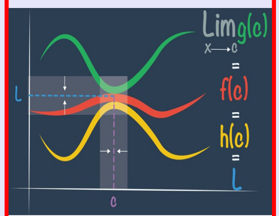
Math 261

Fall 2023

Lecture 24



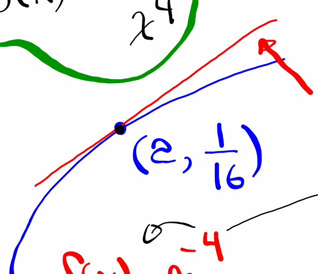
Feb 19-8:47 AM

Class QZ 11

Find eqn of the tan. line to the graph of

$f(x) = \dfrac{1}{x^4}$ at $x = 2$.

$f(2) = \dfrac{1}{2^4} = \dfrac{1}{16}$



$m = f'(2) = \dfrac{-4}{2^5} = \dfrac{-4}{32} = \boxed{\dfrac{-1}{8}}$

$f(x) = x^{-4}$ $\quad f'(x) = -4x^{-5} = \dfrac{-4}{x^5}$

$y - \dfrac{1}{16} = \dfrac{-1}{8}(x-2)$

$y = \dfrac{-1}{8}x + \dfrac{2}{8} + \dfrac{1}{16}$

$y = \dfrac{-1}{8}x + \dfrac{4}{16} + \dfrac{1}{16}$

$\boxed{y = \dfrac{-1}{8}x + \dfrac{5}{16}}$

Oct 9-11:22 AM

More on Chain Rule:

If $y = f(u)$ and $u = g(x)$, then

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

ex: $y = u^5$ and $u = \frac{1}{5}x^4 - \frac{1}{10}x^2$

find $\frac{dy}{dx} = 5u^4 \cdot \left(\frac{4}{5}x^3 - \frac{2}{10}x\right)$

$$= 5\left(\frac{1}{5}x^4 - \frac{1}{10}x^2\right)^4 \cdot \left[\frac{4}{5}x^3 - \frac{1}{5}x\right]$$

$$= \left(\frac{1}{5}x^4 - \frac{1}{10}x^2\right)^4 \cdot \left(4x^3 - x\right)$$

Oct 10-10:34 AM

$y = \sin u^3$ $\quad u = \sqrt{x} - 2$

$$y' = \frac{dy}{du} \cdot \frac{du}{dx}$$

$$= \cos u^3 \cdot 3u^2 \cdot \frac{1}{2\sqrt{x}}$$

$$y' = \cos(\sqrt{x} - 2)^3 \cdot 3(\sqrt{x} - 2)^2 \cdot \frac{1}{2\sqrt{x}}$$

$$\boxed{y' = \frac{3}{2} \cdot \frac{(\sqrt{x} - 2)^2}{\sqrt{x}} \cdot \cos(\sqrt{x} - 2)^3}$$

Oct 10-10:39 AM

$y = \sin u^3 \qquad u = \sqrt{x} - 2$

$y = \sin(\sqrt{x} - 2)^3$

$y' = \cos(\sqrt{x} - 2)^3 \cdot 3(\sqrt{x} - 2)^2 \cdot \frac{1}{2\sqrt{x}}$

$y = \cos^2(4x + 5) = [\cos(4x + 5)]^2$

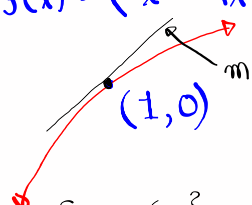
$y' = 2[\cos(4x + 5)]^1 \cdot -\sin(4x + 5) \cdot 4$

$\boxed{y' = -8 \sin(4x + 5) \cdot \cos(4x + 5)}$

Oct 10-10:39 AM

Find eqn of the Tan. line to the graph of

$f(x) = (x^2 - 4x + 3)^5$ at $x = 1$.



$m = f'(1) = 0$

$f(1) = (1^2 - 4(1) + 3)^5$

$= 0^5 = 0$

$y - 0 = 0(x - 1)$

$\boxed{y = 0}$

$f(x) = (x^2 - 4x + 3)^5$

$f'(x) = 5(x^2 - 4x + 3)^4 \cdot (2x - 4)$

$f'(1) = 5(1^2 - 4(1) + 3)^4 \cdot (2(1) - 4) = 0$

$\frac{d}{dx}[x^2 - 4x + 3] = \frac{d}{dx}[x^2] - \frac{d}{dx}[4x] + \frac{d}{dx}[3]$

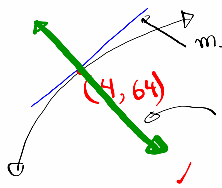
$= 2x - 4 \cdot 1 + 0$

$= \boxed{2x - 4}$

Oct 10-10:48 AM

Find eqn of the normal line to the graph of $f(x) = \left(\frac{x}{x-3}\right)^3$ at $x=4$.

$$f(4) = \left(\frac{4}{4-3}\right)^3 = 4^3 = 64$$



$$m_{\text{tan. line}} = f'(4)$$

$$m_{\text{Normal}} = \frac{-1}{f'(4)} = \frac{-1}{-144} = \boxed{\frac{1}{144}}$$

$$f(x) = \left(\frac{x}{x-3}\right)^3$$

$$f'(x) = 3\left(\frac{x}{x-3}\right)^2 \cdot \frac{d}{dx}\left[\frac{x}{x-3}\right]$$

$$= 3\left(\frac{x}{x-3}\right)^2 \cdot \frac{1(x-3) - x\cdot 1}{(x-3)^2}$$

$$= 3 \cdot \frac{x^2}{(x-3)^2} \cdot \frac{-3}{(x-3)^2}$$

$$= \frac{-9x^2}{(x-3)^4}$$

$$f'(4) = \frac{-9(4)^2}{(4-3)^4} = \frac{-9\cdot 16}{1} = \boxed{-144}$$

$$y - y_1 = m(x - x_1)$$

$$y - 64 = \frac{1}{144}(x-4) \Rightarrow \boxed{y = \quad \checkmark}$$

Slope-Int. Form

Oct 10-10:55 AM

$f(3) = 2$, $g(3) = -5$

$f'(3) = -1$ $\quad g'(3) = 3$

Given $y = \frac{f(x)}{g(x)}$ find $\left.\frac{dy}{dx}\right|_{x=3}$

$$\frac{dy}{dx} = \frac{f'(x)\cdot g(x) - f(x)\cdot g'(x)}{[g(x)]^2}$$

$$\left.\frac{dy}{dx}\right|_{x=3} = \frac{f'(3)\cdot g(3) - f(3)\cdot g'(3)}{[g(3)]^2} = \frac{-1\cdot(-5) - 2\cdot 3}{[-5]^2}$$

$$= \frac{5-6}{25} = \boxed{\frac{-1}{25}}$$

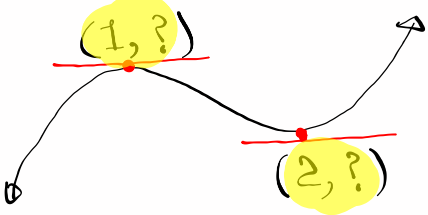
Oct 10-11:03 AM

Find all points where graph of

$\underbrace{y = \frac{1}{3}x^3 - \frac{3}{2}x^2 + 2x}$ has $\underbrace{\text{horizontal tan. lines?}}$

$m = 0$

$y' = 0$



$y' = \frac{1}{3} \cdot 3x^2 - \frac{3}{2} \cdot 2x + 2$

$y' = x^2 - 3x + 2$

$y' = 0$

$x^2 - 3x + 2 = 0$

$(x-1)(x-2) = 0$

$x = 1 \qquad x = 2$

Oct 10-11:08 AM

Find $f'(0)$ for $f(x) = \sqrt{x^3 - 2x + 5}$

$f(x) = \left(x^3 - 2x + 5\right)^{1/2}$

$f'(x) = \frac{1}{2}\left(x^3 - 2x + 5\right)^{\frac{1}{2} - 1} \cdot \left(3x^2 - 2\right)$

$f'(x) = \frac{1}{2} \cdot \left(x^3 - 2x + 5\right)^{-1/2} \cdot \left(3x^2 - 2\right)$

$f'(0) = \frac{1}{2} \cdot \left(0^3 - 2(0) + 5\right)^{-1/2} \cdot \left(3 \cdot 0^2 - 2\right)$

$= \frac{1}{\cancel{2}} \cdot (5)^{-1/2} \cdot (-\cancel{2}) = \frac{-1}{\sqrt{5}} = \boxed{\frac{-\sqrt{5}}{5}}$

Oct 10-11:14 AM

Find $\frac{dy}{dx}$ (Do not Simplify)

1) $y = \sin^3 x = [\sin x]^3$

$y' = 3[\sin x]^2 \cdot \cos x \cdot 1$

2) $y = \sin(x^3)$

$y' = \cos x^3 \cdot 3x^2$

3) $y = \sin^3 x^3 = [\sin x^3]^3$

$y' = 3[\sin x^3]^2 \cdot \cos x^3 \cdot 3x^2$

Oct 10-11:19 AM

Find $\frac{dy}{dx}$

1) $y = \cos(\sin x)$ $\qquad y' = -\sin(\sin x) \cdot \cos x \cdot 1$

2) $y = \cos^2(\sin x^3) = [\cos(\boxed{\sin} x^3)]^2$

$y' = 2[\cos(\sin x^3)]^{2-1} \cdot -\sin(\sin x^3) \cdot \cos x^3 \cdot 3x^2$

If $f(x)$ is differentiable at $x=a$, then $f(x)$ is continuous at $x=a$.

Oct 10-11:26 AM