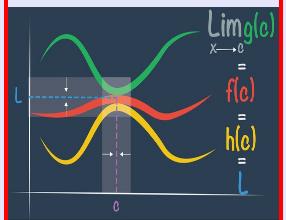
Math 261

Fall 2022

Lecture 3



Given $f(x) = \dfrac{x^3 + 2x^2}{x+2} = \dfrac{x^2(x+2)}{x+2} = x^2$

1) Domain

$x + 2 \neq 0$

$x \neq -2$

$(-\infty, -2) \cup (-2, \infty)$

-2

2) x-Ints

$Y = 0 \rightarrow f(x) = 0$

$\dfrac{x^3 + 2x^2}{x+2} = 0$

$x^3 + 2x^2 = 0$

$x^2(x+2) = 0$

$x^2 = 0 \qquad x + 2 = 0$

$\boxed{x = 0} \qquad \cancel{x = -2}$

$\boxed{(0,0)}$

3) Y-Int

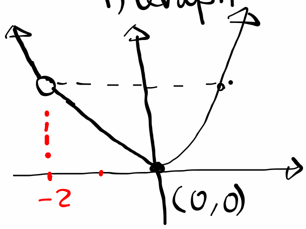
$x = 0$

$f(0) = \dfrac{0^3 + 2(0)}{0 + 2}$

$= 0$

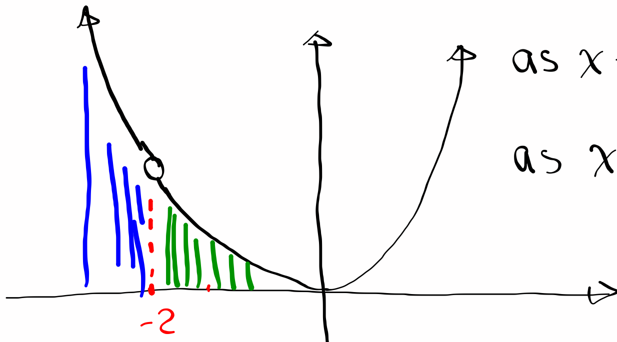
$(0,0)$

4) Graph



5) Discuss range

$[0, \infty)$



as $x \to -2^{+}$, $f(x) \to 4$

as $x \to -2^{-}$, $f(x) \to 4$

$f(x)$ is not defined at $x = -2$

but $f(x) \to 4$ as $x \to -2$.

Consider $f(x) = x^3$

Find the difference quotient, evaluate the final answer for $h=0$.

$$\frac{f(x+h) - f(x)}{h}$$

$f(x+h) - f(x) = (x+h)^3 - x^3$

$= \cancel{x^3} + 3x^2h + 3xh^2 + h^3 \cancel{- x^3}$

$(x+h)(x+h)(x+h)$

OR

Use binomial expansion

$$\frac{f(x+h)-f(x)}{h} = \frac{3x^2h + 3xh^2 + h^3}{h}$$

$$= \frac{\cancel{h}(3x^2 + 3xh + h^2)}{\cancel{h}}$$

$$= 3x^2 + 3xh + h^2$$

Evaluate for $h=0$

$3x^2 + 3x(0) + (0)^2 = 3x^2$

Find difference quotient for

$f(x)=\sqrt[3]{x}$, then evaluate the final answer for $h=0$

$$\frac{f(x+h)-f(x)}{h}=\frac{\sqrt[3]{x+h}-\sqrt[3]{x}}{h}\cdot\frac{(\sqrt[3]{x+h})^2+\sqrt[3]{x+h}\sqrt[3]{x}+(\sqrt[3]{x})^2}{(\sqrt[3]{x+h})^2+\sqrt[3]{x+h}\sqrt[3]{x}+(\sqrt[3]{x})^2}$$

Recall from Algebra

$A^3-B^3=(A-B)(A^2+AB+B^2)$

$$=\frac{(\sqrt[3]{x+h})^3-(\sqrt[3]{x})^3}{h\left[(\sqrt[3]{x+h})^2+\sqrt[3]{x+h}\sqrt[3]{x}+(\sqrt[3]{x})^2\right]}$$

$$=\frac{\cancel{x}+\cancel{h}-\cancel{x}}{\cancel{h}\left[(\sqrt[3]{x+h})^2+\sqrt[3]{x+h}\sqrt[3]{x}+(\sqrt[3]{x})^2\right]}$$

$$=\frac{1}{(\sqrt[3]{x+h})^2+\sqrt[3]{x+h}\sqrt[3]{x}+(\sqrt[3]{x})^2}$$

for $h=0$

$$\frac{1}{(\sqrt[3]{x+0})^2+\sqrt[3]{x+0}\sqrt[3]{x}+(\sqrt[3]{x})^2}=\boxed{\frac{1}{3\sqrt[3]{x^2}}}$$

Introduction to limits

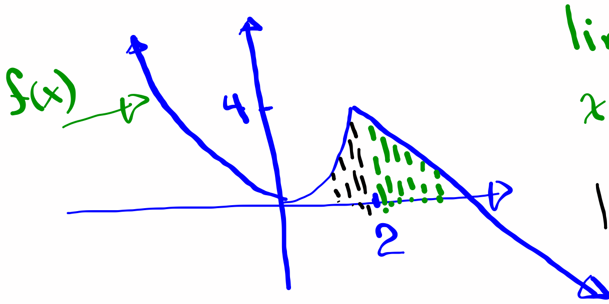
Notation $\lim\limits_{x\to a^+} f(x)=L_1$ x approaches a from right

$\lim\limits_{x\to a^-} f(x)=L_2$ x approaches a from left

If $\underbrace{L_1=L_2}_{L}$, then we simply say

$$\lim_{x\to a} f(x)=L$$

Consider the graph below



$$\lim_{x \to 2^+} f(x) = 4 \leftarrow L_1$$

$$\lim_{x \to 2^-} f(x) = 4 \leftarrow L_2$$

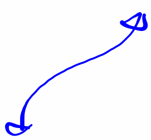
$$L_1 = L_2$$

So $\lim_{x \to 2} f(x) = 4 \leftarrow L$

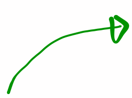
Consider

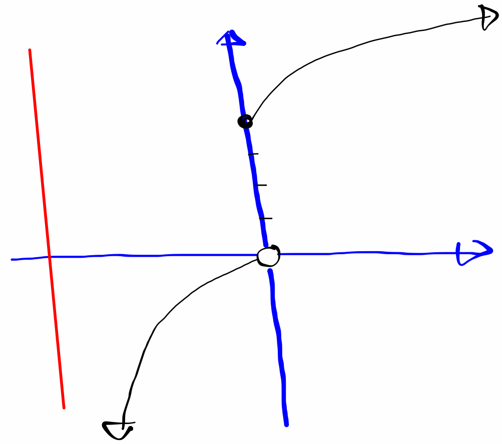
$$f(x) = \begin{cases} x^3 & \text{if } x < 0 \\ \sqrt{x} + 4 & \text{if } x \geq 0 \end{cases}$$

$y = x^3$



$y = \sqrt{x}$





$f(0) = 4$

$\lim_{x \to 0^+} f(x) = 4$

$\lim_{x \to 0^-} f(x) = 0$

$\Rightarrow \lim_{x \to 0} f(x)$ D.N.E.

Suppose $f(x) = \sqrt{|x|}$

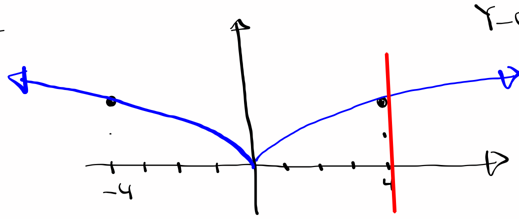
Domain $(-\infty, \infty)$

$f(-x) = \sqrt{|-x|} = \sqrt{|x|} = f(x)$

Since $f(-x) = f(x)$

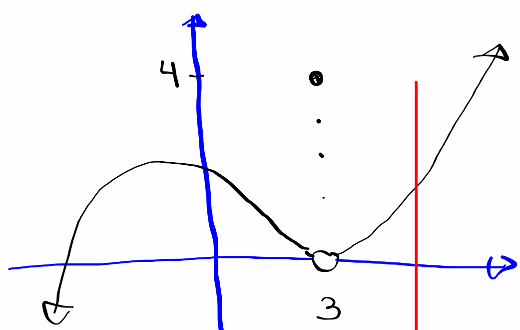
then $f(x)$ is an even function $\Rightarrow$ Symmetric with respect to Y-axis.

x	y
4	2
-4	2



$\lim_{x \to 0^+} f(x) = 0$, $\lim_{x \to 0^-} f(x) = 0$, $\lim_{x \to 0} f(x) = 0$, $f(0) = 0$

Consider the graph below



$\lim_{x \to 3^+} f(x) = 0$

$\lim_{x \to 3^-} f(x) = 0$

$\lim_{x \to 3} f(x) = 0$

$f(3) = 4$

Class QZ 1

Solve $4x^2+5x-9=0$ using quadratic formula.

Express answers in a Solution Set.

$4x^2+5x-9=0$

$ax^2+bx+c=0$

$a=4,\ b=5,\ c=-9$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{-5\pm\sqrt{5^2-4(4)(-9)}}{2(4)}$$

$$=\frac{-5\pm\sqrt{25+144}}{8}=\frac{-5\pm\sqrt{169}}{8}$$

$$=\frac{-5\pm13}{8}$$

$$x=\frac{-5+13}{8}=\frac{8}{8}=1$$

$$x=\frac{-5-13}{8}=\frac{-18}{8}=\frac{-9}{4}$$

$$\Rightarrow \boxed{\text{Solution } \left\{\frac{-9}{4},1\right\}}$$