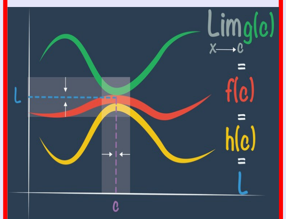
Math 261

Fall 2022

Lecture 11



Evaluate $\lim\limits_{\theta\to 0} \dfrac{\sin\theta}{\theta + \tan\theta} = \dfrac{\sin 0}{0+\tan 0} = \dfrac{0}{0}$

I.F.

Since $\lim\limits_{\theta\to 0} \dfrac{\sin\theta}{\theta} = 1$

Divide everything by $\sin\theta$

$$\lim_{\theta\to 0} \frac{\sin\theta}{\theta+\tan\theta} = \lim_{\theta\to 0} \frac{\frac{\sin\theta}{\sin\theta}}{\frac{\theta}{\sin\theta} + \frac{\tan\theta}{\sin\theta}} = \lim_{\theta\to 0} \frac{1}{\frac{\theta}{\sin\theta} + \frac{1}{\cos\theta}}$$

$$= \frac{\lim\limits_{\theta\to 0} 1}{\lim\limits_{\theta\to 0} \frac{\theta}{\sin\theta} + \lim\limits_{\theta\to 0} \frac{1}{\cos\theta}} = \frac{1}{1+\frac{1}{1}} = \boxed{\frac{1}{2}}$$

Evaluate $\lim_{x \to \frac{\pi}{4}} \frac{1 - \tan x}{\sin x - \cos x} = \frac{1 - \tan \pi/4}{\sin \pi/4 - \cos \pi/4}$

$= \frac{1-1}{\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}} = \frac{0}{0}$ I.F.

$$\lim_{x \to \pi/4} \frac{1 - \tan x}{\sin x - \cos x} =$$

$$\lim_{x \to \pi/4} \frac{1 - \frac{\sin x}{\cos x}}{\sin x - \cos x} = \lim_{x \to \pi/4} \frac{\cos x\left(1 - \frac{\sin x}{\cos x}\right)}{\cos x(\sin x - \cos x)}$$

$$= \lim_{x \to \pi/4} \frac{\overset{-1}{\cancel{\cos x - \sin x}}}{\cos x(\cancel{\sin x - \cos x})} = \lim_{x \to \pi/4} \frac{-1}{\cos x} = \frac{-1}{\frac{\sqrt{2}}{2}}$$

$$= \frac{-\cancel{2}}{\cancel{\sqrt{2}}} \cdot \frac{\sqrt{2}}{\cancel{\sqrt{2}}} = \boxed{-\sqrt{2}}$$

Second way

$$\lim_{x \to \pi/4} \frac{1 - \tan x}{\sin x - \cos x} = \lim_{x \to \pi/4} \frac{1 - \tan x}{-\cos x\left(1 - \frac{\sin x}{\cos x}\right)}$$

$$= \lim_{x \to \pi/4} \frac{\cancel{1 - \tan x}}{-\cos x(\cancel{1 - \tan x})}$$

$$= \lim_{x \to \pi/4} \frac{-1}{\cos x} = \frac{-1}{\frac{\sqrt{2}}{2}} = \cdots \boxed{-\sqrt{2}}$$

Show that $x^5 - x^2 = 4$ has at least one real solution.

$$\underbrace{x^5 - x^2 - 4}_{} = 0$$

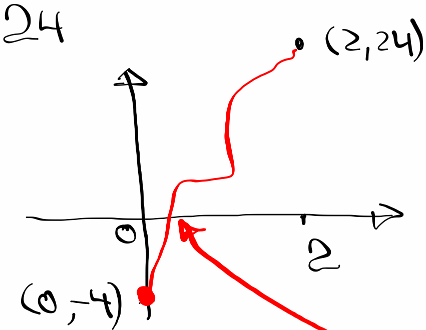
$f(x) = x^5 - x^2 - 4$

Polynomial, cont. everywhere

Consider $[0, 2]$

$f(0) = -4$

$f(2) = +24$



By I.V.T. there is at least one root c where $f(c) = 0$

Show that there is a solution in $(0,1)$ for equation $\sqrt[3]{x} = 1 - x$

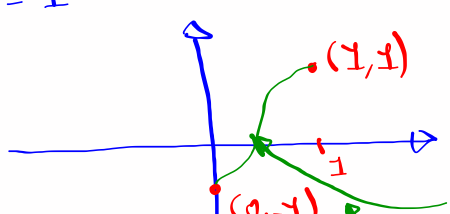
$$\underbrace{\sqrt[3]{x} - 1 + x}_{} = 0$$

$f(x) = \sqrt[3]{x} - 1 + x$

Cont. everywhere

$f(0) = -1$

$f(1) = 1$



there is a C in $(0,1)$ where $f(C) = 0$ by I.V.T.

Show $\sin x = x^2 - x$ has at least one solution in $(1,2)$.

$$\underbrace{\sin x - x^2 + x}_{f(x)} = 0$$

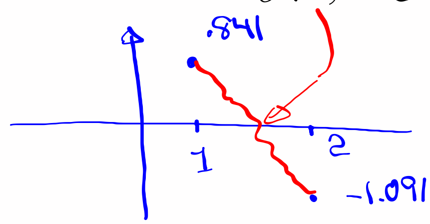
Cont. everywhere

Make sure you are in radians

$f(1) = \sin 1 - 1^2 + 1 = \sin 1 = \boxed{.841}$

$f(2) = \sin 2 - 2^2 + 2 = \sin 2 - 2 = \boxed{-1.091}$

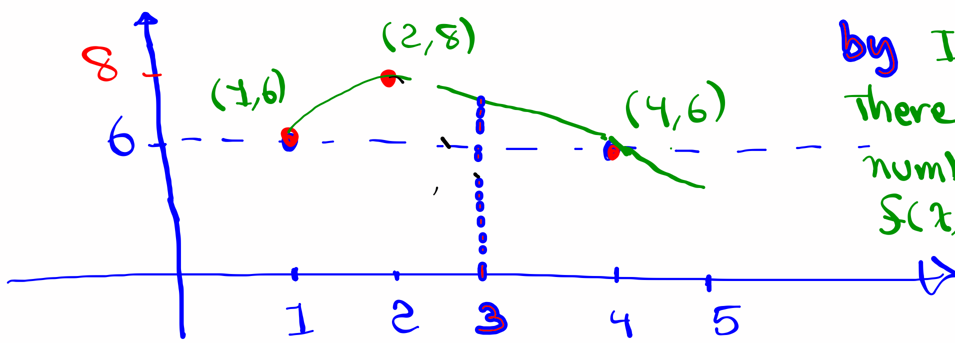
by I.V.T. there is a number C in $(1,2)$ such that $f(c) = 0$, So C is a solution.



Suppose $f(x)$ is cont. on $[1,5]$

and $f(x) = 6$ only for $\boxed{x=1}$ and $\boxed{x=4}$

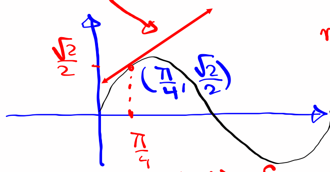
Show that $f(3) > 6$ if $f(2) = 8$.



by I.V.T. there is a number $f(x) > 6$

$f(3) > 6$

Find slope of the tan. line at $x = \frac{\pi}{4}$ to the graph of $f(x) = \sin x$.



$m_{\text{tan. line}} = \lim\limits_{h\to 0} \frac{f(x+h)-f(x)}{h}$

$m = \lim\limits_{h\to 0} \frac{\sin(x+h) - \sin x}{h} = \frac{0}{0}$ I.F.

Recall
$\sin(A+B) =$
$\sin A \cos B +$
$\cos A \sin B$

$= \lim\limits_{h\to 0} \frac{\sin x \cosh + \cos x \sin h - \sin x}{h}$

$= \lim\limits_{h\to 0} \frac{\sin x[\cos h - 1]}{h} + \lim\limits_{h\to 0} \frac{\cos x \sin h}{h}$

$= \sin x \cdot \lim\limits_{h\to 0} \frac{\cos h - 1}{h} + \cos x \lim\limits_{h\to 0} \frac{\sin h}{h}$

$\lim\limits_{h\to 0} \frac{\cos h - 1}{h} \to 0$, $\quad \lim\limits_{h\to 0} \frac{\sin h}{h} \to 1$

$= \sin x \cdot 0 + \cos x \cdot 1$

$m_{\text{tan. line}} = \cos x$

at $x = \pi/4$

$m_{\text{tan. line}} = \cos\frac{\pi}{4} = \boxed{\frac{\sqrt{2}}{2}}$

Class QZ 3

For $\varepsilon > 0$, find a $\delta > 0$ such that

1) $\lim\limits_{x\to 2} (5x-3) = 7$

$|5x-3-7| < \varepsilon$ whenever $|x-2| < \delta$

$|5x-10| < \varepsilon$ " "

$5|x-2| < \varepsilon$ " "

$|x-2| < \frac{\varepsilon}{5}$ $\quad \boxed{\delta = \frac{\varepsilon}{5}}$

2) $\lim\limits_{x\to 2} (x^2-2x) = 0$

$|x^2-2x-0| < \varepsilon$ whenever $|x-2| < \delta$

$|x(x-2)| < \varepsilon$

$|x|\,|x-2| < \varepsilon$

$\delta \le 1$

$|x-2| < 1$

$-1 < x-2 < 1$

$x < 3$

$|x| < 3$

$|x-2| < \frac{\varepsilon}{3}$

Pick $\delta = \min\{1, \frac{\varepsilon}{3}\}$