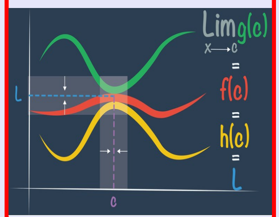
Math 261

Fall 2023

Lecture 10



Feb 19-8:47 AM

1) Evaluate $\lim_{x \to 4} \dfrac{5+\sqrt{x}}{\sqrt{x+5}} = \dfrac{5+\sqrt{4}}{\sqrt{4+5}} = \dfrac{5+2}{\sqrt{9}} = \boxed{\dfrac{7}{3}}$ ✓

Direct Subs.

$f(x) = \dfrac{5+\sqrt{x}}{\sqrt{x+5}}$

$f(4) = \dfrac{5+\sqrt{4}}{\sqrt{4+5}} = \dfrac{7}{3}$

Since $\lim_{x \to 4} \dfrac{5+\sqrt{x}}{\sqrt{x+5}} = f(4)$, then $f(x)$ is Cont. at $x=4$.

Sep 13-10:26 AM

2) Evaluate $\lim_{x \to \pi} \sin(x + \sin x) = \sin(\pi + \sin \pi)$

Direct Subs.

$= \sin(\pi + 0)$

$= \sin \pi = \boxed{0}$

$f(x) = \sin(x + \sin x)$

$f(\pi) = \sin(\pi + \sin \pi)$

$= \boxed{0}$

Since $\lim_{x \to \pi} f(x) = f(\pi)$, then $f(x) = \sin(x + \sin x)$ is cont. at $x = \pi$.

Sep 13-10:29 AM

3) Is $f(x)$ given below cont. at $x = 0$?

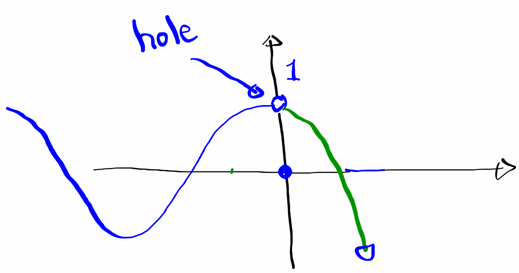
$$f(x) = \begin{cases} \boxed{\cos x \quad \text{if } x < 0} \\ 0 \quad \text{if } x = 0 \\ \boxed{1 - x^2 \quad \text{if } x > 0} \end{cases}$$

$\lim_{x \to 0^-} f(x) = 1$

$\lim_{x \to 0^+} f(x) = 1$

So $\lim_{x \to 0} f(x) = 1$

$f(0) = 0$



Since $\lim_{x \to 0} f(x) \neq f(0)$

then $f(x)$ is not cont. at $x = 0$.

Sep 13-10:33 AM

4) Is $f(x)$ given below cont. at $x=3$?

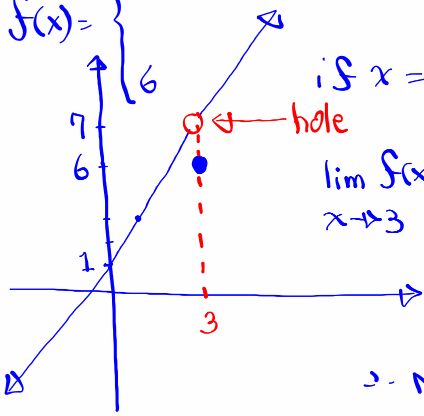
$$f(x)=\begin{cases} \dfrac{2x^2-5x-3}{x-3} & \text{if } x\neq 3 \\ 6 & \text{if } x=3 \end{cases}$$

Can this be simplified?

$$\frac{2x^2-5x-3}{x-3} = \frac{\cancel{(x-3)}(2x+1)}{\cancel{x-3}} = 2x+1$$

$f(x)=2x+1$

$$f(x)=\begin{cases} 2x+1 & \text{if } x\neq 3 \\ 6 & \text{if } x=3 \end{cases}$$



$\lim_{x\to 3} f(x)=7$

$f(3)=6$

$\lim_{x\to 3} f(x)\neq f(3)$

$\therefore$ Not cont. at $x=3$

Sep 13-10:39 AM

$f(x)=\dfrac{1}{x^2}$, Evaluate $\lim_{x\to a}\dfrac{f(x)-f(a)}{x-a}$

$$\lim_{x\to a}\frac{f(x)-f(a)}{x-a} = \lim_{x\to a}\frac{\frac{1}{x^2}-\frac{1}{a^2}}{x-a} = \lim_{x\to a}\frac{a^2-x^2}{(x-a)\cdot x^2a^2}$$

LCD: x^2a^2

$$=\lim_{x\to a}\frac{(a+x)\overset{-1}{\cancel{(a-x)}}}{\cancel{(x-a)}\cdot x^2a^2} = \lim_{x\to a}\frac{-1(a+x)}{x^2a^2}$$

Now direct Subs. $=\dfrac{-1(a+a)}{a^2a^2}=\dfrac{-2a}{a^4}=\boxed{\dfrac{-2}{a^3}}$

Sep 13-10:46 AM

Use ε and δ definition to prove $\lim_{x \to 2} (5x+3) = 13$

$f(x) = 5x+3$

$L = 13$

$a = 2$

1) verify the limit

$\lim_{x \to 2} (5x+3) = 5(2)+3 = 13 \checkmark$

2) for any $\varepsilon > 0$, there is a $\delta > 0$ such that

$|f(x) - L| < \varepsilon$ whenever $|x - a| < \delta$

$|5x+3-13| < \varepsilon$ $\qquad$ $|x-2| < \delta$

$|5x - 10| < \varepsilon$

$|5(x-2)| < \varepsilon$

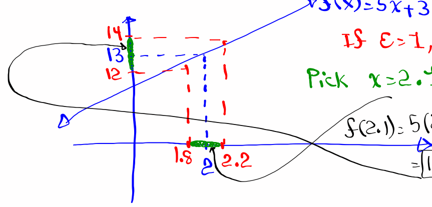
$|ab| = |a|\,|b|$

$|5|\,|x-2| < \varepsilon$ $\rightarrow$ $5|x-2| < \varepsilon$

Divide by 5

$|x-2| < \frac{\varepsilon}{5}$

$\boxed{\text{Pick } \delta = \frac{\varepsilon}{5}}$



If $\varepsilon = 1$, $\delta = \frac{1}{5} = .2$

Pick $x = 2.1$

$f(2.1) = 5(2.1)+3 = \boxed{13.5}$

Sep 13-10:55 AM

Use ε and δ definition to prove $\lim_{x \to 3} x^2 = 9$

$f(x) = x^2$

$L = 9$

$a = 3$

1) verify the limit

$\lim_{x \to 3} x^2 = 3^2 = 9 \checkmark$

2) for every $\varepsilon > 0$, there is a $\delta > 0$ such that

$|f(x) - L| < \varepsilon$ whenever $|x-a| < \delta$

$|x^2 - 9| < \varepsilon$ whenever $|x-3| < \delta$

$x^2 - 9 = (x+3)(x-3)$

$|x^2 - 9| = |(x+3)(x-3)|$

$= |x+3|\,|x-3| < \varepsilon$ $\rightarrow$ $|x+3|\,|x-3| < \varepsilon$

Keep

$c\,|x-3| < \varepsilon$

Let's say $|x+3| < c$

$|x-3| < \frac{\varepsilon}{c}$

Suppose we like δ no more than 1.

$|x-3| < 1$

$-1 < x-3 < 1$

$2 < x < 4$

$5 < x+3 < 7$

$|x+3| < 7$

$\boxed{c = 7}$

So $\delta = \frac{\varepsilon}{7}$

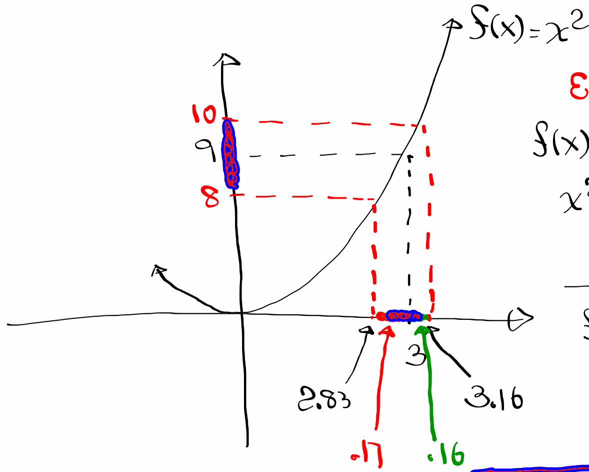
$$\boxed{\delta = \min\left\{1, \frac{\varepsilon}{7}\right\}}$$

if $\varepsilon = 1$ $\rightarrow$ $\delta = \min\{1, \frac{1}{7}\} = \frac{1}{7}$

If $\varepsilon = 2$ $\rightarrow$ $\delta = \min\{1, \frac{2}{7}\} = \frac{2}{7}$

If $\varepsilon = 10$ $\rightarrow$ $\delta = \min\{1, \frac{10}{7}\} = 1$

Sep 13-11:03 AM



$\varepsilon = 1$

$f(x) = 10$

$x^2 = 10$

$x = 3.16$

$f(x) = 8$

$x^2 = 8$

$x = 2.83$

$\delta = \min\{.17, .16\} = .16$

$\boxed{\delta = \min\{1, \frac{\varepsilon}{7}\}}$

for $\varepsilon = 1$

$\delta = \min\{1, \frac{1}{7}\}$

$= \frac{1}{7} \approx \boxed{.14}$

$3 - .14 \;,\; 3 + .14$

$(2.86 \;,\; 3.14)$

Sep 13-11:18 AM

Use ε & δ definition to prove $\lim_{x \to 2} (x^2 - 4x + 5) = 1$

$f(x) = x^2 - 4x + 5$

$L = 1$

$a = 2$

1) Verify L

$\lim_{x \to 2} (x^2 - 4x + 5) = 2^2 - 4(2) + 5 = 1$ ✓

2) For $\varepsilon > 0$, there is a $\delta > 0$ such that

$|f(x) - L| < \varepsilon$ whenever $|x - a| < \delta$

$|x^2 - 4x + 5 - 1| < \varepsilon$ " $|x - 2| < \delta$

$|x^2 - 4x + 4| < \varepsilon$ " $|x - 2| < \delta$

$|(x-2)(x-2)| < \varepsilon$ $\quad |x - 2| < \sqrt{\varepsilon}$

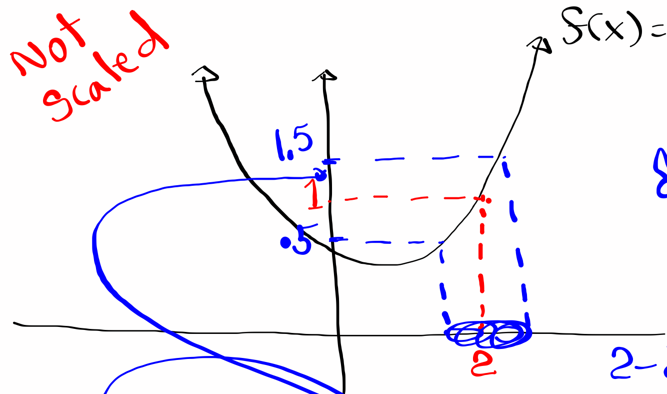
$|(x-2)^2| < \varepsilon$ $\quad \delta = \sqrt{\varepsilon}$

$|x - 2|^2 < \varepsilon$

If $\varepsilon = 1 \rightarrow \delta = 1$

If $\varepsilon = 4 \rightarrow \delta = 2$

Sep 13-11:25 AM



$f(x) = x^2 - 4x + 5$

$\varepsilon = .5$

$\delta = \sqrt{\varepsilon}$

$= \sqrt{.5} \approx .71$

$2 - .71 \quad , 2 + .71$

$(1.29 \quad , 2.71)$

$x = 2.5$

$f(2.5) = 2.5^2 - 4(2.5) + 5$

$= \boxed{1.25}$

Sep 13-11:31 AM