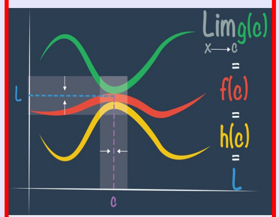
Math 261

Fall 2023

Lecture 11



Feb 19-8:47 AM

Given $f(x) = x^3$, evaluate

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}.$$

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{(x+h)^3 - x^3}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{x^3} + 3x^2h + 3xh^2 + h^3 - \cancel{x^3}}{h} = \lim_{h \to 0} \frac{\cancel{h}(3x^2 + 3xh + h^2)}{\cancel{h}}$$

$$= \lim_{h \to 0} \left[3x^2 + 3xh + h^2\right] = 3x^2 + 3x(0) + 0^2 = \boxed{3x^2}$$

Sep 14-10:25 AM

For $\boxed{\varepsilon = .1}$, Find $\delta > 0$ Such that $\lim_{x \to 2} \left(\frac{1}{2}x + 3\right) = 4$

1) Verify the limit

$\lim_{x \to 2} \left(\frac{1}{2}x + 3\right) = \frac{1}{2}(2) + 3 = 4 \checkmark$

2) $|f(x) - L| < \varepsilon$ whenever $|x - a| < \delta$

$\left|\frac{1}{2}x + 3 - 4\right| < .1$ $\qquad$ $|x - 2| < \delta$

$\left|\frac{1}{2}x - 1\right| < .1$

Multiply both sides by 2

$2\left|\frac{1}{2}x - 1\right| < 2(.1)$ $\qquad$ $\boxed{\delta = .2}$

$|x - 2| < .2$

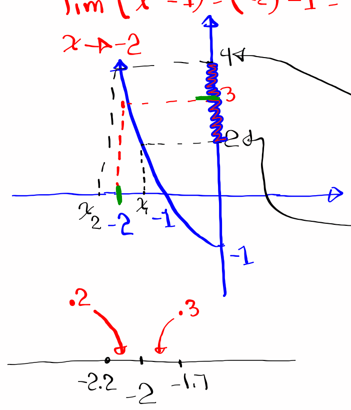
Sep 14-10:29 AM

For $\varepsilon = 1$, Find $\delta > 0$ Such that $\lim_{x \to -2} (x^2 - 1) = 3 \checkmark$

1) Verify the limit

$\lim_{x \to -2} (x^2 - 1) = (-2)^2 - 1 = 4 - 1 = 3 \checkmark$

$f(x) = x^2 - 1$



For $\varepsilon = 1$

$f(x) = 4$

$x^2 - 1 = 4$

$x^2 = 5$ $\qquad$ $\boxed{x = -2.2}$

$f(x) = 2$

$x^2 - 1 = 2$

$x^2 = 3$ $\qquad$ $\boxed{x = -1.7}$

Pick δ to be smaller distance

Not Symmetric about $x = -2$

For $\varepsilon = 1$, there is $\delta = .2$ Such that $\boxed{\delta = .2} \checkmark$

$\lim_{x \to -2} (x^2 - 1) = 3$

Sep 14-10:33 AM

For $\varepsilon > 0$, there is a $\delta > 0$ such that

$|f(x) - L| < \varepsilon$ whenever $|x - a| < \delta$

$|x^2 - 1 - 3| < \varepsilon$ " $|x - (-2)| < \delta$

$|x^2 - 4| < \varepsilon$ " $|x + 2| < \delta$

$|(x-2)(x+2)| < \varepsilon$

$\underbrace{|x-2|}_{\text{Bound}} \; \underbrace{|x+2|}_{\text{Keep}} < \varepsilon$ $\qquad \delta = \frac{\varepsilon}{C}$

Let's say $|x - 2| < C$, then $|x+2| < \frac{\varepsilon}{C}$

Say δ has to be no more than 1.

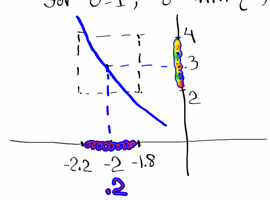
$|x + 2| < 1$ $\qquad -5 < x - 2 < -3 < 5$

$-1 < x + 2 < 1$ $\qquad -5 < x - 2 < 5$

Subtract -4 $\qquad |x - 2| < 5$

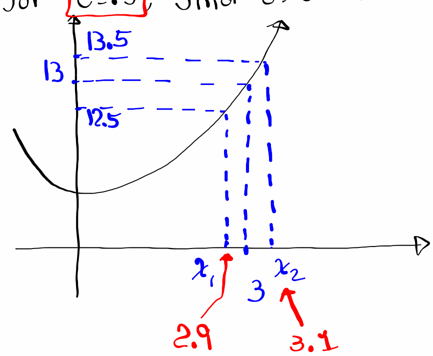
$-1 - 4 < x + 2 - 4 < 1 - 4$ $\qquad \delta = \min\left\{1, \frac{\varepsilon}{5}\right\}$

For $\varepsilon = 1$, $\delta = \min\left\{1, \frac{1}{5}\right\} = \frac{1}{5} = \boxed{.2}$



Sep 14-10:42 AM

For $\boxed{\varepsilon = .5}$, find $\delta > 0$ such that $\lim_{x \to 3} (x^2 + 2x - 2) = 13$. ✓



$f(x) = 12.5$

$x^2 + 2x - 2 = 12.5$

$x^2 + 2x + 1 = 14.5 + 1$

$(x+1)^2 = 15.5$

$x + 1 = \sqrt{15.5}$

$x = \sqrt{15.5} - 1$

$\boxed{x = 2.9}$

$f(x) = 13.5$

$x^2 + 2x - 2 = 13.5$

$x^2 + 2x + 1 = 15.5 + 1$

$(x+1)^2 = 16.5$

$x + 1 = \sqrt{16.5}$

$x = \sqrt{16.5} - 1$

$\boxed{x = 3.1}$

.1 .1

2.9 3 3.1

$\boxed{\text{Pick } \delta = .1}$ ✓

Sep 14-10:54 AM

For $\varepsilon = .5$, there is a $\delta > 0$ Such that $\lim_{x\to 3}(x^2+2x-2) = 13$

For $\varepsilon > 0$, there is a $\delta > 0$ Such that

$|f(x) - L| < \varepsilon$ whenever $|x-a| < \delta$

$|x^2+2x-2-13| < \varepsilon$ " $|x-3| < \delta$

$|x^2+2x-15| < \varepsilon$ " $|x-3| < \delta$

$|(x+5)(x-3)| < \varepsilon$

$|x+5|\,|x-3| < \varepsilon$

Bound Keep

$\delta = \frac{\varepsilon}{c}$

If $|x+5| < c$, then $|x-3| < \frac{\varepsilon}{c}$

let's agree that δ to be no more than 1.

$|x-3| < 1$

$-1 < x-3 < 1$

Add 8

$7 < x+5 < 9$

$-9 < 7 < x+5 < 9$

$-9 < x+5 < 9$

$|x+5| < 9$

For $\varepsilon = .5$

$\delta = \min\{1, \frac{.5}{9}\}$

$= \min\{1, \frac{1}{18}\} = \frac{1}{18} = .05 \approx \boxed{.1}$

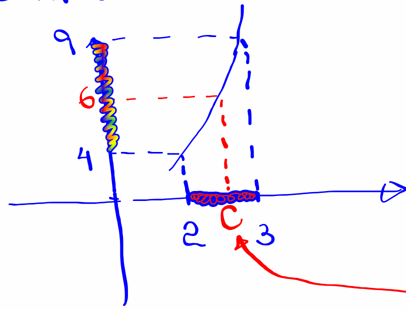
$\delta = \min\{1, \frac{\varepsilon}{9}\}$

Sep 14-11:02 AM

Intermediate Value Theorem: (Google for more information)

Suppose $f(x)$ is a continuous function on $[a, b]$ and let N be a number between $f(a)$ and $f(b)$, where $f(a) \neq f(b)$ there exists a number c in (a, b) Such that $f(c) = N$.

Consider $f(x) = x^2$ over $[2, 3]$



If $N = 6$

$f(x) = x^2 = 6$

$x = \sqrt{6}$

$x = 2.45$

Sep 14-11:12 AM

Class QZ 7

Class room for exam : G5-118

Use ε and δ definition to prove $\lim_{x\to 6}\left(\frac{1}{6}x+4\right)=5$

$f(x)=\frac{1}{6}x+4$

$L=5$

$a=6$

1) verify the limit

$\lim_{x\to 6}\left(\frac{1}{6}x+4\right)=\frac{1}{6}(6)+4=1+4=5$ ✓

for $\varepsilon>0$, there is a $\delta>0$ such that

$|f(x)-L|<\varepsilon$ whenever $|x-a|<\delta$

$\left|\frac{1}{6}x+4-5\right|<\varepsilon$

$|x-6|<\delta$

$\left|\frac{1}{6}x-1\right|<\varepsilon$

Multiply by 6

$6\left|\frac{1}{6}x-1\right|<6\varepsilon$

$|x-6|<6\varepsilon$

Choose

$\boxed{\delta=6\varepsilon}$

Sep 14-11:20 AM