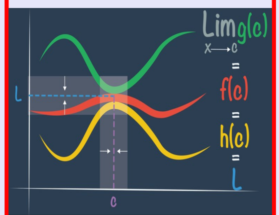
Math 261

Fall 2023

Lecture 14



Feb 19-8:47 AM

Given $f(x) = x^3 + 2x^2 - 3$

1) find $f(1)$ $\quad f(1) = 1^3 + 2(1)^2 - 3 = 0 \rightarrow (1,0)$

2) Use limit definition to find $f'(x)$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{(x+h)^3 + 2(x+h)^2 - 3 - x^3 - 2x^2 + 3}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{x^3} + 3x^2h + 3xh^2 + h^3 + \cancel{2x^2} + 4xh + 2h^2 \cancel{-3} \cancel{-x^3} \cancel{-2x^2} \cancel{+3}}{h}$$

$$= \lim_{h\to 0} \frac{3x^2h + 3xh^2 + h^3 + 4xh + 2h^2}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h}(3x^2 + 3x\cancel{h}^{0} + \cancel{h^2}^{0} + 4x + \cancel{2h}^{0})}{\cancel{h}} = \boxed{3x^2 + 4x}$$

$f'(x)$

Sep 20-10:30 AM

3) Find $f'(1)$

$f'(x) = 3x^2 + 4x$

$f'(1) = 3(1)^2 + 4(1) = \boxed{7}$

4) Simplify $y - f(1) = f'(1)(x-1)$

$y - 0 = 7(x-1)$

$y = 7(x-1)$ $\quad \boxed{y = 7x - 7}$

Sep 20-10:38 AM

For $\varepsilon = .01$, find a $\delta > 0$ such that

$\lim\limits_{x \to 1} (2x^2 - 3x + 1) = 0$ ✓

$f(x) = 2x^2 - 3x + 1$ $\qquad \lim\limits_{x \to 1} (2x^2 - 3x + 1) = 2(1)^2 - 3(1) + 1$

$L = 0$ ✓ $\qquad = 2 - 3 + 1 = 0$ ✓

$a = 1$

For every $\varepsilon > 0$, there is a $\delta > 0$ such that

$|f(x) - L| < \varepsilon$ whenever $|x - a| < \delta$

$|2x^2 - 3x + 1 - 0| < \varepsilon$ whenever $|x - 1| < \delta$

$|2x^2 - 3x + 1| < \varepsilon$

$|(2x-1)(x-1)| < \varepsilon$ $\qquad \delta = \frac{\varepsilon}{C}$

$|2x - 1||x - 1| < \varepsilon$

If $|2x-1| < C$, then $|x - 1| < \frac{\varepsilon}{C}$

If we wish $\delta \le 1$, then

$|x - 1| < \delta \le 1$

$|x-1| < 1 \to -1 < x - 1 < 1$

Add 1

$0 < x < 2$

Multiply by 2 $\qquad$ Subtract 1

$0 < 2x < 4 \qquad -1 < 2x - 1 < 3$

$2x - 1 < 3 \to |2x - 1| < 3$ ← C $\qquad$ for $\varepsilon = .01$ δ

Now $\delta = \min\left\{1, \frac{\varepsilon}{3}\right\}$

For $\varepsilon = .01$, $\delta = \min\left\{1, \frac{.01}{3}\right\} = \min\left\{1, \frac{1}{300}\right\} = \boxed{\frac{1}{300}}$

Sep 20-10:40 AM

$$f(x)=\begin{cases}\dfrac{x^4-16}{x^2-4} & \text{if } x\neq -2\\[4pt] Kx+5 & \text{if } x=-2\end{cases}$$

Find K such that $f(x)$ is Cont. at $x=-2$.

$f(-2)=K(-2)+5=\boxed{-2K+5}$ $\quad \lim\limits_{x\to -2} f(x)=f(-2)$ ✓

$$\lim_{x\to -2} f(x)=\lim_{x\to -2}\frac{x^4-16}{x^2-4}=\frac{(-2)^4-16}{(-2)^2-4}=\frac{16-16}{4-4}=\frac{0}{0}\ \text{I.F.}$$

$$=\lim_{x\to -2}\frac{(x^2+4)\cancel{(x^2-4)}}{\cancel{x^2-4}}=\lim_{x\to -2}(x^2+4)=(-2)^2+4=\boxed{8}$$

$-2K+5=8$

$-2K=3 \to \boxed{K=\frac{-3}{2}}$

Sep 20-10:50 AM

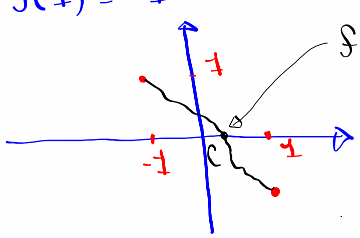
Show $x^3+x^2-2x=1$ has a solution in the interval $[-1,1]$.

$\underbrace{x^3+x^2-2x-1}_{\text{Polynomial Function}}=0$ → Continuous everywhere.

Use Intermediate Value Theorem

$f(-1)=(-1)^3+(-1)^2-2(-1)-1=1$

$f(1)=1^3+1^2-2(1)-1=-1$



$f(c)=0$

By I.V.T., equation $x^3+x^2-2x=1$ in $(-1,1)$

Sep 20-10:56 AM

Suppose $2\cos x - 1 \le f(x) \le x^3 + 1$ Near $x=0$

Find $\lim\limits_{x \to 0} f(x)$.

$$\lim_{x\to 0}(2\cos x - 1) = 2\cos 0 - 1 = 2(1) - 1 = \boxed{1}$$

$$\lim_{x\to 0}(x^3+1) = 0^3 + 1 = 0 + 1 = \boxed{1}$$

By S.T., $\lim\limits_{x\to 0} f(x) = 1$

Sep 20-11:02 AM

Evaluate $\lim\limits_{x\to 0} \dfrac{\sqrt{x^2+9} - 3}{x} = \dfrac{\sqrt{0^2+9} - 3}{0} = \dfrac{\sqrt{9}-3}{0} = \dfrac{0}{0}$ I.F.

$$= \lim_{x\to 0} \frac{(\sqrt{x^2+9} - 3)(\sqrt{x^2+9} + 3)}{x(\sqrt{x^2+9} + 3)}$$

$$= \lim_{x\to 0} \frac{x^2 \cancel{+9} \cancel{-9}}{x(\sqrt{x^2+9} + 3)} = \lim_{x\to 0} \frac{\cancel{x^2}^{\,x}}{\cancel{x}(\sqrt{x^2+9} + 3)}$$

$$= \lim_{x\to 0} \frac{x}{\sqrt{x^2+9} + 3}$$

$$= \frac{0}{\sqrt{0^2+9} + 3} = \frac{0}{6}$$

$$= \boxed{0}$$

Sep 20-11:07 AM

Class QZ 8

For $\varepsilon = .1$, find a $\delta > 0$ such that

$\lim_{x \to 2} (x^2 + 4x) = 12$.

$f(x) = x^2 + 4x$

$L = 12$ ✓

$a = 2$

$\lim_{x \to 2} (x^2 + 4x) = 2^2 + 4(2) = 4 + 8 = 12$ ✓

For any $\varepsilon > 0$, there is a $\delta > 0$ such that

$|f(x) - L| < \varepsilon$ whenever $|x - a| < \delta$

$|x^2 + 4x - 12| < \varepsilon$ " $|x - 2| < \delta$

$|(x+6)(x-2)| < \varepsilon$ " $|x - 2| < \delta$

$|x + 6|\,|x - 2| < \varepsilon$

If $|x+6| < C$, then $|x - 2| < \frac{\varepsilon}{C}$

If we wish $\delta \le 1$, then

$|x - 2| < 1$

$-1 < x - 2 < 1$

Add 8

$7 < x + 6 < 9$

$|x + 6| < 9 \leftarrow C$

Pick

$\delta = \min\left\{1, \frac{\varepsilon}{9}\right\}$

For $\varepsilon = .1$

$\delta = \min\left\{1, \frac{.1}{9}\right\}$

$= \boxed{\frac{1}{90}}$

Sep 20-11:12 AM