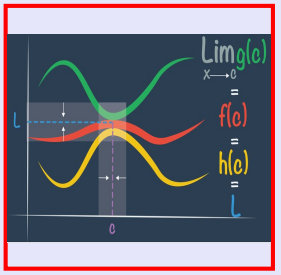
Math 261

Fall 2023

Lecture 27



Feb 19-8:47 AM

Class QZ 12

Find $f'(x)$ **Do not Simplify**

1) $f(x) = (x^4 - 4x^2)^3$ $\quad f'(x) = 3(x^4-4x^2)^2 \cdot (4x^3 - 8x)$

2) $f(x) = \cos(\sqrt{x})$ $\quad f'(x) = -\sin(\sqrt{x}) \cdot \frac{1}{2\sqrt{x}}$

3) $x^4 - y^2 = 8x$ where $y = f(x)$

$4x^3 - 2y\frac{dy}{dx} = 8$ $\quad 4x^3 - 8 = 2y\frac{dy}{dx}$ $\quad \frac{dy}{dx} = \frac{4x^3-8}{2y}$

Oct 12-11:14 AM

Find $\frac{dy}{dx}$ where $y = f(x)$

$x^2y + xy^2 = xy$

$\frac{d}{dx}[x^2y] + \frac{d}{dx}[xy^2] = \frac{d}{dx}[xy]$

$2xy + x^2\frac{dy}{dx} + 1\cdot y^2 + x\cdot 2y\cdot\frac{dy}{dx} = 1\cdot y + x\cdot\frac{dy}{dx}$

$x^2\frac{dy}{dx} + 2xy\frac{dy}{dx} - x\frac{dy}{dx} = y - y^2 - 2xy$

$(x^2 + 2xy - x)\frac{dy}{dx} = y - y^2 - 2xy$

$\frac{dy}{dx} = \frac{y - y^2 - 2xy}{x^2 + 2xy - x} = \frac{-y(y + 2x - 1)}{x(x + 2y - 1)}$

$x(x + 2y - 1) \neq 0$

$x \neq 0 \quad , \quad x + 2y \neq 1$

Oct 16-10:32 AM

Find $\frac{dy}{dx}$ where $y = f(x)$

$x^2y + xy^2 = xy$

$xy(y + x) = xy$

$xy \neq 0 \rightarrow x \neq 0, y \neq 0$

$y + x = 1$

$\frac{dy}{dx} + 1 = 0 \rightarrow \frac{dy}{dx} = -1$

Oct 16-10:40 AM

Find $\frac{dy}{dx}$ where $y = f(x)$ for $x = \sin y$

$x = \sin y$

$\frac{d}{dx}[x] = \frac{d}{dx}[\sin y]$

$1 = \cos y \cdot \frac{dy}{dx}$

$\frac{dy}{dx} = \frac{1}{\cos y} = \boxed{\sec y}$

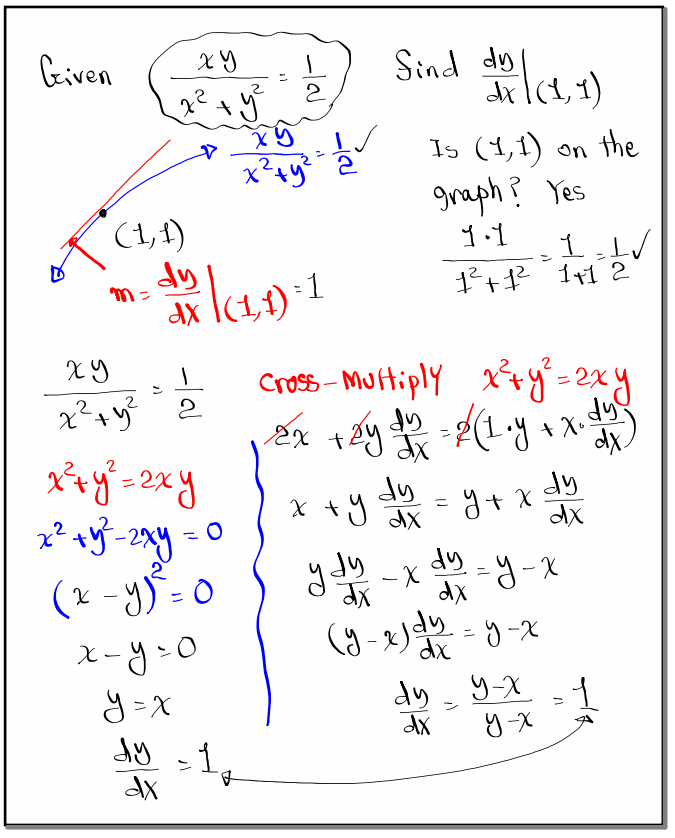
Oct 16-10:43 AM

$\frac{d}{dx}\left[(f(x^3))^2\right]$

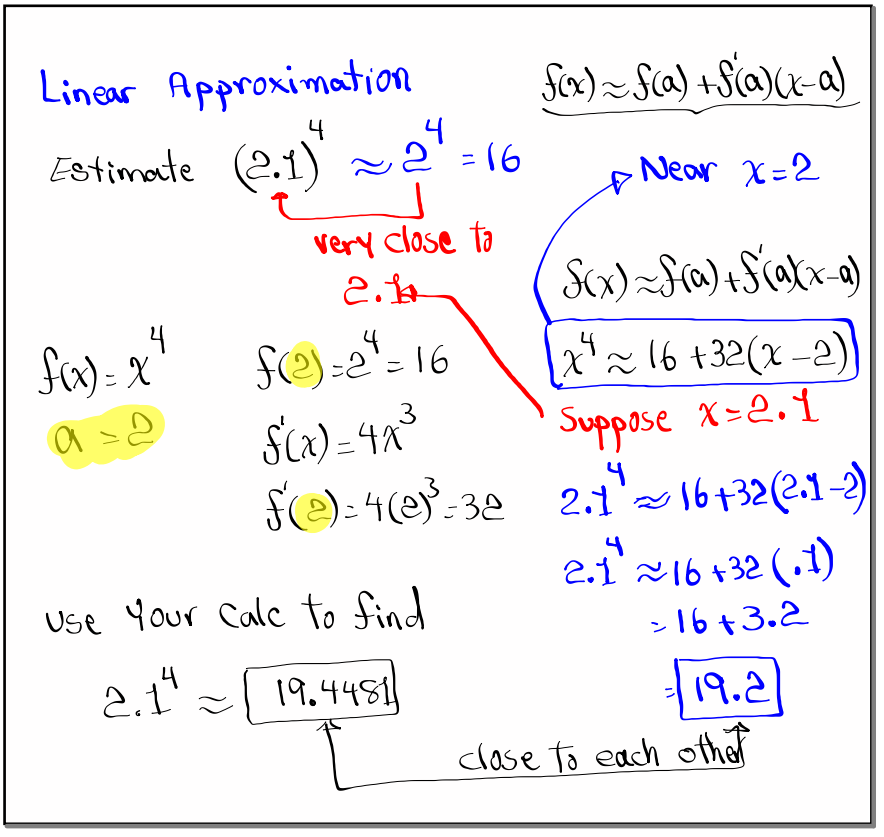
$= 2\left(f(x^3)\right)^{2-1} \cdot f'(x^3) \cdot 3x^2$

$= \boxed{6x^2 \cdot f(x^3) \cdot f'(x^3)}$

Oct 16-10:46 AM



Oct 16-10:48 AM



Oct 16-10:57 AM

Use linear approximation to

estimate $\sqrt{26}$

$$f(x) \approx f(a) + f'(a)(x-a)$$

$$\sqrt{26} \approx \sqrt{25} \approx 5$$

$$\sqrt{x} \approx 5 + \frac{1}{10}(x-25)$$

Near $x=25$

$f(x) = \sqrt{x}$ $\quad f(25) = \sqrt{25} = 5$

$a = 25$ $\quad f'(x) = \frac{1}{2\sqrt{x}}$

$$f'(25) = \frac{1}{2\sqrt{25}} = \frac{1}{10}$$

$$\sqrt{26} \approx 5 + \frac{1}{10}(26-25)$$

$$\approx 5 + \frac{1}{10}\cdot(1)$$

$$\approx 5 + .1$$

$$\sqrt{26} \approx 5.1$$

Use your Calc

$\sqrt{26} \approx 5.0990\ldots$

Oct 16-11:04 AM

Use linear Approximation to

$$f(x) \approx f(a) + f'(a)\cdot(x-a)$$

estimate $\sin 31^\circ$

$\sin 31^\circ \approx \sin 30^\circ = \frac{1}{2}$

$f(x) = \sin x$

$a = 30^\circ$

$$\sin x \approx \frac{1}{2} + \frac{\sqrt{3}}{2}(x - 30^\circ)$$

Near 30°

$\frac{\pi}{6}$

$f(30^\circ) = \sin 30^\circ = \frac{1}{2}$

$f'(x) = \cos x$

$f'(30^\circ) = \cos 30^\circ = \frac{\sqrt{3}}{2}$

$180^\circ = \pi$ Rad

$1^\circ = \frac{\pi}{180}$

$$\sin 31^\circ \approx \frac{1}{2} + \frac{\sqrt{3}}{2}(31^\circ - 30^\circ)$$

$$= \frac{1}{2} + \frac{\sqrt{3}}{2}(1^\circ) = \frac{1}{2} + \frac{\sqrt{3}}{2}\left(\frac{\pi}{180}\right) = \frac{1}{2} + \frac{\pi\sqrt{3}}{360}$$

$\approx .515$

Use your calc.

to find $\sin 31^\circ \approx .515$

Oct 16-11:09 AM

Use linear approximation to estimate $\tan 46°$.

$\tan 46° \approx \tan 45° = 1$

$f(x) = \tan x$ $\quad f(45°) = \tan 45° = 1$

$a = 45°$ $\quad f'(45°) = \sec^2 45° = 2$

$f'(x) = \sec^2 x$

$f(x) \approx f(a) + f'(a)(x-a)$

$\tan x \approx 1 + 2(x - 45°)$

$\tan 46° \approx 1 + 2(46° - 45°)$

$\approx 1 + 2 \cdot 1°$

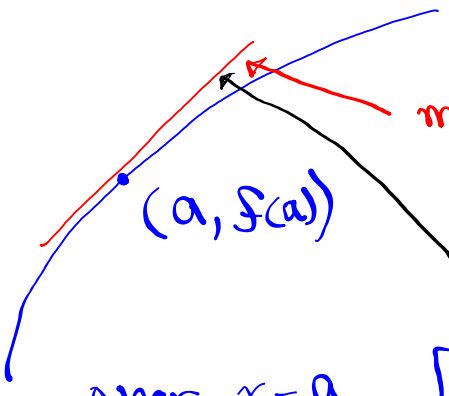
$= 1 + 2 \cdot \frac{\pi}{180}$

$= 1 + \frac{\pi}{90} \approx 1.035$

Now use your calc to find

$\tan 46° \approx 1.036$

Oct 16-11:18 AM



$y = f(x)$

$m = f'(a)$

$(a, f(a))$

$y - f(a) = f'(a)(x - a)$

$y = f(a) + f'(a)(x-a)$

Near $x = a$

$y \approx f(x)$

$f(x) \approx f(a) + f'(a)(x-a)$

Linear Approximation

SG 12

Oct 16-11:29 AM