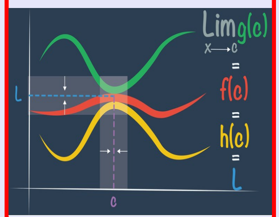
Math 261

Fall 2023

Lecture 26



Feb 19-8:47 AM

If $f(x)$ is differentiable at $x=a$, then $f(x)$ is continuous at $x=a$.

Let's begin with

$$f(x)-f(a)=\frac{f(x)-f(a)}{x-a}\cdot(x-a)$$

Now take the limit of both sides as $x\to a$

$$\lim_{x\to a}\left[f(x)-f(a)\right]=\lim_{x\to a}\left[\frac{f(x)-f(a)}{x-a}\cdot(x-a)\right]$$

$$=\boxed{\lim_{x\to a}\frac{f(x)-f(a)}{x-a}}\cdot\boxed{\lim_{x\to a}(x-a)}$$

$$=f'(a)\cdot(a-a)$$

$$=f'(a)\cdot 0$$

$$=\boxed{0}$$

Now $\lim_{x\to a}f(x)=\lim_{x\to a}\left[f(x)+f(a)-f(a)\right]$

$$=\lim_{x\to a}\left[f(a)+f(x)-f(a)\right]$$

$$=\boxed{\lim_{x\to a}f(a)}+\boxed{\lim_{x\to a}\left[f(x)-f(a)\right]}$$

$$=f(a)+0$$

$$=f(a)$$

$\lim_{x\to a}f(x)=f(a)\implies f(x)$ is Cont. at $x=a$.

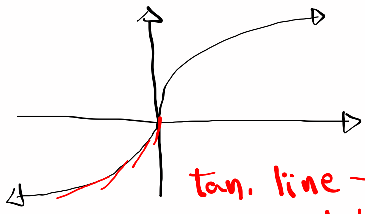
Oct 10-11:26 AM

Given $f(x) = \sqrt[3]{x}$ Domain $(-\infty, \infty)$

$$f(-x) = \sqrt[3]{-x} = \sqrt[3]{-1} \cdot \sqrt[3]{x} = -\sqrt[3]{x} = -f(x)$$

Since $f(-x) = -f(x)$

$f(x)$ is odd, and symmetric with respect to the origin.



$$f(x) = x^{1/3}$$

$$f'(x) = \frac{1}{3}x^{-2/3}$$

$$f'(x) = \frac{1}{3\sqrt[3]{x^2}}$$

tan. line →
vertical line →
No slope

Not differentiable at $x=0$

$f(x)$ is cont. everywhere but not diff. at $x=0$.

Oct 12-10:35 AM

Given $f(x) = \dfrac{1-6x}{x+3}$ Cont. everywhere except at $x=-3$.

$$f'(x) = \frac{-6(x+3)-(1-6x)\cdot 1}{(x+3)^2} = \frac{\cancel{-6x}-18-1\cancel{+6x}}{(x+3)^2}$$

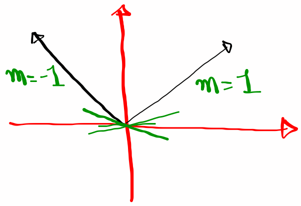
$$= \frac{-19}{(x+3)^2}$$

$f'(x)$ is undefined at $x=-3$.

So $f(x)$ is not differentiable at $x=-3$.

Oct 12-10:41 AM

Show $f(x) = |x|$ is not differentiable at $x=0$.



$$|x| = \begin{cases} -x & \text{if } x<0 \\ x & \text{if } x \geq 0 \end{cases}$$

$$\frac{d}{dx}\left[|x|\right] = \begin{cases} -1 & \text{if } x<0 \\ 1 & \text{if } x>0 \end{cases}$$

$|x| = \sqrt{x^2}$

$f(x) = \sqrt{x^2} = (x^2)^{1/2}$

$$f'(x) = \frac{1}{\cancel{2}}\left(x^2\right)^{\frac{1}{2}-1} \cdot \cancel{2}x$$

$$= \frac{x}{(x^2)^{1/2}} = \frac{x}{\sqrt{x^2}}$$

Not defined at $x=0$

$f(x)$ is not differentiable at $x=0$

Oct 12-10:45 AM

$$f(x) = \sqrt{\frac{t}{t^2+4}} \qquad \text{Find } f'(x) = 0$$

Variable

No x there $\rightarrow$ Constant

$$f(t) = \sqrt{\frac{t}{t^2+4}}$$ ← Not a constant

$$f(t) = \left(\frac{t}{t^2+4}\right)^{1/2}$$

$$f'(t) = \frac{1}{2}\left(\frac{t}{t^2+4}\right)^{\frac{1}{2}-1} \cdot \frac{1(t^2+4) - t \cdot 2t}{(t^2+4)^2}$$

$$= \frac{1}{2} \cdot \frac{t^{-1/2}}{(t^2+4)^{-1/2}} \cdot \frac{4-t^2}{(t^2+4)^2}$$

$$= \frac{(4-t^2) \cdot \sqrt{t^2+4}}{2\sqrt{t}\,(t^2+4)^2}$$

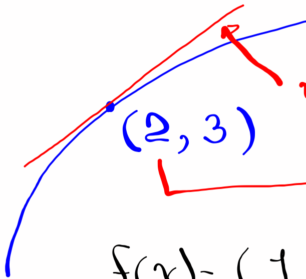
Oct 12-10:50 AM

Find eqn of the tangent line to the graph of $f(x)=\sqrt{1+x^3}$ at $x=2$.

$y-y_1=m(x-x_1)$

$y-3=2(x-2)$

$\boxed{y= \checkmark}$



$m=f'(2)$

$f(x)=(1+x^3)^{1/2}$ $\quad f'(x)=\frac{1}{2}(1+x^3)^{-1/2}\cdot 3x^2$

$$f'(x)=\frac{3x^2}{2\sqrt{1+x^3}} \qquad f'(2)=\frac{\cancel{3}\cdot 2^{\cancel{2}}}{\cancel{2}\cancel{\sqrt{1+2^3}}}=\boxed{2}$$

Oct 12-10:56 AM

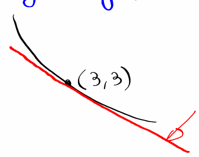
1) Find a point in QI on the graph of the equation $x^3+y^3-6xy=0$ where $x=3$.

$3^3+y^3-6(3)y=0$

$y^3-18y+27=0$

3	1	0	−18	27
		3	9	−27
	1	3	−9	0

3 is the answer



$m=\frac{dy}{dx}\Big|_{(3,3)}$

$x^3+y^3-6xy=0$

$\frac{d}{dx}[x^3+y^3-6xy]=\frac{d}{dx}[0]$

$\frac{d}{dx}[x^3]+\frac{d}{dx}[y^3]-6\frac{d}{dx}[xy]=0$

Product

$3x^2+3y^2\cdot\frac{dy}{dx}-6\left[1\cdot y+x\cdot\frac{dy}{dx}\right]=0$

at $(3,3)$, $\frac{dy}{dx}\Big|_{(3,3)}=m_{\text{tan. line}}$

$3(3)^2+3(3)^2\cdot m-6[1\cdot 3+3\cdot m]=0$

Solve for m ← Slope of tan. line at (3,3)

Find eqn. of tan. line at (3,3) $\boxed{y=-x+6}$

Oct 12-11:02 AM

Class QZ 12

Find $f'(x)$ Do not Simplify

1) $f(x) = (x^4 - 4x^2)^3$ $\boxed{f'(x) = 3(x^4 - 4x^2)^2 \cdot (4x^3 - 8x)}$

2) $f(x) = \cos(\sqrt{x})$ $\boxed{f'(x) = -\sin(\sqrt{x}) \cdot \frac{1}{2\sqrt{x}}}$

3) $x^4 - y^2 = 8x$ where $y = f(x)$

$4x^3 - 2y\frac{dy}{dx} = 8$ $4x^3 - 8 = 2y\frac{dy}{dx}$ $\boxed{\frac{dy}{dx} = \frac{4x^3 - 8}{2y}}$

Oct 12-11:14 AM