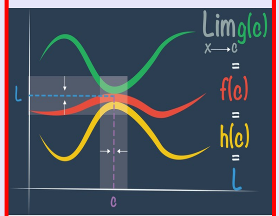
Math 261

Fall 2023

Lecture 21



Feb 19-8:47 AM

1) Evaluate $\lim_{\theta \to 0} \frac{\theta}{\cos\theta} = \frac{0}{\cos 0} = \frac{0}{1} = \boxed{0}$ ✓

2) Evaluate $\lim_{\theta \to 0} \frac{\sin^2\theta}{\theta} = \frac{\sin^2 0}{0} = \frac{0^2}{0} = \frac{0}{0}$ I.F.

$$\lim_{\theta \to 0} \left(\frac{\sin\theta}{\theta} \cdot \sin\theta \right) = \boxed{\lim_{\theta \to 0} \frac{\sin\theta}{\theta}} \cdot \lim_{\theta \to 0} \sin\theta$$

$$= 1 \cdot \sin 0$$

$$= 1 \cdot 0 = \boxed{0}$$

Oct 4-10:25 AM

3) Evaluate $\lim_{x\to 0} \dfrac{x}{\cos(\frac{\pi}{2}-x)} = \dfrac{0}{\cos(\frac{\pi}{2}-0)}$

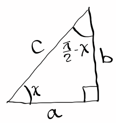
$= \dfrac{0}{\cos\frac{\pi}{2}} = \dfrac{0}{0}$ I.F.

Recall

$\cos(A-B) = \cos A\cos B + \sin A \sin B$

$$\lim_{x\to 0} \frac{x}{\underbrace{\cos\frac{\pi}{2}\cos x}_{0} + \underbrace{\sin\frac{\pi}{2}\sin x}_{\sin x}}$$

$$= \lim_{x\to 0} \frac{x}{\sin x} = \lim_{x\to 0} \frac{1}{\frac{\sin x}{x}} = \frac{1}{1} = \boxed{1}$$



$\cos\left(\frac{\pi}{2}-x\right) = \frac{b}{c}$

$\sin x = \frac{b}{c}$

$\Rightarrow \cos\left(\frac{\pi}{2}-x\right) = \sin x$

$$\lim_{x\to 0} \frac{x}{\cos(\frac{\pi}{2}-x)} = \lim_{x\to 0} \frac{x}{\sin x} = \cdots = 1$$

✓ Co functions of Complementary angles are equal

Total = 90°

Sin & Cos

Sec & Csc

tan & cot

ex: $\boxed{\tan}\, 30° = \boxed{\cot}\, 60°$

$\boxed{\sec}\, 25° = \boxed{\csc}\, 65°$

$\boxed{\sin}\, x = \boxed{\cos}(90° - x)$

Oct 4-10:29 AM

Evaluate $\lim_{x\to 0} \dfrac{x^2}{1-\cos^2 x} = \dfrac{0^2}{1-\cos^2 0} = \dfrac{0}{0}$ I.F.

$$= \lim_{x\to 0} \frac{x^2}{\sin^2 x} = \lim_{x\to 0} \left(\frac{x}{\sin x}\right)^2$$

$$= \left[\lim_{x\to 0} \frac{x}{\sin x}\right]^2$$

$$= 1^2 = \boxed{1}$$

Oct 4-10:39 AM

Evaluate $\lim_{x\to\infty} \frac{2x-3}{3x-2} = \frac{\infty}{\infty}$ I.F.

$$= \lim_{x\to\infty} \frac{\frac{2x}{x} - \frac{3}{x}}{\frac{3x}{x} - \frac{2}{x}} = \lim_{x\to\infty} \frac{2-\frac{3}{x}}{3-\frac{2}{x}}$$

$$= \frac{2 - \cancel{\lim_{x\to\infty}\frac{3}{x}}^{\to 0}}{3 - \cancel{\lim_{x\to\infty}\frac{2}{x}}^{\to 0}} = \frac{2-0}{3-0} = \boxed{\frac{2}{3}}$$

Evaluate $\lim_{x\to\infty} \frac{\pi x}{2-3x} = \frac{\infty}{-\infty}$ I.F.

$$\lim_{x\to\infty} \frac{\frac{\pi x}{x}}{\frac{2}{x} - \frac{3x}{x}} = \lim_{x\to\infty} \frac{\pi}{\cancel{\frac{2}{x}}^{\to 0} - 3} = -\frac{\pi}{3}$$

what about

$$\lim_{x\to\infty} \sin\left(\frac{\pi x}{2-3x}\right) = \sin\left(\frac{-\pi}{3}\right) = -\boxed{\sin\frac{\pi}{3}} = \boxed{-\frac{\sqrt{3}}{2}}$$

Recall $\sin(-\alpha) = -\sin\alpha$

Oct 4-10:44 AM

Evaluate

$$\lim_{x\to\infty}\left(\sqrt{x^2+10x} - \sqrt{x^2-10x}\right) = \infty - \infty \text{ I.F.} \quad \to 10$$

If $x = 100$, $\sqrt{100^2+10(100)} - \sqrt{100^2-10(100)} \approx 10.012555$

If $x = 1000$, $\sqrt{1000^2+10(1000)} - \sqrt{1000^2-10(1000)} \approx 10.000125\ldots$

$$\lim_{x\to\infty}\left(\sqrt{x^2+10x} - \sqrt{x^2-10x}\right) = \lim_{x\to\infty} \frac{(\overbrace{\sqrt{x^2+10x}}^{A} - \overbrace{\sqrt{x^2-10x}}^{B})(\overbrace{\sqrt{x^2+10x}}^{A} + \overbrace{\sqrt{x^2-10x}}^{B})}{\sqrt{x^2+10x} + \sqrt{x^2-10x}}$$

$$= \lim_{x\to\infty} \frac{(\sqrt{x^2+10x})^2 - (\sqrt{x^2-10x})^2}{\sqrt{x^2+10x} + \sqrt{x^2-10x}} = \lim_{x\to\infty} \frac{x^2+10x - (x^2-10x)}{\sqrt{x^2+10x} + \sqrt{x^2-10x}}$$

$$= \lim_{x\to\infty} \frac{20x}{\sqrt{x^2+10x} + \sqrt{x^2-10x}}$$

$$= \lim_{x\to\infty} \frac{\frac{20x}{x}}{\sqrt{\frac{x^2+10x}{x^2}} + \sqrt{\frac{x^2-10x}{x^2}}}$$

$$= \lim_{x\to\infty} \frac{20}{\sqrt{1+\frac{10}{x}} + \sqrt{1-\frac{10}{x}}} = \frac{20}{\sqrt{1}+\sqrt{1}} = \frac{20}{2} = \boxed{10}$$

Oct 4-10:52 AM

$$\lim_{x \to \infty} \frac{\cos\left(\frac{1}{x}\right)}{x}$$

Radian

Pick $x = 100$

Evaluate $\dfrac{\cos\left(\frac{1}{100}\right)}{100} = .0099995$

Pick $x = 1000$

$\dfrac{\cos\left(\frac{1}{1000}\right)}{1000} = 9.9 \times 10^{-4}$

$= .00099\ldots$

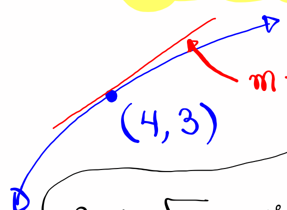
as $x \to \infty$ $\quad \dfrac{\cos \frac{1}{x}}{x} \to 0$

Try to evaluate

$$\lim_{x \to \infty} \frac{\cos\left(\frac{1}{x}\right)}{x}$$

Oct 4-11:04 AM

Find eqn of the tan. line to the graph of $f(x) = \sqrt{x} + 1$ at $x = 4$.



$m = f'(4) = \dfrac{1}{2\sqrt{4}} = \dfrac{1}{4}$

$f(x) = \sqrt{x} + 1 \qquad f(x) = x^{1/2} + 1$

$f'(x) = \dfrac{d}{dx}\left[x^{1/2} + 1\right] = \dfrac{d}{dx}\left[x^{1/2}\right] + \dfrac{d}{dx}[1]$

$= \dfrac{1}{2} x^{\frac{1}{2} - 1} + 0 = \dfrac{1}{2} x^{-1/2}$

$= \dfrac{1}{2\sqrt{x}}$

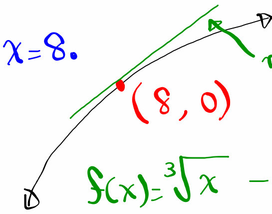
$y - y_1 = m(x - x_1)$

$y - 3 = \dfrac{1}{4}(x - 4) \longrightarrow \boxed{y = \dfrac{1}{4}x + 2}$

Oct 4-11:08 AM

Class QZ 10

Find equation of the tangent line in Slope-Int. form to the graph of $f(x)=\sqrt[3]{x}-2$ at $x=8$.



$$m = f'(8) = \frac{1}{3\sqrt[3]{8^2}} = \frac{1}{3\sqrt[3]{64}} = \boxed{\frac{1}{12}}$$

$$f(x)=\sqrt[3]{x}-2 \qquad f(x)=x^{1/3}-2$$

$$f'(x)=\frac{1}{3}x^{1/3-1}-0=\frac{1}{3}x^{-2/3}=\frac{1}{3x^{2/3}}=\boxed{\frac{1}{3\sqrt[3]{x^2}}}$$

$$y-y_1=m(x-x_1)$$

$$y-0=\frac{1}{12}(x-8) \rightarrow y=\frac{1}{12}x-\frac{8}{12} \rightarrow \boxed{y=\frac{1}{12}x-\frac{2}{3}}$$

Final Answer

Oct 4-11:16 AM