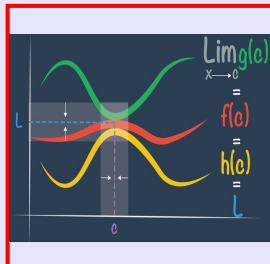


**Math 261**  
**Fall 2023**  
**Lecture 27**



Feb 19-8:47 AM

Class QZ 12

find  $f'(x)$ 

Do not Simplify

$$1) f(x) = (x^4 - 4x^2)^3$$

$$f'(x) = 3(x^4 - 4x^2)^2 \cdot (4x^3 - 8x)$$

$$2) f(x) = \cos(\sqrt{x})$$

$$f'(x) = -\sin(\sqrt{x}) \cdot \frac{1}{2\sqrt{x}}$$

$$3) x^4 - y^2 = 8x$$

$$\text{where } y = f(x)$$

$$4x^3 - 2y \frac{dy}{dx} = 8$$

$$4x^3 - 8 = 2y \frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{4x^3 - 8}{2y}$$

Oct 12-11:14 AM

Find  $\frac{dy}{dx}$  where  $y = f(x)$

$$x^2 y + x y^2 = x y$$

$$\frac{d}{dx}[x^2 y] + \frac{d}{dx}[x y^2] = \frac{d}{dx}[x y]$$

$$2xy + x^2 \frac{dy}{dx} + 1 \cdot y^2 + x \cdot 2y \cdot \frac{dy}{dx} = 1 \cdot y + x \cdot \frac{dy}{dx}$$

$$x^2 \frac{dy}{dx} + 2xy \frac{dy}{dx} - x \frac{dy}{dx} = y - y^2 - 2xy$$

$$(x^2 + 2xy - x) \frac{dy}{dx} = y - y^2 - 2xy$$

$$\frac{dy}{dx} = \frac{y - y^2 - 2xy}{x^2 + 2xy - x} = \frac{-y(y + 2x - 1)}{x(x + 2y - 1)}$$

$$x(x + 2y - 1) \neq 0$$

$$x \neq 0, \quad x + 2y \neq 1$$

Oct 16-10:32 AM

Find  $\frac{dy}{dx}$  where  $y = f(x)$

$$x^2 y + x y^2 = x y$$

$$xy(y + x) = xy$$

$$y + x = 1$$

$$\frac{dy}{dx} + 1 = 0 \rightarrow \frac{dy}{dx} = -1$$

$$xy \neq 0 \rightarrow x \neq 0, y \neq 0$$

Oct 16-10:40 AM

Find  $\frac{dy}{dx}$  where  $y = f(x)$  for  $x = \sin y$

$$x = \sin y$$

$$\frac{d}{dx}[x] = \frac{d}{dx}[\sin y]$$

$$1 = \cos y \cdot \frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{1}{\cos y} = \boxed{\sec y}$$

Oct 16-10:43 AM

$$\frac{d}{dx}[(f(x^3))^2]$$

$$= 2(f(x^3))^{2-1} \cdot f'(x^3) \cdot 3x^2$$

$$= \boxed{6x^2 \cdot f(x^3) \cdot f'(x^3)}$$

Oct 16-10:46 AM

Given  $\frac{xy}{x^2+y^2} = \frac{1}{2}$  Find  $\frac{dy}{dx} \Big|_{(1,1)}$

Is  $(1,1)$  on the graph? Yes  
 $\frac{1 \cdot 1}{1^2 + 1^2} = \frac{1}{1+1} = \frac{1}{2} \checkmark$

$m = \frac{dy}{dx} \Big|_{(1,1)} = 1$

$\frac{xy}{x^2+y^2} = \frac{1}{2}$  cross-multiply  $x^2 + y^2 = 2xy$

$x^2 + y^2 - 2xy = 0$

$(x-y)^2 = 0$

$x - y = 0$

$y = x$

$\frac{dy}{dx} = 1$

$\cancel{2x} + \cancel{2y} \frac{dy}{dx} = \cancel{2}(1 \cdot y + x \cdot \frac{dy}{dx})$

$x + y \frac{dy}{dx} = y + x \frac{dy}{dx}$

$y \frac{dy}{dx} - x \frac{dy}{dx} = y - x$

$(y-x) \frac{dy}{dx} = y - x$

$\frac{dy}{dx} = \frac{y-x}{y-x} = 1$

Oct 16-10:48 AM

Linear Approximation

Estimate  $(2.1)^4 \approx 2^4 = 16$

very close to 2.1

$f(x) = x^4$

$a = 2$

$f(2) = 2^4 = 16$

$f'(x) = 4x^3$

$f'(2) = 4(2)^3 = 32$

use your calc to find

$2.1^4 \approx 19.4481$

$f(x) \approx f(a) + f'(a)(x-a)$

Near  $x=2$

$f(x) \approx f(a) + f'(a)(x-a)$

$x^4 \approx 16 + 32(x-2)$

Suppose  $x=2.1$

$2.1^4 \approx 16 + 32(2.1-2)$

$2.1^4 \approx 16 + 32(.1)$

$\approx 16 + 3.2$

$\approx 19.2$

close to each other

Oct 16-10:57 AM

Use linear approximation to

estimate  $\sqrt{26}$

$$f(x) \approx f(a) + f'(a)(x-a)$$

$$\sqrt{26} \approx \sqrt{25} \approx 5$$

$$\sqrt{x} \approx 5 + \frac{1}{10}(x-25)$$

Near  $x=25$

$$f(x) = \sqrt{x} \quad f(25) = \sqrt{25} = 5$$

$$a = 25$$

$$f'(x) = \frac{1}{2\sqrt{x}}$$

$$\sqrt{26} \approx 5 + \frac{1}{10}(26-25)$$

$$f'(25) = \frac{1}{2\sqrt{25}} = \frac{1}{10}$$

$$\approx 5 + \frac{1}{10} \cdot (1)$$

Use Your Calc

$$\sqrt{26} \approx 5.0990 \dots$$

$$\sqrt{26} \approx 5.1$$

Oct 16-11:04 AM

Use linear approximation to

estimate  $\sin 31^\circ$

$$f(x) \approx f(a) + f'(a)(x-a)$$

$$\sin 31^\circ \approx \sin 30^\circ = \frac{1}{2}$$

$$\sin x \approx \frac{1}{2} + \frac{\sqrt{3}}{2}(x-30^\circ)$$

Near  $30^\circ$

$$f(x) = \sin x$$

$$a = 30^\circ$$

$$f(30^\circ) = \sin 30^\circ = \frac{1}{2}$$

$$f'(x) = \cos x$$

$$f'(30^\circ) = \cos 30^\circ = \frac{\sqrt{3}}{2}$$

$$180^\circ = \pi \text{ Rad}$$

$$1^\circ = \frac{\pi}{180}$$

$$\sin 31^\circ \approx \frac{1}{2} + \frac{\sqrt{3}}{2}(31^\circ - 30^\circ)$$

$$= \frac{1}{2} + \frac{\sqrt{3}}{2}(1^\circ) = \frac{1}{2} + \frac{\sqrt{3}}{2}\left(\frac{\pi}{180}\right) = \frac{1}{2} + \frac{\pi\sqrt{3}}{360}$$

$$\approx .515$$

use your calc.

$$\text{to find } \sin 31^\circ \approx .515$$

Oct 16-11:09 AM

Use linear approximation

to estimate  $\tan 46^\circ$ .

$$\tan 46^\circ \approx \tan 45^\circ = 1$$

$$f(x) = \tan x \quad f(45^\circ) = \tan 45^\circ = 1$$

$$a = 45^\circ \quad f'(45^\circ) = \sec^2 45^\circ = 2$$

$$f'(x) = \sec^2 x$$

$$f(x) \approx f(a) + f'(a)(x-a)$$

$$\tan x \approx 1 + 2(x - 45^\circ)$$

$$\tan 46^\circ \approx 1 + 2(46^\circ - 45^\circ)$$

$$\approx 1 + 2 \cdot 1^\circ$$

$$= 1 + 2 \cdot \frac{\pi}{180}$$

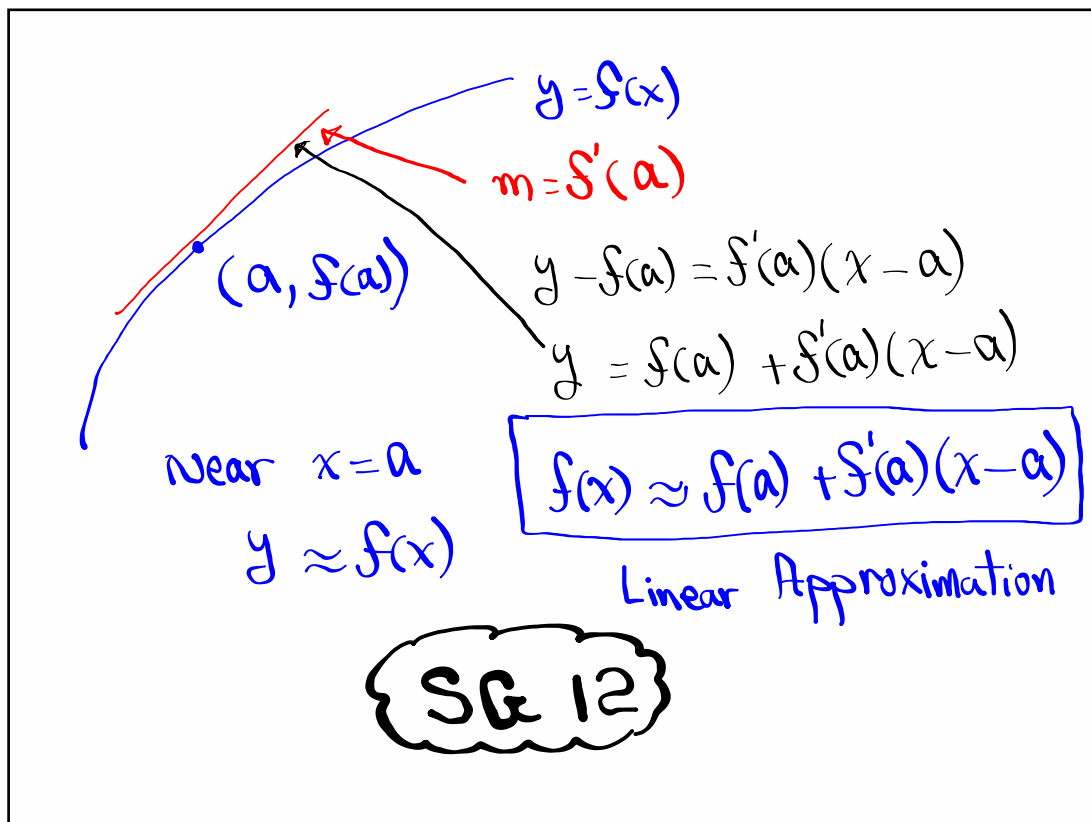
$$= 1 + \frac{\pi}{90} \approx 1.035$$

Now use your calc

to find

$$\tan 46^\circ \approx \boxed{1.036}$$

Oct 16-11:18 AM



Oct 16-11:29 AM