10:58 AM

Use linear approximation to estimate $\sqrt{80.9}$

$\to f(x) \approx f(a) + f'(a)(x-a)$

Near a

Ans in reduced fraction.

$\sqrt{x} \approx 9 + \frac{1}{18}(x - 81)$

$\sqrt{80.9} \approx \sqrt{81} = 9$

For $x = 80.9$

$f(x) = \sqrt{x}$ $\quad f(81) = \sqrt{81} = 9$

$\sqrt{80.9} \approx 9 + \frac{1}{18}(80.9 - 81)$

$a = 81$

$f'(81) = \frac{1}{2\sqrt{81}} = \frac{1}{2\cdot 9} = \frac{1}{18}$

$\approx 9 + \frac{1}{18}(-.1)$

$f'(x) = \frac{1}{2\sqrt{x}}$

$= 9 - \frac{1}{180}$

Use your calc. to find

$= \frac{1619}{180}$

$\sqrt{80.9} = \boxed{8.994}$

$= \boxed{8.994}$

Oct 17-11:10 AM

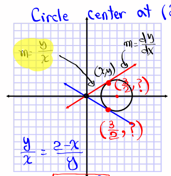
Find eqn for two tan. lines that contain the origin and are tan. to the graph of $x^2-4x+y^2+3=0$.

$$\underbrace{x^2-4x+4}+y^2+3=4$$

$$(x-2)^2+(y-0)^2=4-3$$

$$(x-2)^2+(y-0)^2=1$$

Circle center at $(2,0)$ with radius 1



$$\frac{y}{x}=\frac{2-x}{y}$$

$$y^2=\boxed{2x-x^2}$$

$$x^2-4x+\boxed{y^2}+3=0$$

$$\cancel{x^2}-4x+2x-\cancel{x^2}+3=0$$

$$-2x+3=0$$

$$x=\frac{3}{2}$$

Tangent points

$$\left(\frac{3}{2},\frac{\sqrt{3}}{2}\right),\ \left(\frac{3}{2},\frac{-\sqrt{3}}{2}\right)$$

$$m=\frac{\frac{\sqrt{3}}{2}}{\frac{3}{2}}$$

$$m=\frac{\sqrt{3}}{3} \qquad m=\frac{-\sqrt{3}}{3}$$

Make Sure to review this Problem.

$$x^2-4x+y^2+3=0$$

$$2x-4+2y\frac{dy}{dx}=0$$

$$\frac{dy}{dx}=\frac{4-2x}{2y}$$

$$\frac{dy}{dx}=\frac{2-x}{y}$$

$$x^2-4x+y^2+3=0$$

$$\frac{9}{4}-\frac{\cancel{12}^{6}}{\cancel{2}}+y^2+3=0$$

$$\frac{9}{4}-3+y^2=0$$

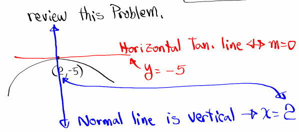
$$y^2=3-\frac{9}{4}=\frac{3}{4}$$

$$y^2=\frac{3}{4}$$

$$\boxed{y=\pm\frac{\sqrt{3}}{2}}$$

$$y-0=\pm\frac{\sqrt{3}}{3}(x-0)$$

$$\boxed{y=\pm\frac{\sqrt{3}}{3}x}$$



Oct 17-11:21 AM