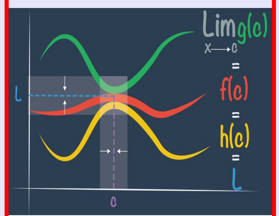
Math 261

Fall 2023

Lecture 17



Feb 19-8:47 AM

Evaluate $\lim_{x \to 0} \frac{4x - \sin kx}{x} = \frac{4(0) - \sin 0}{0} = \frac{0}{0}$ I.F.

$$= \lim_{x \to 0} \left[\frac{4x}{x} - \frac{k \sin kx}{kx} \right]$$

$$= \lim_{x \to 0} \left[4 - k \cdot \frac{\sin kx}{kx} \right]$$

$$= 4 - k \cdot \underbrace{\lim_{x \to 0} \frac{\sin kx}{kx}}_{\to 1} = 4 - k \cdot 1$$

$$= \boxed{4 - k}$$

Sep 27-10:25 AM

Evaluate $\lim\limits_{x\to 0}\dfrac{x\sin x}{1-\cos x} = \dfrac{0\sin 0}{1-\cos 0} = \dfrac{0}{0}$ I.F.

$$=\lim_{x\to 0}\frac{x\sin x\cdot(1+\cos x)}{(1-\cos x)\cdot(1+\cos x)} \qquad 1-\cos^2 x = \sin^2 x$$

$$=\lim_{x\to 0}\frac{x\cancel{\sin x}(1+\cos x)}{\sin^{\cancel{2}} x}$$

Divide top & Bottom by x

$$=\lim_{x\to 0}\frac{1+\cos x}{\frac{\sin x}{x}}$$

$$=\frac{\lim\limits_{x\to 0}(1+\cos x)}{\lim\limits_{x\to 0}\frac{\sin x}{x}} = \frac{1+\cos 0}{1} = \frac{1+1}{1} = \boxed{2}$$

Sep 27-10:28 AM

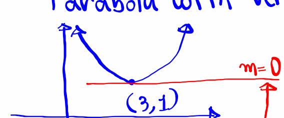
Find $\boxed{\text{points}}$ on the graph of $f(x)=x^2-6x+10$ $\to (3,1)$
that we have horizontal tan. line.

Non-Calculus method:

$$f(x)=x^2-6x+10 = \underline{x^2-6x+9}+1 = (x-3)^2+1$$

$$f(x)=(x-3)^2+1$$

Parabola with vertex at $(3,1)$



$2x-6=0$

$\boxed{x=3}$ $\quad f(3)=3^2-6(3)+10 = \boxed{1}$

$\boxed{\text{Point } (3,1)}$

Calculus Method

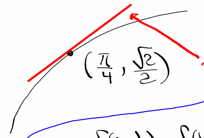
$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\frac{(x+h)^2-6(x+h)+10-x^2+6x-10}{h}$$

$$=\lim_{h\to 0}\frac{\cancel{x^2}+2xh+h^2-\cancel{6x}-6h+\cancel{10}-\cancel{x^2}+\cancel{6x}-\cancel{10}}{h}$$

$$=\lim_{h\to 0}\frac{\cancel{h}(2x+h-6)}{\cancel{h}} = \lim_{h\to 0}(2x+h-6) = \boxed{2x-6}$$

Sep 27-10:35 AM

Find equation of the tan. line to the graph of $f(x) = \sin x$ at the point with $x = \pi/4$.



$f(\frac{\pi}{4}) = \sin \pi/4 = \frac{\sqrt{2}}{2}$

$m = f'(\frac{\pi}{4}) = \cos \pi/4 = \boxed{\frac{\sqrt{2}}{2}}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{\sin(x+h) - \sin x}{h}$$

Recall $\sin(A+B) = \dots$

$$= \lim_{h \to 0} \frac{\sin x \cos h + \cos x \sin h - \sin x}{h}$$

$$= \lim_{h \to 0} \frac{\sin x [\cos h - 1] + \cos x \sin h}{h}$$

$$= \lim_{h \to 0} \left[\frac{\sin x (\cos h - 1)}{h} + \frac{\cos x \sin h}{h} \right]$$

$$= \sin x \cdot \lim_{h \to 0} \frac{\cos h - 1}{h} + \cos x \cdot \lim_{h \to 0} \frac{\sin h}{h}$$

$$= \sin x \cdot 0 + \cos x \cdot 1 = \boxed{\cos x}$$

Point $(\pi/4, \frac{\sqrt{2}}{2})$

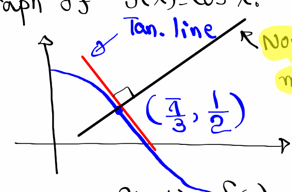
slope $\frac{\sqrt{2}}{2}$

$\Rightarrow y - y_1 = m(x - x_1)$

$y - \frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{2}(x - \frac{\pi}{4})$ $\Rightarrow$ $\boxed{y = \quad}$

Sep 27-10:44 AM

Find eqn. of the $\boxed{\text{normal line}}$ at $x = \frac{\pi}{3}$ on the graph of $f(x) = \cos x$.



$f(\frac{\pi}{3}) = \cos \frac{\pi}{3} = \frac{1}{2}$

$m = \frac{-1}{f'(\frac{\pi}{3})} = \frac{-1}{-\sin \frac{\pi}{3}} = \frac{1}{\frac{\sqrt{3}}{2}} = \frac{2}{\sqrt{3}} = \boxed{\frac{2\sqrt{3}}{3}}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{\cos(x+h) - \cos x}{h}$$

$$= \lim_{h \to 0} \frac{\cos x \cos h - \sin x \sin h - \cos x}{h}$$

$$= \lim_{h \to 0} \left[\frac{\cos x [\cos h - 1]}{h} - \frac{\sin x \sin h}{h} \right]$$

$$= \cos x \lim_{h \to 0} \frac{\cos h - 1}{h} - \sin x \lim_{h \to 0} \frac{\sin h}{h} = \boxed{-\sin x}$$

Point $(\frac{\pi}{3}, \frac{1}{2})$

Slope $\frac{2\sqrt{3}}{3}$

$y - y_1 = m(x - x_1)$

$y - \frac{1}{2} = \frac{2\sqrt{3}}{3}(x - \frac{\pi}{3})$ $\Rightarrow$ $\boxed{y = \quad}$

Sep 27-10:54 AM

$f(x) = \sin x \rightarrow f'(x) = \cos x$

$f(x) = \cos x \rightarrow f'(x) = -\sin x$

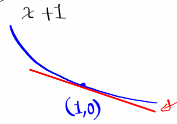
Important

Find eqn of tan. line at the x-Int of

$f(x) = \frac{x-1}{x+1}$.

$y=0 \rightarrow f(x)=0$

$\frac{x-1}{x+1} = 0 \rightarrow \boxed{x=1}$



$m = f'(1)$

$= \frac{2}{(1+1)^2} = \frac{2}{4} = \frac{1}{2}$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{\frac{x+h-1}{x+h+1} - \frac{x-1}{x+1}}{h}$$

LCD $(x+h+1)(x+1)$

$$= \lim_{h\to 0} \frac{(x+1)(x+h-1)-(x-1)(x+h+1)}{h(x+h+1)(x+1)}$$

$$= \lim_{h\to 0} \frac{\overbrace{\cancel{x^2}+\cancel{xh}-\cancel{x}+\cancel{x}+h-\cancel{1}-\cancel{x^2}-\cancel{xh}-\cancel{x}+\cancel{x}+h+\cancel{1}}^{2\cancel{h}}}{\cancel{h}(x+h+1)(x+1)}$$

$$= \lim_{h\to 0} \frac{2}{(x+h+1)(x+1)} = \boxed{\frac{2}{(x+1)^2}} \rightarrow f'(x)$$

Point $(1,0)$

Slope $\frac{1}{2}$

$y - y_1 = m(x - x_1)$

$y - 0 = \frac{1}{2}(x-1) \rightarrow \boxed{y=\quad}$

Sep 27-11:05 AM

Class QZ 9

Use Calculus method to find all points on the graph of $f(x) = x^2+4x+8$ with horizontal Tangent line.

$m=0$, $f'(x)=0$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{(x+h)^2+4(x+h)+8-x^2-4x-8}{h}$$

$$= \lim_{h\to 0} \frac{x^2+2xh+h^2+4x+4h+8-x^2-4x-8}{h}$$

$$= \lim_{h\to 0} \frac{h(2x+h+4)}{h} = \lim_{h\to 0}(2x+h+4) = \boxed{2x+4}$$

$f'(x)=0 \rightarrow 2x+4=0$ $\boxed{x=-2}$

$f(-2) = (-2)^2+4(-2)+8 = 4-8+8 = \boxed{4}$

$\boxed{\text{Point } (-2,4)}$

Sep 27-11:16 AM