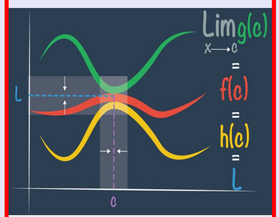
Math 261

Fall 2023

Lecture 16



Feb 19-8:47 AM

More work with Trig.:

1) $f(x) = \dfrac{x}{1-2\sin x}$ over $[0, 2\pi]$

Domain $1 - 2\sin x \neq 0$

$\sin x \neq \frac{1}{2}$

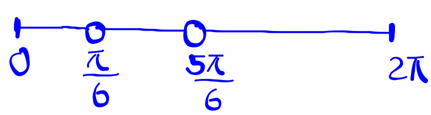
$\sin x = \frac{1}{2}$ in QI & QII

Ref. Angle is $30^\circ = \frac{\pi}{6}$

QI $x = \frac{\pi}{6}$

QII $x = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$

Domain $[0, \frac{\pi}{6}) \cup (\frac{\pi}{6}, \frac{5\pi}{6}) \cup (\frac{5\pi}{6}, 2\pi]$



$f(x)$ is cont. on $[0, 2\pi]$ except at $x = \frac{\pi}{6}$ and $x = \frac{5\pi}{6}$

Sep 26-10:26 AM

Evaluate $\lim_{x\to 0} \frac{\tan 8x}{\sin 5x} = \frac{\tan 8(0)}{\sin 5(0)} = \frac{\tan 0}{\sin 0} = \frac{0}{0}$ I.F.

$$= \lim_{x\to 0} \frac{\frac{\sin 8x}{\cos 8x}}{\sin 5x} = \lim_{x\to 0} \frac{1}{\cos 8x} \cdot \frac{8\frac{\sin 8x}{8x}}{5\frac{\sin 5x}{5x}}$$

$$= \frac{8}{5} \cdot \lim_{x\to 0} \frac{1}{\cos 8x} \cdot \lim_{x\to 0} \frac{\frac{\sin 8x}{8x}}{\frac{\sin 5x}{5x}}$$

$$= \frac{8}{5} \cdot \frac{1}{1} \cdot \frac{\lim_{x\to 0} \frac{\sin 8x}{8x} \to 1}{\lim_{x\to 0} \frac{\sin 5x}{5x} \to 1} = \frac{8}{5} \cdot \frac{1}{1} \cdot \frac{1}{1} = \boxed{\frac{8}{5}}$$

Sep 26-10:31 AM

Find K such that $f(x) = \begin{cases} \frac{\tan Kx}{x} & \text{if } x<0 \\ 3x+2K^2 & \text{if } x\geq 0 \end{cases}$

will be continuous at $x=0$.

$$\lim_{x\to 0} f(x) = f(0)$$

$f(0) = 3(0)+2K^2 = 2K^2$

$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x)$$

$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} \frac{\tan Kx}{x} = \lim_{x\to 0} \frac{\sin Kx}{\cos Kx \cdot x}$$

$$= \lim_{x\to 0} \frac{1}{\cos Kx} \to 1 \cdot \lim_{x\to 0} \frac{K\sin Kx}{Kx}$$

$$= 1 \cdot K \cdot \lim_{x\to 0} \frac{\sin Kx}{Kx} \to 1 = \boxed{K}$$

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} (3x+2K^2) = \boxed{2K^2}$$

$2K^2 = K$

$2K^2 - K = 0$

$K(2K-1) = 0$

$\boxed{K=0}$ $\boxed{K=\frac{1}{2}}$

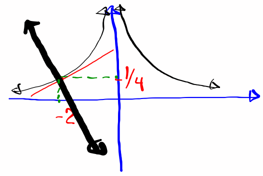
If K is nonzero, then $\boxed{K=\frac{1}{2}}$

Sep 26-10:37 AM

Find equation of the normal line to the graph of $f(x)=\frac{1}{x^2}$ at $x=-2$.

$f(x)=\frac{1}{x^2}$ is an even function $f(-x)=\frac{1}{(-x)^2}=\frac{1}{x^2}=f(x)$

even functions are symmetric with respect to Y-axis.



$f(x)=\frac{1}{x^2}$, $x\neq 0$

$f(-2)=\frac{1}{(-2)^2}=\frac{1}{4}$

$$m_{\text{Normal line}}=\frac{-1}{m_{\text{tan. line}}}=\frac{-1}{f'(-2)}$$

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\frac{\frac{1}{(x+h)^2}-\frac{1}{x^2}}{h}=\cdots=\boxed{\frac{-2}{x^3}}$$

$$f'(-2)=\frac{-2}{(-2)^3}=\frac{-2}{-8}=\frac{1}{4}\qquad m_{\text{Normal Line}}=\frac{-1}{\frac{1}{4}}=\boxed{-4}$$

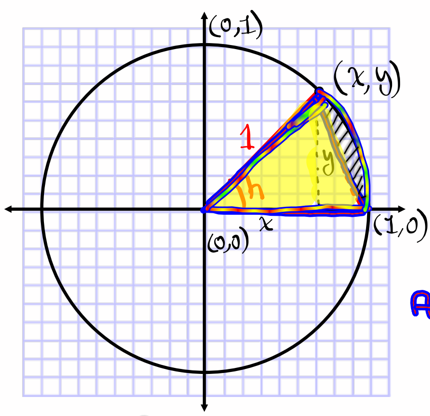
$$y-y_1=m(x-x_1)$$

$$y-\frac{1}{4}=-4(x-{}^{-}2)\Longrightarrow \boxed{y=\quad}$$

Slope-Int. Form

Sep 26-10:45 AM

Consider a unit circle, centered at $(0,0)$.



$\sin h=\frac{y}{1}$, $\cos h=\frac{x}{1}$

$y=\sin h$, $x=\cos h$

$$\text{Area}=\frac{b\cdot h}{2}=\frac{x\cdot y}{2}=\frac{\sin h\cdot\cos h}{2}$$

$$\text{Area}=\frac{b\cdot h}{2}=\frac{1\cdot\sin h}{2}$$

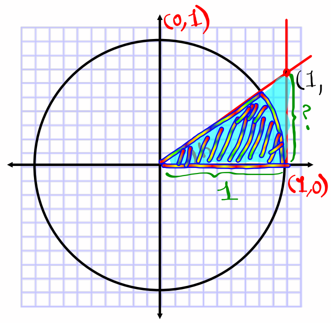
Area of Sector with Central angle h.

$$A_{\text{sector}}=\frac{1}{2}r^2\theta=\frac{1}{2}\cdot 1^2\cdot h=\frac{h}{2}$$

Area of Purple triangle < Area of Sector

$$\frac{1}{2}\sin h<\frac{h}{2}$$

Sep 26-10:54 AM



$\tan h = \frac{\text{Opposite}}{\text{Adjacent}} = \frac{?}{1}$

$\tan h = ?$

Area of Blue triangle $= \frac{bh}{2}$

$= \frac{1 \cdot \tan h}{2}$

$= \frac{\tan h}{2}$

Yellow Triangle < Sector < Blue Triangle

$\frac{1}{2}\sin h < \frac{1}{2}h < \frac{1}{2}\tan h$

multiply by 2, and replace $\tan h = \frac{\sin h}{\cos h}$

$\sin h < h < \frac{\sin h}{\cos h}$ → $1 < \frac{h}{\sin h} < \frac{1}{\cos h}$

Divide by Sinh

Do reciprocal $\quad \cos h < \frac{\sin h}{h} < 1$

$\lim_{h \to 0} \cos h = 1$, $\lim_{h \to 0} 1 = 1$

By S.T.,

$$\boxed{\lim_{h \to 0} \frac{\sin h}{h} = 1}$$

Sep 26-11:05 AM

Prove $\lim_{h \to 0} \frac{1 - \cos h}{h} = 0$ ✓

By direct Subs. $\lim_{h \to 0} \frac{1 - \cos h}{h} = \frac{1 - \cos 0}{0}$

$= \frac{1-1}{0}$

$= \frac{0}{0}$ I.F.

$$\lim_{h \to 0} \frac{1 - \cos h}{h} = \lim_{h \to} \frac{1 - \cos h}{h} \cdot \frac{1 + \cos h}{1 + \cos h}$$

$$= \lim_{h \to 0} \frac{1 - \cos^2 h}{h(1 + \cos h)} = \lim_{h \to 0} \frac{\sin^2 h}{h(1 + \cos h)}$$

$$= \lim_{h \to 0} \left(\frac{\sin h}{h} \cdot \frac{1}{1 + \cos h} \cdot \sin h \right)$$

$$= \lim_{h \to 0} \frac{\sin h}{h} \cdot \lim_{h \to 0} \frac{1}{1 + \cos h} \cdot \lim_{h \to 0} \sin h$$

$$= 1 \cdot \frac{1}{1+1} \cdot 0 = \boxed{0}$$

Sep 26-11:17 AM