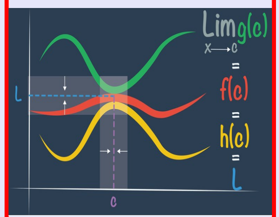
Math 261

Fall 2023

Lecture 18



Feb 19-8:47 AM

Class QZ 9

Use Calculus method to find all points on the graph of $\boxed{f(x) = x^2+4x+8}$ with $\underbrace{\text{horizontal Tangent line.}}_{m=0,\ f'(x)=0}$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{(x+h)^2+4(x+h)+8-(x^2+4x+8)}{h}$$

$$= \lim_{h\to 0} \frac{x^2+2xh+h^2+4x+4h+8-x^2-4x-8}{h}$$

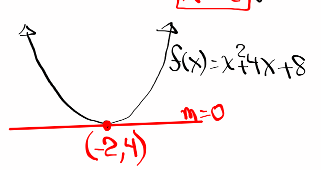
$$= \lim_{h\to 0} \frac{\cancel{h}(2x+h+4)}{\cancel{h}} = \lim_{h\to 0}(2x+h+4) = \overset{f'(x)}{\boxed{2x+4}}$$

$f'(x) = 0 \to 2x+4=0$

$\boxed{x=-2}$ ✓

$f(-2) = (-2)^2+4(-2)+8 = 4-8+8 = \boxed{4}$

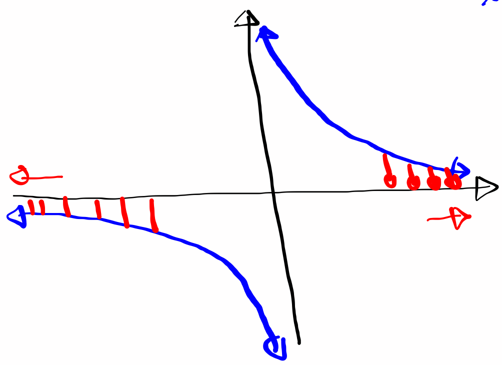
$\boxed{\text{Point } (-2,4)}$



Sep 27-11:16 AM

Now limits at $\pm\infty$

$f(x) = \frac{1}{x}$



$\lim_{x\to 0^+} \frac{1}{x} = \infty$

$\lim_{x\to 0^-} \frac{1}{x} = -\infty$

what about

$\boxed{\lim_{x\to\infty} \frac{1}{x} = 0}$ $\boxed{\lim_{x\to -\infty} \frac{1}{x} = 0}$

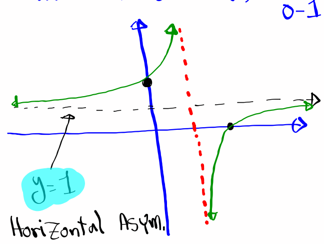
Sep 28-10:29 AM

$f(x) = \frac{x-2}{x-1}$ → $x=1$ is a vertical asymptote

Domain : $x \neq 1$

x-Int → $y=0$ → $f(x)=0$ → $\frac{x-2}{x-1} = 0$ → $x-2=0$ $\boxed{x=2}$

Y-Int → $x=0$ → $f(0) = \frac{0-2}{0-1} = \frac{-2}{-1} = \boxed{2}$



If let $x=100$

$f(100) = \frac{100-2}{100-1} = \frac{98}{99} \approx 1$

$x \to \infty$ → $f(x) \to 1$

If we let $x=-100$

$f(-100) = \frac{-102}{-101} \approx 1$

$\lim_{x\to\infty} \frac{x-2}{x-1} = \frac{\infty}{\infty}$ I.F.

Divide by x

$$\lim_{x\to\infty} \frac{\frac{x}{x} - \frac{2}{x}}{\frac{x}{x} - \frac{1}{x}} = \lim_{x\to\infty} \frac{1-\frac{2}{x}}{1-\frac{1}{x}} = \frac{\overbrace{\lim_{x\to\infty} 1}^{1} - \overbrace{\lim_{x\to\infty} \frac{2}{x}}^{0}}{\underbrace{\lim_{x\to\infty} 1}_{1} - \underbrace{\lim_{x\to\infty} \frac{1}{x}}_{0}}$$

$$= \frac{1-0}{1-0} = \boxed{1}$$

Sep 28-10:32 AM

Evaluate $\lim\limits_{x\to\infty} \dfrac{x^2-5x+8}{2x^2+3x-4} = \dfrac{\infty}{\infty}$ I.F.

Divide everything by highest power of x from the denominator. $\Rightarrow x^2$

$$= \lim_{x\to\infty} \frac{\frac{x^2}{x^2} - \frac{5x}{x^2} + \frac{8}{x^2}}{\frac{2x^2}{x^2} + \frac{3x}{x^2} - \frac{4}{x^2}} = \lim_{x\to\infty} \frac{1 - \frac{5}{x} + \frac{8}{x^2}}{2 + \frac{3}{x} - \frac{4}{x^2}}$$

$$= \frac{\lim\limits_{x\to\infty} 1 - \lim\limits_{x\to\infty} \frac{5}{x} + \lim\limits_{x\to\infty} \frac{8}{x^2}}{\lim\limits_{x\to\infty} 2 + \lim\limits_{x\to\infty} \frac{3}{x} - \lim\limits_{x\to\infty} \frac{4}{x^2}} = \frac{1-0+0}{2+0-0} = \boxed{\frac{1}{2}}$$

Sep 28-10:42 AM

Evaluate $\lim\limits_{x\to\infty} \dfrac{3x+5}{\sqrt{4x^2-1}} = \dfrac{\infty}{\infty}$ I.F.

Divide everything by x, Keep in mind $x=\sqrt{x^2}$ as $x\to\infty$

$$= \lim_{x\to\infty} \frac{\frac{3x}{x} + \frac{5}{x}}{\frac{\sqrt{4x^2-1}}{x}} = \lim_{x\to\infty} \frac{3 + \frac{5}{x}}{\sqrt{\frac{4x^2-1}{x^2}}}$$

$$= \lim_{x\to\infty} \frac{3 + \frac{5}{x}}{\sqrt{\frac{4x^2}{x^2} - \frac{1}{x^2}}} = \lim_{x\to\infty} \frac{3 + \cancel{\frac{5}{x}}^{\to 0}}{\sqrt{4 - \cancel{\frac{1}{x^2}}^{\to 0}}}$$

$$= \frac{3}{\sqrt{4}} = \boxed{\frac{3}{2}}$$

Sep 28-10:51 AM

Evaluate $\lim_{x\to\infty} x\sin\frac{1}{x} = \infty\cdot 0$ I.F.

$\frac{1}{x}\to 0$ as $x\to\infty$

$\sin\frac{1}{x}\to 0$ as $x\to\infty$

Let $h=\frac{1}{x}$

as $x\to\infty$, $h\to 0$

$hx=1 \to x=\frac{1}{h}$

$\lim_{x\to\infty} x\sin\frac{1}{x} =$

$\lim_{h\to 0} \frac{1}{h}\sin h$

$=\lim_{h\to 0}\frac{\sin h}{h} = \boxed{1}$

Make sure you are in radians,

evaluate $1000\cdot\sin\frac{1}{1000} = \boxed{.9999998333}$

x

Evaluate $1000000\cdot\sin\frac{1}{1000000} = 1$

Sep 28-11:01 AM

Evaluate $\lim_{x\to\infty}\left(\sqrt{x^2+4x}-x\right) = \infty-\infty$ I.F.

$$=\lim_{x\to\infty}\frac{\left(\sqrt{x^2+4x}-x\right)\left(\sqrt{x^2+4x}+x\right)}{1\left(\sqrt{x^2+4x}+x\right)}$$

I.F.

$$=\lim_{x\to\infty}\frac{\cancel{x^2}+4x-\cancel{x^2}}{\sqrt{x^2+4x}+x} = \lim_{x\to\infty}\frac{4x}{\sqrt{x^2+4x}+x} = \frac{\infty}{\infty}$$

as $x\to\infty$

$x=\sqrt{x^2}$

$$=\lim_{x\to\infty}\frac{\frac{4x}{x}}{\sqrt{\frac{x^2+4x}{x^2}}+\frac{x}{x}}$$

$$=\lim_{x\to\infty}\frac{4}{\sqrt{1+\cancel{\frac{4}{x}}^{\to 0}}+1} = \frac{4}{\sqrt{1}+1} = \frac{4}{2} = \boxed{2}$$

If $x=1000$

$\sqrt{1000^2+4(1000)}-1000 = 1.99800399$

If $x=1000000$

$\sqrt{1000000^2+4(1000000)}-1000000 = 1.999998$

Sep 28-11:11 AM

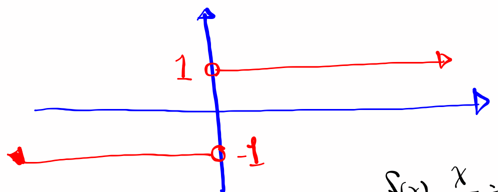
Evaluate $\lim_{x \to \infty} \cos \frac{1}{x}$

$= \cos\left(\lim_{x \to \infty} \frac{1}{x}\right)$

$= \cos 0 = \boxed{1}$

Evaluate $\lim_{x \to 0^+} \frac{x}{|x|} = \lim_{x \to 0^+} \frac{x}{x} = \boxed{1}$

Evaluate $\lim_{x \to 0^-} \frac{x}{|x|} = \lim_{x \to 0^-} \frac{x}{-x} = \boxed{-1}$



$$f(x) = \frac{x}{|x|} = \begin{cases} -1 & \text{if } x<0 \\ 1 & \text{if } x>0 \end{cases}$$