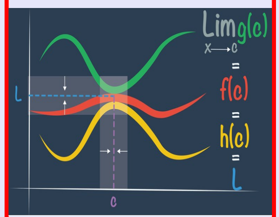
Math 261

Fall 2023

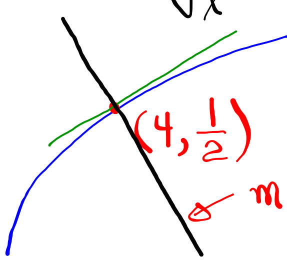
Lecture 23



Feb 19-8:47 AM

Find eqn of the normal line to the graph of $f(x) = \frac{1}{\sqrt{x}}$ at $x = 4$.

$f(4) = \frac{1}{2}$



$(4, \frac{1}{2})$

$m = \frac{-1}{f'(4)}$

$f(x) = \frac{1}{\sqrt{x}}$

$f(x) = x^{-1/2}$

$f'(x) = -\frac{1}{2} \cdot x^{-3/2} = \frac{-1}{2x\sqrt{x}}$

$f'(4) = \frac{-1}{2(4)\sqrt{4}} = \frac{-1}{16}$

$m = \frac{-1}{\frac{-1}{16}} = 16$

$y - \frac{1}{2} = 16(x - 4)$

$\boxed{y = \quad \checkmark}$

Oct 9-10:26 AM

Prove $\dfrac{d}{dx}\left[\dfrac{f(x)}{g(x)}\right] = \dfrac{f'(x)\cdot g(x) - f(x)\cdot g'(x)}{[g(x)]^2}$ $\quad g(x) \neq 0$

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \lim_{h \to 0} \frac{\dfrac{f(x+h)}{g(x+h)} - \dfrac{f(x)}{g(x)}}{h}$$

LCD $= g(x+h)\cdot g(x)$

$$= \lim_{h\to 0} \frac{f(x+h)\cdot g(x) - f(x)\cdot g(x+h)}{h\, g(x+h)\cdot g(x)}$$

$$= \lim_{h\to 0} \frac{f(x+h)\cdot g(x) - f(x)\cdot g(x) - f(x)\cdot g(x+h) + f(x)g(x)}{h\, g(x+h)\cdot g(x)}$$

$$= \lim_{h\to 0} \frac{g(x)\left[f(x+h) - f(x)\right]}{h\, g(x+h)\cdot g(x)} - \lim_{h\to 0} \frac{f(x)\left[g(x+h) - g(x)\right]}{h\, g(x+h)\cdot g(x)}$$

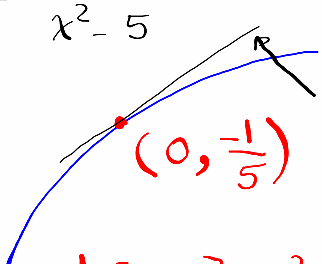
$$= \frac{f'(x)\cdot g(x)}{[g(x)]^2} - \frac{f(x)\cdot g'(x)}{[g(x)]^2} = \boxed{\frac{f'(x)\cdot g(x) - f(x)\cdot g'(x)}{[g(x)]^2}}$$

Oct 9-10:31 AM

Find eqn of the tan. line to the graph of

$y = \dfrac{4x+1}{x^2-5}$ at $x = 0$.

$y - \left(-\frac{1}{5}\right) = -\frac{4}{5}(x - 0)$



$m = y'\big|_{x=0}$

$\boxed{y = \quad ✓}$

$$y' = \frac{\frac{d}{dx}[4x+1]\cdot(x^2-5) - (4x+1)\cdot\frac{d}{dx}[x^2-5]}{[x^2-5]^2}$$

$$= \frac{4(x^2-5) - (4x+1)\cdot 2x}{(x^2-5)^2}$$

$$y'\big|_{x=0} = \frac{4(-5) - (1)\cdot 0}{(0^2-5)^2} = \frac{-20}{25} = \boxed{-\frac{4}{5}}$$

Oct 9-10:41 AM

$y' \to \frac{dy}{dx}$ First derivative

$y'' \to \frac{d^2y}{dx^2}$ Second derivative

$y''' \to \frac{d^3y}{dx^3}$ Third derivative

Show for $x \neq 0$, $y = \frac{1}{x}$ satisfies

$x^3 y'' + x^2 y' - xy = 0.$ ✓

$y = \frac{1}{x} = x^{-1}$

$y' = -1\, x^{-2}$

$y'' = (-1)(-2) \cdot x^{-3}$

$y' = \frac{-1}{x^2}$

$y'' = \frac{2}{x^3}$

$\cancel{x^3}\left(\frac{2}{\cancel{x^3}}\right) + \cancel{x^2}\left(\frac{-1}{\cancel{x^2}}\right) - \cancel{x}\left(\frac{1}{\cancel{x}}\right) =$

$2 - 1 - 1 = 0$ ✓

Oct 9-10:46 AM

Given $y = \sin x \cos x$

find

1) $y' = \cos x \cdot \cos x + \sin x \cdot (-\sin x) = \cos^2 x - \sin^2 x$

$= \cos x \cos x - \sin x \sin x$

2) $y'' = -\sin x \cdot \cos x + \cos x \cdot -\sin x - \left[\cos x \cdot \sin x + \sin x \cdot \cos x\right]$

$= -\sin x \cos x - \sin x \cos x - \sin x \cos x - \sin x \cos x$

$= \boxed{-4 \sin x \cos x}$

Oct 9-10:51 AM

Find $\frac{d^2y}{dx^2}$ for $y = \boxed{x^2}\boxed{\sin x}$

$$\frac{dy}{dx} = 2x \cdot \sin x + x^2 \cdot \cos x$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left[\frac{dy}{dx}\right]$$

$$= \frac{d}{dx}\left[2\underbrace{x\sin x} + \underbrace{x^2\cos x}\right]$$

$$= 2\left[1 \cdot \sin x + x \cdot \cos x\right] + \left[2x \cdot \cos x + x^2 \cdot -\sin x\right]$$

$$= 2\sin x + 2x\cos x + 2x\cos x - x^2\sin x$$

$$= \boxed{2\sin x + 4x\cos x - x^2\sin x}$$

Oct 9-10:56 AM

Introduction to chain Rule:

If $y = f(u)$ and $u = g(x)$, then

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

Ex. $y = \sin u$, $u = x^4$ $\rightarrow$ $y = \sin(x^4)$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = \cos u \cdot 4x^3$$

$y = \sin(x^4)$ $\rightarrow$ $y' = \cos x^4 \cdot 4x^3$

$$\boxed{y' = 4x^3 \cdot \cos x^4}$$

Oct 9-11:02 AM

$y = \tan u \quad , \quad u = 4x^3 - 1 \longrightarrow y = \tan(4x^3 - 1)$

$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$

$= \sec^2 u \cdot 12x^2$

$\boxed{\frac{dy}{dx} = 12x^2 \sec^2(4x^3 - 1)}$

Oct 9-11:06 AM

$y = u^3 \quad , \quad u = x^2 + 5x - 4 \rightarrow y = (x^2 + 5x - 4)^3$

$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$

$= 3u^2 \cdot (2x + 5) = 3(x^2 + 5x - 4)^2 \cdot (2x + 5)$

find $\frac{d}{dx}\left[(x^3 - 6x^2 + 8)^{40}\right]$ $\quad y = u^{40} \quad , \quad u = x^3 - 6x^2 + 8$

$= \frac{dy}{du} \cdot \frac{du}{dx} = 40u^{39} \cdot (3x^2 - 12x)$

$= \boxed{40(3x^2 - 12x) \cdot (x^3 - 6x^2 + 8)^{39}}$

$= 120x(x - 4)(x^3 - 6x^2 + 8)^{39}$

Oct 9-11:10 AM

$$y = \cos\left(\frac{x}{x-1}\right)$$

$$y' = -\sin\left(\frac{x}{x-1}\right) \cdot \frac{\overbrace{1(x-1) - x(1)}^{-1}}{(x-1)^2}$$

$$= -\sin\left(\frac{x}{x-1}\right) \cdot \frac{-1}{(x-1)^2}$$

$$\boxed{y' = \sin\left(\frac{x}{x-1}\right) \cdot \frac{1}{(x-1)^2}}$$

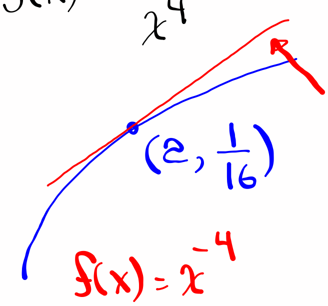
Oct 9-11:19 AM

Class QZ 11

Find eqn of the tan. line to the graph of

$f(x) = \frac{1}{x^4}$ at $x = 2$.

$f(2) = \frac{1}{2^4} = \frac{1}{16}$



$m = f'(2) = \frac{-4}{2^5} = \frac{-4}{32} = \frac{-1}{8}$

$f(x) = x^{-4}$ $\quad f'(x) = -4x^{-5} = \frac{-4}{x^5}$

$y - \frac{1}{16} = \frac{-1}{8}(x-2)$

$y = \frac{-1}{8}x + \frac{2}{8} + \frac{1}{16}$

$\rightarrow y = \frac{-1}{8}x + \frac{4}{16} + \frac{1}{16}$

$$\boxed{y = \frac{-1}{8}x + \frac{5}{16}}$$

Oct 9-11:22 AM