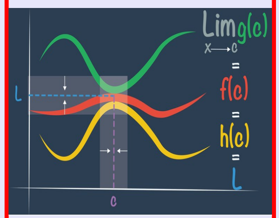
Math 261

Fall 2023

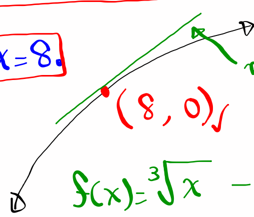
Lecture 22



Feb 19-8:47 AM

Class QZ 10

Find equation of the tangent line in Slope - Int. form to the graph of $f(x) = \sqrt[3]{x} - 2$ at $x = 8$.



$(8, 0)$ ✓

$m = f'(8) = \frac{1}{3\sqrt[3]{8^2}} = \frac{1}{3\sqrt[3]{64}} = \boxed{\frac{1}{12}}$ ✓

$f(x) = \sqrt[3]{x} - 2 \qquad f(x) = x^{1/3} - 2$

$f'(x) = \frac{1}{3}x^{1/3 - 1} - 0 = \frac{1}{3}x^{-2/3} = \frac{1}{3x^{2/3}} = \boxed{\frac{1}{3\sqrt[3]{x^2}}}$ ✓

$y - y_1 = m(x - x_1)$

$y - 0 = \frac{1}{12}(x - 8) \rightarrow y = \frac{1}{12}x - \frac{8}{12} \rightarrow \boxed{y = \frac{1}{12}x - \frac{2}{3}}$ ✓

Final Answer

Oct 4-11:16 AM

Prove $\frac{d}{dx}[x^4] = 4x^3$

$f(x) = x^4$

$$\frac{d}{dx}[x^4] = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{(x+h)^4 - x^4}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{x^4} + 4x^3h + 6x^2h^2 + 4xh^3 + h^4 \cancel{- x^4}}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(4x^3 + 6x^2h + 4xh^2 + h^3)}{\cancel{h}}$$

$$= \lim_{h \to 0} (4x^3 + \underbrace{6x^2h}_{\to 0} + \underbrace{4xh^2}_{\to 0} + \underbrace{h^3}_{\to 0})$$

$$= \boxed{4x^3}$$

Oct 5-10:31 AM

Prove $\frac{d}{dx}[\csc x] = -\csc x \cot x$

$f(x) = \csc x = \frac{1}{\sin x}$

$$\frac{d}{dx}[\csc x] = \frac{d}{dx}\left[\frac{1}{\sin x}\right] = \lim_{h \to 0} \frac{\frac{1}{\sin(x+h)} - \frac{1}{\sin x}}{h}$$

LCD $= \sin(x+h) \cdot \sin x$

$$= \lim_{h \to 0} \frac{\sin x - \sin(x+h)}{h \sin x \cdot \sin(x+h)}$$

$\sin(x+h) \to \sin x \cos h + \cos x \sin h$

$$= \lim_{h \to 0} \frac{\sin x - \sin x \cos h - \cos x \sin h}{h \sin x \sin(x+h)}$$

$$= \lim_{h \to 0} \left[\frac{\cancel{\sin x}(1 - \cos h)}{h \cancel{\sin x} \sin(x+h)} - \frac{\cos x \sin h}{h \sin x \sin(x+h)} \right]$$

$$= \lim_{h \to 0} \underbrace{\frac{1 - \cos h}{h}}_{\to 0} \cdot \lim_{h \to 0} \frac{1}{\sin(x+h)} - \lim_{h \to 0} \underbrace{\frac{\sin h}{h}}_{\to 1} \cdot \lim_{h \to 0} \frac{\cos x}{\sin x \cdot \sin(x+h)}$$

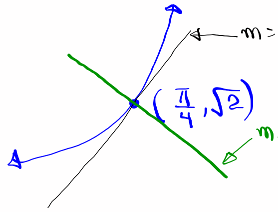
$$= 0 \cdot \underbrace{\frac{1}{\sin x}}_{\to 0} - 1 \cdot \frac{\cos x}{\sin x \cdot \sin x} = -\frac{\cos x}{\sin x} \cdot \frac{1}{\sin x}$$

$$= \boxed{-\csc x \cdot \cot x}$$

$\frac{d}{dx}[\sin x] =$ ✓	$\frac{d}{dx}[\csc x] =$ ✓
$\frac{d}{dx}[\cos x] =$ ✓	* * $\boxed{\frac{d}{dx}[\sec x] =}$
$\frac{d}{dx}[\tan x] =$ ✓	$\frac{d}{dx}[\cot x] =$ ✓

Oct 5-10:36 AM

Find the equation of the normal line to the graph of $f(x) = \sec x$ at $x = \pi/4$.



$m = f'(\frac{\pi}{4})$

$(\frac{\pi}{4}, \sqrt{2})$

$m = \dfrac{-1}{f'(\frac{\pi}{4})} = \dfrac{-1}{\sqrt{2}} = \boxed{\dfrac{-\sqrt{2}}{2}}$

$f(\frac{\pi}{4}) = \sec\frac{\pi}{4}$
$= \sec 45°$
$= \dfrac{1}{\cos 45°}$
$= \dfrac{1}{\frac{\sqrt{2}}{2}}$
$= \dfrac{2}{\sqrt{2}} \cdot \dfrac{\sqrt{2}}{\sqrt{2}} = \sqrt{2}$

$\frac{d}{dx}[\sec x] = \sec x \tan x$ ← Make sure to prove this for yourself.

$f(x) = \sec x$

$f'(x) = \sec x \tan x$

$f'(\frac{\pi}{4}) = \sec\frac{\pi}{4} \cdot \tan\frac{\pi}{4} = \sqrt{2} \cdot 1 = \sqrt{2}$

$y - y_1 = m(x - x_1)$

$y - \sqrt{2} = \frac{-\sqrt{2}}{2}(x - \frac{\pi}{4})$

$$\boxed{y = -\frac{\sqrt{2}}{2}x + \frac{\pi\sqrt{2}}{8} + \sqrt{2}}$$

Oct 5-10:48 AM

Evaluate $\displaystyle\lim_{x \to -\infty} \frac{x}{\sqrt{x^2+4}}$ → -1

If $x = -100$

$\Rightarrow \dfrac{-100}{\sqrt{(-100)^2+4}} \approx -.99980006$

If $x = -1000$

$\Rightarrow \dfrac{-1000}{\sqrt{(-1000)^2+4}} \approx -.999998$

as $x \to \infty$, $x = \sqrt{x^2}$ but as $x \to -\infty$, $x = -\sqrt{x^2}$

If $x = -10$, $-10 = -\sqrt{(-10)^2} = -\sqrt{100} = -10$ ✓

$$\lim_{x\to-\infty} \frac{x}{\sqrt{x^2+4}} = \lim_{x\to-\infty} \frac{\frac{x}{x}}{-\sqrt{\frac{x^2+4}{x^2}}} = \lim_{x\to-\infty} \frac{1}{-\sqrt{1+\cancel{\frac{4}{x^2}}}}$$

$$= \frac{1}{-\sqrt{1+0}} = \frac{1}{-1} = \boxed{-1}$$

Oct 5-10:55 AM

Evaluate $\lim_{x \to -\infty} \dfrac{8-3x}{\sqrt{36x^2-10x}}$ $\quad \dfrac{\infty}{\infty}$ I.F.

Since $x \to -\infty$, then $x = -\sqrt{x^2}$

$$= \lim_{x \to -\infty} \frac{\frac{8-3x}{x}}{\frac{\sqrt{36x^2-10x}}{x}} = \lim_{x \to -\infty} \frac{\frac{8}{x} - \frac{3x}{x}}{\frac{\sqrt{36x^2-10x}}{-\sqrt{x^2}}}$$

$$= \lim_{x \to -\infty} \frac{\frac{8}{x} - 3}{-\sqrt{\frac{36x^2-10x}{x^2}}} = \lim_{x \to -\infty} \frac{\cancel{\frac{8}{x}}^{\to 0} - 3}{-\sqrt{36 - \cancel{\frac{10}{x}}^{\to 0}}}$$

$$= \frac{-3}{-\sqrt{36}} = \frac{3}{6} = \boxed{\frac{1}{2}}$$

If $x = -100$

$$\Rightarrow \frac{8-3(-100)}{\sqrt{36\cdot(-100)^2 - 10(-100)}} = \boxed{.5}140477851$$

try $x = -1000 \Rightarrow$ $= \boxed{.5}012637182$

Oct 5-11:04 AM

Prove $\dfrac{d}{dx}[f(x)\cdot g(x)] = f'(x)\cdot g(x) + f(x)\cdot g'(x)$

$$\frac{d}{dx}[f(x)\cdot g(x)] = \lim_{h \to 0} \frac{f(x+h)g(x+h) - f(x)g(x)}{h}$$

$$= \lim_{h \to 0} \frac{f(x+h)\boxed{g(x+h)} - f(x)\boxed{g(x+h)} - \boxed{f(x)}g(x) + \boxed{f(x)}g(x+h)}{h}$$

$$= \lim_{h \to 0} \frac{g(x+h)\cdot[f(x+h) - f(x)]}{h} + \lim_{h \to 0} \frac{f(x)[-g(x) + g(x+h)]}{h}$$

$$= \boxed{\lim_{h \to 0} g(x+h)} \cdot \boxed{\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}} + \boxed{\lim_{h \to 0} f(x)} \cdot \boxed{\lim_{h \to 0} \frac{g(x+h) - g(x)}{h}}$$

$$= g(x+0)\cdot f'(x) + f(x)\cdot g'(x)$$

$$= \boxed{f'(x)\cdot g(x) + f(x)\cdot g'(x)}$$

make sure to try to Prove

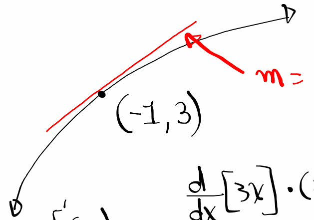
$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{f'(x)\cdot g(x) - f(x)\cdot g'(x)}{[g(x)]^2}$$

Oct 5-11:12 AM

Find eqn of tan. line to the graph of

$f(x) = \dfrac{3x}{2x+1}$ at $x = -1$.

$f(-1) = \dfrac{3(-1)}{2(-1)+1} = \dfrac{-3}{-1} = \boxed{3}$



$m = f'(-1) = \dfrac{3}{(2(-1)+1)^2} = \dfrac{3}{1} = 3$

$$f'(x) = \frac{\frac{d}{dx}[3x]\cdot(2x+1) - 3x\cdot\frac{d}{dx}[2x+1]}{(2x+1)^2}$$

$$= \frac{3(2x+1) - 3x(2)}{(2x+1)^2} = \boxed{\frac{3}{(2x+1)^2}}$$

$y - y_1 = m(x - x_1)$

$y - 3 = 3(x - {-1})$ $\quad$ $y - 3 = 3x + 3$

$\boxed{y = 3x + 6}$