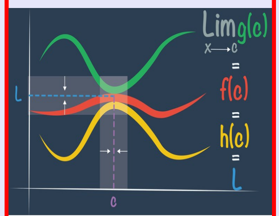
Math 261

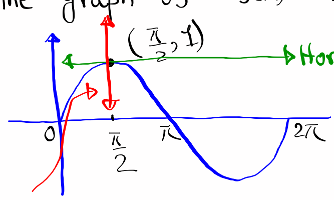
Fall 2023

Lecture 19



Feb 19-8:47 AM

find eqn of tan. line and normal line to the graph of $f(x) = \sin x$ at $x = \frac{\pi}{2}$.



→ Horizontal line $m = 0$

$y = 1$ ✓

Normal line → Vertical → No slope → $x = \frac{\pi}{2}$

$f(x) = \sin x$ $\qquad$ $f(\frac{\pi}{2}) = \sin\frac{\pi}{2} = \sin 90^\circ = 1 \rightarrow (\frac{\pi}{2}, 1)$

$f'(x) = \cos x$ $\qquad$ $f'(\frac{\pi}{2}) = \cos\frac{\pi}{2} = \cos 90^\circ = 0$

$m_{\text{tan. line}} = 0 \rightarrow$ Horizontal

$y - 1 = 0(x - \frac{\pi}{2}) \rightarrow \boxed{y = 1}$

$\underbrace{m_{\text{Normal line}}}_{\text{Vertical line}}$ undefined

$x = a$ $\qquad$ $\boxed{x = \frac{\pi}{2}}$

Oct 2-10:26 AM

More notation

$f(x)$ Function $y = f(x)$

$f'(x)$ f-Prime of x → First derivative

y' y-Prime → 〃

y'' y-double Prime → Second derivative

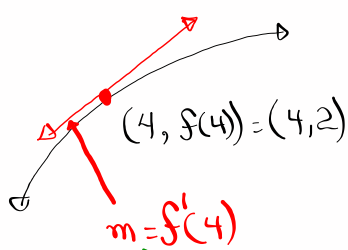
$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h} = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

→ m of Tan. line to graph of $f(x)$ at $(a, f(a))$

Oct 2-10:34 AM

Find $f'(4)$ for $f(x) = \sqrt{x}$.



$$f'(4) = \lim_{x \to 4} \frac{f(x) - f(4)}{x - 4}$$

$$= \lim_{x \to 4} \frac{\sqrt{x} - 2}{x - 4}$$

$$= \lim_{x \to 4} \frac{(\sqrt{x} - 2)(\sqrt{x} + 2)}{(x - 4)(\sqrt{x} + 2)} = \lim_{x \to 4} \frac{\overset{1}{\cancel{x - 4}}}{\cancel{(x-4)}(\sqrt{x} + 2)}$$

$$= \lim_{x \to 4} \frac{1}{\sqrt{x} + 2} = \frac{1}{\sqrt{4} + 2} = \boxed{\frac{1}{4}}$$

$y - y_1 = m(x - x_1)$

$y - 2 = \frac{1}{4}(x - 4)$

$y - 2 = \frac{1}{4}x - 1$

$$\boxed{y = \frac{1}{4}x + 1}$$

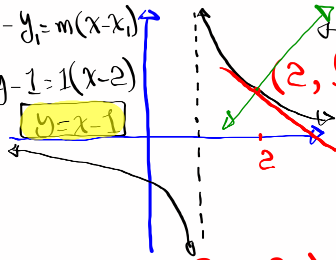
Oct 2-10:38 AM

Find eqn of the normal line to the graph of $f(x)=\frac{1}{x-1}$ at $x=2$.

$y-y_1=m(x-x_1)$

$y-1=1(x-2)$

$\boxed{y=x-1}$



$(2, f(2))=(2,1)$

$m=f'(2)$

$m=\frac{-1}{f'(2)}=\frac{-1}{-1}=\boxed{1}$

$$f'(2)=\lim_{x\to 2}\frac{f(x)-f(2)}{x-2}=\lim_{x\to 2}\frac{\frac{1}{x-1}-1}{x-2}$$

LCD $= x-1$

$$=\lim_{x\to 2}\frac{\cancel{(x-1)}\cdot\frac{1}{\cancel{x-1}}-1(x-1)}{(x-1)(x-2)}=\lim_{x\to 2}\frac{1-x+1}{(x-1)(x-2)}$$

$$=\lim_{x\to 2}\frac{\overset{-1}{\cancel{2-x}}}{(x-1)\cancel{(x-2)}}=\lim_{x\to 2}\frac{-1}{x-1}=\frac{-1}{2-1}=\boxed{-1}$$

Oct 2-10:45 AM

$y=f(x)$

$y'=f'(x)$

New notation $\quad y'=\frac{dy}{dx}=\frac{d}{dx}[y]=\frac{d}{dx}[f(x)]$

Derivative Rules:

$$\frac{d}{dx}\left[f(x)+g(x)\right]=\frac{d}{dx}[f(x)]+\frac{d}{dx}[g(x)]$$

$$\frac{d}{dx}\left[f(x)-g(x)\right]=\frac{d}{dx}[f(x)]-\frac{d}{dx}[g(x)]$$

ex: $\frac{d}{dx}\left[\sin x-\cos x\right]=\frac{d}{dx}[\sin x]-\frac{d}{dx}[\cos x]$

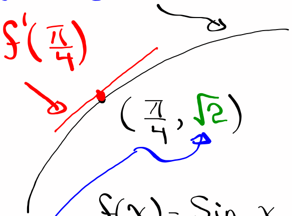
$=\cos x-(-\sin x)$

$=\boxed{\cos x+\sin x}$

Oct 2-10:53 AM

Find eqn of tan. line to the graph of $f(x) = \sin x + \cos x$ at $x = \frac{\pi}{4}$.

$m = f'\left(\frac{\pi}{4}\right)$



$\left(\frac{\pi}{4}, \sqrt{2}\right)$

$f\left(\frac{\pi}{4}\right) = \sin\frac{\pi}{4} + \cos\frac{\pi}{4}$

$= \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}$

$= \sqrt{2}$

$f(x) = \sin x + \cos x$

$f'(x) = \cos x + -\sin x$

$= \cos x - \sin x$

$f'\left(\frac{\pi}{4}\right) = \cos\frac{\pi}{4} - \sin\frac{\pi}{4} = \boxed{0}$

Horizontal tan. line

$y - \sqrt{2} = 0\left(x - \frac{\pi}{4}\right)$

$y - \sqrt{2} = 0$ $\quad \boxed{y = \sqrt{2}}$ $\quad \boxed{y = \sqrt{2}}$

Oct 2-10:58 AM

$$\frac{d}{dx}\left[f(x) \cdot g(x)\right] = \frac{d}{dx}\left[f(x)\right] \cdot g(x) + f(x) \cdot \frac{d}{dx}\left[g(x)\right]$$

$$= f'(x) \cdot g(x) + f(x) \cdot g'(x)$$

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{\frac{d}{dx}\left[f(x)\right] \cdot g(x) - f(x) \cdot \frac{d}{dx}\left[g(x)\right]}{\left[g(x)\right]^2}$$

$$= \frac{f'(x) \cdot g(x) - f(x) \cdot g'(x)}{\left[g(x)\right]^2}$$

Oct 2-11:04 AM

Find $f'(x)$ for $f(x) = \underbrace{\sin x \cos x}_{\text{Product}}$

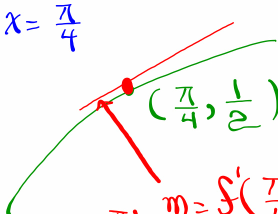
$$f'(x) = \frac{d}{dx}[\sin x] \cdot \cos x + \sin x \cdot \frac{d}{dx}[\cos x]$$

$$= \cos x \cdot \cos x + \sin x \cdot -\sin x$$

$$= \cos^2 x - \sin^2 x = \boxed{\cos 2x}$$

Find eqn of tan. line to $f(x) = \sin x \cos x$ at $x = \frac{\pi}{4}$

$$f\left(\tfrac{\pi}{4}\right) = \sin\tfrac{\pi}{4} \cdot \cos\tfrac{\pi}{4} = \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{2}}{2} = \frac{1}{2}$$



$$m = f'\left(\tfrac{\pi}{4}\right) = \cos 2\left(\tfrac{\pi}{4}\right) = \cos\tfrac{\pi}{2} = \boxed{0}$$

$$y - \tfrac{1}{2} = 0\left(x - \tfrac{\pi}{4}\right)$$

$$\boxed{y = \tfrac{1}{2}}$$

Oct 2-11:08 AM

Find $f'(x)$ for $f(x) = \tan x$

✓ $\frac{d}{dx}[\sin x] = \cos x$

$\Rightarrow \tan x = \frac{\sin x}{\cos x}$

✓ $\frac{d}{dx}[\cos x] = -\sin x$

$$\frac{d}{dx}[\tan x] = \frac{d}{dx}\left[\frac{\sin x}{\cos x}\right] = \frac{\frac{d}{dx}[\sin x] \cdot \cos x - \sin x \cdot \frac{d}{dx}[\cos x]}{[\cos x]^2}$$

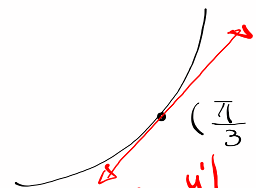
$$= \frac{\cos x \cdot \cos x - \sin x \cdot -\sin x}{\cos^2 x} = \frac{\cos^2 x + \sin^2 x}{\cos^2 x}$$

$$= \frac{1}{\cos^2 x} = \left[\frac{1}{\cos x}\right]^2 = \boxed{\sec^2 x}$$

✓ $\boxed{\frac{d}{dx}[\tan x] = \sec^2 x}$

Oct 2-11:16 AM

Find eqn of tan. line to the graph of $y = \tan x$ at $x = \frac{\pi}{3}$.



$\left(\frac{\pi}{3}, \sqrt{3}\right)$

$m = y'\Big|_{x=\frac{\pi}{3}} = f'\left(\frac{\pi}{3}\right) = 4$

at $x = \frac{\pi}{3}$

$y = \tan\frac{\pi}{3}$

$= \tan 60^\circ$

$= \sqrt{3}$

$y = \tan x$

$y' = \sec^2 x$

$m = \sec^2\frac{\pi}{3} = \left[\frac{1}{\cos\frac{\pi}{3}}\right]^2 = \left[\frac{1}{\frac{1}{2}}\right]^2 = 2^2 = 4$

$y - y_1 = m(x - x_1)$

$y - \sqrt{3} = 4\left(x - \frac{\pi}{3}\right) \Rightarrow \boxed{y = 4x - \frac{4\pi}{3} + \sqrt{3}}$

Oct 2-11:23 AM

Evaluate $\lim\limits_{x \to \pi/4} \dfrac{\tan x - 1}{x - \pi/4}$

Hint: $f(x) = \tan x$

$f'(a) = \lim\limits_{x \to a} \dfrac{f(x) - f(a)}{x - a}$

Oct 2-11:28 AM