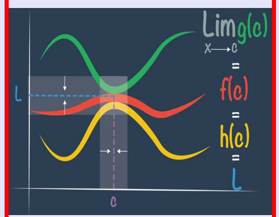
Math 261

Fall 2023

Lecture 25



Feb 19-8:47 AM

If $f(x)$ is differentiable at $x=a$, then $f(x)$ is continuous at $x=a$.

we need to show $f(x)$ is cont. at $x=a$.

$$\lim_{x \to a} f(x) = f(a)$$

we have $f(x)$ is differentiable at $x=a$.

$f'(a)$ exists.

Recall $$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

If we let $x = a+h \to x-a = h$

as $h \to 0$

$x - a \to 0$

$x \to a$

$$\boxed{f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x-a}}$$

Oct 10-11:26 AM

find $f'(x)$ for $f(x) = \sqrt[3]{x^4 - 2x^2 + 8}$

$f(x) = (x^4 - 2x^2 + 8)^{1/3}$ Use chain Rule

$f'(x) = \frac{1}{3}(x^4 - 2x^2 + 8)^{\frac{1}{3}-1} \cdot (4x^3 - 4x)$ ✓

$= \frac{1}{3}(x^4 - 2x^2 + 8)^{-2/3} \cdot (4x^3 - 4x)$

$= \frac{4x^3 - 4x}{3(x^4 - 2x^2 + 8)^{2/3}} = \boxed{\frac{4x^3 - 4x}{3\sqrt[3]{(x^4 - 2x^2 + 8)^2}}}$

Oct 11-10:34 AM

find $\frac{dy}{dx}$ for $y = \tan^2(\sin^4\sqrt{x}) = [\tan(\sin^4\sqrt{x})]^2$

$y' = \cancel{2}[\tan(\sin^4\sqrt{x})]^{(2-1)\,1} \cdot \sec^2(\sin^4\sqrt{x})$

$\cdot 4\sin^3\sqrt{x} \cdot \cos\sqrt{x} \cdot \frac{1}{\cancel{2}\sqrt{x}}$

$= \frac{4\tan(\sin^4\sqrt{x}) \cdot \sec^2(\sin^4\sqrt{x}) \cdot \sin^3\sqrt{x} \cdot \cos\sqrt{x}}{\sqrt{x}}$

Oct 11-10:38 AM

find $f'(x)$ for $f(x) = \cos^3\left(\frac{x}{x+1}\right)^2$

$$f(x) = \left[\cos\left(\frac{x}{x+1}\right)^2\right]^3 \qquad u = \cos\left(\frac{x}{x+1}\right)^2$$

$$f(u) = u^3$$

$$f'(x) = \frac{df}{du}\cdot\frac{du}{dx} = 3u^2 \cdot \frac{d}{dx}\left[\cos\left(\frac{x}{x+1}\right)^2\right]$$

$$= 3\cdot\left[\cos\left(\frac{x}{x+1}\right)^2\right]^2 \cdot -\sin\left(\frac{x}{x+1}\right)^2 \cdot 2\left(\frac{x}{x+1}\right)^1 \cdot \frac{\overbrace{1(x+1)-x\cdot 1}^{1}}{(x+1)^2}$$

$$= -6\cos^2\left(\frac{x}{x+1}\right)^2 \cdot \sin\left(\frac{x}{x+1}\right)^2 \cdot \frac{x}{(x+1)^3}$$

Oct 11-10:44 AM

find y' for $y = x^2\tan\left(\frac{1}{x}\right)$

Product Rule

$$\frac{d}{dx}\left[\frac{1}{x}\right] =$$

$$\frac{d}{dx}\left[x^{-1}\right] =$$

$$-1x^{-2} = \frac{-1}{x^2}$$

$$y' = \frac{d}{dx}\left[x^2\right]\cdot\tan\left(\frac{1}{x}\right) + x^2\cdot\frac{d}{dx}\left[\tan\left(\frac{1}{x}\right)\right]$$

$$= 2x\tan\frac{1}{x} + \cancel{x^2}\cdot\sec^2\frac{1}{x}\cdot\frac{-1}{\cancel{x^2}}$$

$$= \boxed{2x\tan\frac{1}{x} - \sec^2\frac{1}{x}}$$

Oct 11-10:50 AM

If $\frac{d}{dx}\left[f(x^2)\right] = x^2$, find $f'(x^2)$

$$\frac{d}{dx}\left[f(x^2)\right] = f'(x^2)\cdot 2x = x^2$$

$$f'(x^2) = \frac{x^2}{2x} \qquad x \neq 0$$

$$\boxed{f'(x^2) = \frac{x}{2}}$$

Oct 11-10:55 AM

$f'(x)$ is even if $f'(-x) = f'(x)$

$f'(x)$ is odd if $f'(-x) = -f'(x)$

Show that $f'(x)$ is even if $f(x)$ is odd.

If $f(x)$ is odd,

$$f(-x) = -f(x)$$

Take the derivative of both sides.

$$\frac{d}{dx}\left[f(-x)\right] = \frac{d}{dx}\left[-f(x)\right]$$

$$f'(-x)\cdot -1 = -\frac{d}{dx}\left[f(x)\right]$$

$$-f'(-x) = -f'(x)$$

$$f'(-x) = f'(x)$$

$f'(x)$ is even.

Oct 11-10:58 AM

Implicit Differentiation:

$y = \frac{1}{x}$ $\quad y' = \frac{-1}{x^2}$

Cross-Multiply

$xy = 1$ Let's take derivative of both Sides

$\frac{d}{dx}[xy] = \frac{d}{dx}[1]$

Product Rule

$\frac{d}{dx}[x] \cdot y + x \cdot \frac{d}{dx}[y] = 0$

$1 \cdot y + x \cdot y' = 0$

$xy' = -y$

$y' = \frac{-y}{x}$

but $y = \frac{1}{x}$

$y' = \frac{-\frac{1}{x}}{x}$

$\boxed{y' = \frac{-1}{x^2}}$

Oct 11-11:04 AM

$y = \sqrt{4 - x^2}$

Square both Sides

$y^2 = 4 - x^2$

Take derivative of both Sides

$\frac{d}{dx}[y^2] = \frac{d}{dx}[4 - x^2]$

$2y \cdot \frac{dy}{dx} = -2x$

$\frac{dy}{dx} = \frac{-x}{y} = \frac{-x}{\sqrt{4 - x^2}}$

$y = (4 - x^2)^{1/2}$

$y' = \frac{1}{\cancel{2}}(4 - x^2)^{\frac{1}{2} - 1 = -\frac{1}{2}} \cdot (-\cancel{2}x)$

$y' = \frac{-x}{\sqrt{4 - x^2}}$

Oct 11-11:09 AM

Find eqn of tan. line at (2, -1) for the graph of $y^2 - x + 1 = 0$

1) verify the point ✓

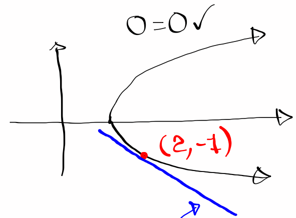
$(-1)^2 - 2 + 1 = 0$

$1 - 2 + 1 = 0$

$0 = 0$ ✓

$y^2 + 1 = x$

$x = y^2 + 1$



$y^2 - x + 1 = 0$

$\frac{d}{dx}\left[y^2 - x + 1\right] = \frac{d}{dx}[0]$

$m = \frac{dy}{dx}\Big|_{(2,-1)}$

$\frac{d}{dx}[y^2] - \frac{d}{dx}[x] + \frac{d}{dx}[1] = 0$

$y - (-1) = \frac{-1}{2}(x - 2)$

$\boxed{y =}$

$2y \cdot \frac{dy}{dx} - 1 + 0 = 0$

$\frac{dy}{dx} = \frac{1}{2y}$ $\quad m = \frac{dy}{dx}\Big|_{(2,-1)} = \frac{1}{2(-1)} = \boxed{\frac{-1}{2}}$

Oct 11-11:15 AM

Given $\sqrt{x} + \sqrt{y} = 8$

find $\frac{dy}{dx}$.

$x^{1/2} + y^{1/2} = 8$

$\frac{d}{dx}\left[x^{1/2} + y^{1/2}\right] = \frac{d}{dx}[8]$

$\frac{d}{dx}\left[x^{1/2}\right] + \frac{d}{dx}\left[y^{1/2}\right] = 0$

$\frac{1}{2}x^{-1/2} + \frac{1}{2}y^{-1/2} \cdot \frac{dy}{dx} = 0$

multiply by 2

$x^{-1/2} + y^{-1/2} \cdot \frac{dy}{dx} = 0 \rightarrow \frac{dy}{dx} = \frac{-x^{-1/2}}{y^{-1/2}}$

$\frac{dy}{dx} = -\frac{y^{1/2}}{x^{1/2}}$ $\quad \boxed{\frac{dy}{dx} = -\sqrt{\frac{y}{x}}}$

Oct 11-11:23 AM

Find $\left.\frac{dy}{dx}\right|_{(3,2)}$ for $y^2 - 3xy + 2x^2 = 4$

$$\frac{d}{dx}\left[y^2\right] - 3\frac{d}{dx}\left[xy\right] + 2\frac{d}{dx}\left[x^2\right] = \frac{d}{dx}\left[4\right]$$

Product Rule

$$2y \cdot \frac{dy}{dx} - 3\left[1 \cdot y + x \cdot \frac{dy}{dx}\right] + 2 \cdot 2x = 0$$

$$4\frac{dy}{dx} - 3\left[2 + 3\frac{dy}{dx}\right] + 12 = 0$$

$$4\frac{dy}{dx} - 6 - 9\frac{dy}{dx} + 12 = 0$$

$$-5\frac{dy}{dx} = -6$$

$$\boxed{\left.\frac{dy}{dx}\right|_{(3,2)} = \frac{6}{5}}$$

Oct 11-11:29 AM