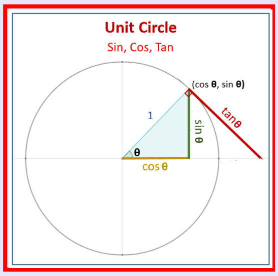
Math 241

Winter 2024

Lecture 4



Feb 19-8:47 AM

Class QZ 2

Solve $9x^2+4=12x$ by quadratic formula

$9x^2+4-12x=0$ $\qquad ax^2+bx+c=0$

Solution Set

$9x^2-12x+4=0$

$a=9 \qquad b=-12 \qquad c=4$

$b^2-4ac=(-12)^2-4(9)(4)=144-144=\boxed{0}$

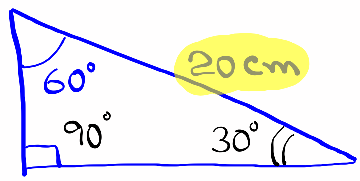
$$\boxed{x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}}=\frac{-(-12)\pm\sqrt{0}}{2(9)}=\frac{12\pm0}{18}=\frac{12}{18}=\boxed{\frac{2}{3}}$$

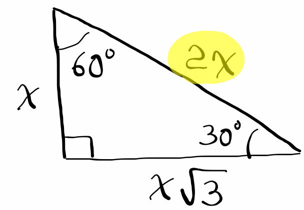
$\boxed{\left\{\frac{2}{3}\right\}}$ Repeated Solution

Jan 4-12:06 PM

Solve the triangle below:

$30° - 60° - 90°$

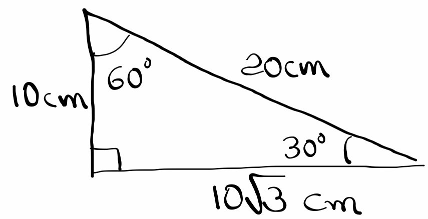




↳ find all missing information (sides or angles).

$20 = 2x$

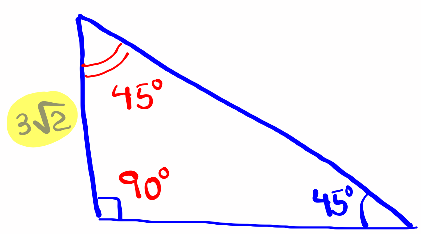
$\boxed{x = 10}$

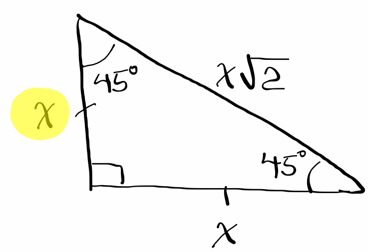


Jan 8-8:09 AM

Solve the triangle below

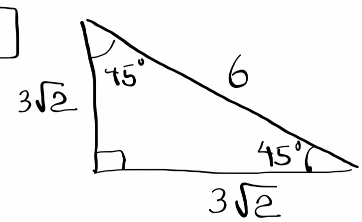
$45° - 45° - 90°$





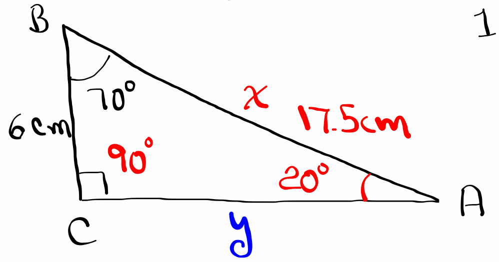
$x = 3\sqrt{2}$

$x\sqrt{2} = 3\sqrt{2}\sqrt{2} = 3\sqrt{4} = 3 \cdot 2 = \boxed{6}$



Jan 8-8:15 AM

Solve the triangle below. Round answers to 1-decimal place.



$\sin 20° = \frac{6}{x}$

$x \sin 20° = 6$

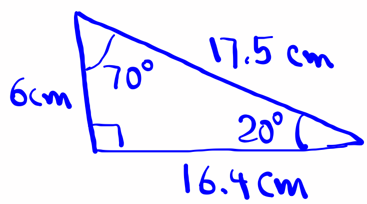
$x = \frac{6}{\sin 20°}$

$\boxed{x \approx 17.5}$

$\cos 20° = \frac{y}{17.5}$

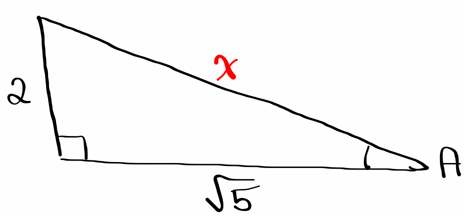
$y = 17.5 \cos 20°$

$\approx \boxed{16.4}$



Jan 8-8:18 AM

Find exact values for all six trig. functions for angle A below:



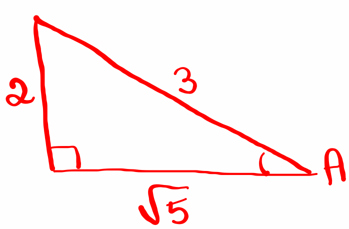
Pythagorean Thrm

$a^2 + b^2 = c^2$

$2^2 + (\sqrt{5})^2 = c^2$

$4 + 5 = c^2$

$\boxed{c = 3}$



$\sin A = \frac{2}{3}$

$\cos A = \frac{\sqrt{5}}{3}$

$\tan A = \frac{2}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$

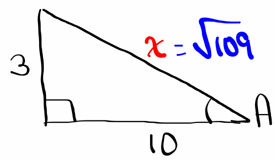
$\csc A = \frac{3}{2}$

$\sec A = \frac{3}{\sqrt{5}} = \frac{3\sqrt{5}}{5}$

$\cot A = \frac{\sqrt{5}}{2}$

Jan 8-8:24 AM

In Right triangle ABC, $\tan A = \frac{3}{10}$, find the exact values for the remaining trig. functions.



$\tan A = \frac{3}{10}$ $\frac{\text{opp.}}{\text{adj}}$

$3^2 + 10^2 = x^2$

$109 = x^2 \rightarrow x = \sqrt{109}$

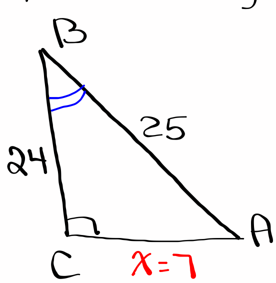
$\sin A = \frac{3}{\sqrt{109}} = \frac{3\sqrt{109}}{109}$ $\csc A = \frac{\sqrt{109}}{3}$

$\cos A = \frac{10}{\sqrt{109}} = \frac{10\sqrt{109}}{109}$ $\sec A = \frac{\sqrt{109}}{10}$

$\tan A = \frac{3}{10}$ $\cot A = \frac{10}{3}$

Jan 8-8:31 AM

Find exact value of all trig. functions using right triangle below for angle B.



$24^2 + x^2 = 25^2$

$x^2 = 49$

$x = 7$

$\sin B = \frac{7}{25}$ $\csc B = \frac{25}{7}$

$\cos B = \frac{24}{25}$ $\sec B = \frac{25}{24}$

$\tan B = \frac{7}{24}$ $\cot B = \frac{24}{7}$

Jan 8-8:41 AM

Known identities:

$$\sin^2 x + \cos^2 x = 1$$

$$\tan x = \frac{\sin x}{\cos x}$$

$$1 + \tan^2 x = \sec^2 x$$

$$1 + \cot^2 x = \csc^2 x$$

$$\csc x = \frac{1}{\sin x}$$

$$\sec x = \frac{1}{\cos x}$$

$$\cot x = \frac{1}{\tan x}$$

$$\cot x = \frac{\cos x}{\sin x}$$

Jan 8-8:47 AM

Simplify

$$(\sin x + \cos x)^2 - \tan x \cot x$$

$$= (\sin x + \cos x)(\sin x + \cos x) - \tan x \cdot \cot x$$

$$= \sin^2 x + \boxed{\sin x \cos x} + \boxed{\cos x \sin x} + \cos^2 x - \tan x \cdot \cot x$$

$$= \cancel{1} + 2 \sin x \cos x - \cancel{1} = \boxed{2 \sin x \cos x}$$

Jan 8-8:50 AM

Simplify

$$(\tan x - \cot x)^2 + 2\underbrace{\sin x \csc x} - \sec^2 x$$

$$= (\tan x - \cot x)(\tan x - \cot x) + 2\cdot 1 - \sec^2 x$$

$$= \tan^2 x - \underbrace{\tan x \cot x - \cot x \tan x} + \cot^2 x + 2 - \sec^2 x$$

$$= \tan^2 x \cancel{-2} + \cot^2 x \cancel{+2} - \sec^2 x$$

$$= \tan^2 x + \cot^2 x - (1 + \tan^2 x)$$

$$= \cancel{\tan^2 x} + \cot^2 x - 1 \cancel{- \tan^2 x} = \boxed{\cot^2 x - 1}$$

Jan 8-8:53 AM

Simplify $\dfrac{\sin^3 x - \cos^3 x}{\sin^2 x - \cos^2 x}$

Hint:

$A^3 - B^3 = (A-B)(A^2 + AB + B^2)$

$A^2 - B^2 = (A+B)(A-B)$

$$= \frac{\cancel{(\sin x - \cos x)}(\sin^2 x + \sin x \cos x + \cos^2 x)}{(\sin x + \cos x)\cancel{(\sin x - \cos x)}}$$

$$= \boxed{\frac{1 + \sin x \cos x}{\sin x + \cos x}}$$

Jan 8-9:01 AM

Simplify

Hint: LCD $= (1-\sin x)(1+\sin x)$

$$\frac{1}{1-\sin x} + \frac{1}{1+\sin x}$$

$$= \frac{1}{1-\sin x}\cdot\frac{1+\sin x}{1+\sin x} + \frac{1}{1+\sin x}\cdot\frac{1-\sin x}{1-\sin x}$$

$$= \frac{1+\sin x}{(1-\sin x)(1+\sin x)} + \frac{1-\sin x}{(1+\sin x)(1-\sin x)}$$

$$= \frac{1+\cancel{\sin x}+1-\cancel{\sin x}}{(1-\sin x)(1+\sin x)} = \frac{2}{1-\sin^2 x} = \frac{2}{\cos^2 x} = \frac{2\cdot 1}{\cos^2 x}$$

$(A-B)(A+B)$

$A^2 - B^2$

$$= 2\left(\frac{1}{\cos x}\right)^2$$

$$= 2\cdot\sec^2 x = \boxed{2\sec^2 x}$$

Recall

$\sin^2 x + \cos^2 x = 1$

$\cos^2 x = 1 - \sin^2 x$

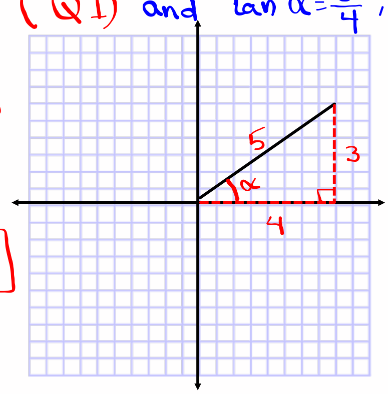
Jan 8-9:06 AM

$\sin(-\alpha) = -\sin\alpha$ $\qquad$ $\csc(-\alpha) = -\csc\alpha$

$\cos(-\alpha) = \cos\alpha$ $\qquad$ $\sec(-\alpha) = \sec\alpha$

$\tan(-\alpha) = -\tan\alpha$ $\qquad$ $\cot(-\alpha) = -\cot\alpha$

Suppose $0^\circ < \alpha < 90^\circ$ (QI) and $\tan\alpha = \frac{3}{4}$, find



$\sin(-\alpha) = -\sin\alpha = \boxed{-\frac{3}{5}}$

$\cos(-\alpha) = \cos\alpha = \boxed{\frac{4}{5}}$

$\tan(-\alpha) = -\tan\alpha = \boxed{\frac{-3}{4}}$

$\csc(-\alpha) = \boxed{\frac{-5}{3}}$

$\sec(-\alpha) = \frac{5}{4}$

$\cot(-\alpha) = \boxed{-\frac{4}{3}}$

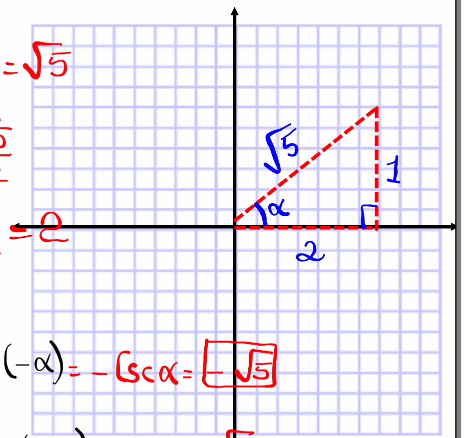
Jan 8-9:15 AM

$\cos\alpha = \frac{2}{\sqrt{5}}$, $0° < \alpha < 90°$, Complete the chart below. Make sure all answers are rationalized.

$\sin\alpha = \frac{1}{\sqrt{5}} = \frac{\sqrt{5}}{5}$ $\quad$ $\csc\alpha = \frac{\sqrt{5}}{1} = \sqrt{5}$

$\cos\alpha = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$ $\quad$ $\sec\alpha = \frac{\sqrt{5}}{2}$

$\tan\alpha = \frac{1}{2}$ $\quad$ $\cot\alpha = \frac{2}{1} = 2$



Now find

$\sin(-\alpha) = -\sin\alpha = \boxed{-\frac{\sqrt{5}}{5}}$ $\quad$ $\csc(-\alpha) = -\csc\alpha = \boxed{-\sqrt{5}}$

$\cos(-\alpha) = \cos\alpha = \frac{2\sqrt{5}}{5}$ $\quad$ $\sec(-\alpha) = \sec\alpha = \frac{\sqrt{5}}{2}$

$\tan(-\alpha) = -\tan\alpha = -\frac{1}{2}$ $\quad$ $\cot(-\alpha) = -\cot\alpha = \boxed{-2}$

Jan 8-9:40 AM

Simplify

$\sin^2 A + \tan^2 A + \cos^2 A$

$= 1 + \tan^2 A$

$= \boxed{\sec^2 A}$

Simplify

$\cos^2 x \cdot \tan^2 x \cdot \csc x$

$= \cancel{\cos^2 x} \cdot \frac{\cancel{\sin^2} x}{\cancel{\cos^2 x}} \cdot \frac{1}{\cancel{\sin x}} = \boxed{\sin x}$

Jan 8-9:49 AM

Simplify

$$\frac{1}{1+\tan x} + \frac{\cot x}{1+\tan x} = \frac{1+\cot x}{1+\tan x}$$

$$= \frac{1 + \frac{\cos x}{\sin x}}{1 + \frac{\sin x}{\cos x}}$$

Multiply by LCD

LCD = $\sin x \cos x$

$$= \frac{\sin x \cos x \cdot 1 + \cancel{\sin x} \cos x \cdot \frac{\cos x}{\cancel{\sin x}}}{\sin x \cos x \cdot 1 + \sin x \cancel{\cos x} \cdot \frac{\sin x}{\cancel{\cos x}}}$$

$$= \frac{\sin x \cos x + \cos^2 x}{\sin x \cos x + \sin^2 x} = \frac{\cos x \cancel{(\sin x + \cos x)}}{\sin x \cancel{(\cos x + \sin x)}}$$

$$= \frac{\cos x}{\sin x} = \boxed{\cot x}$$

Jan 8-9:54 AM

Simplify $(\sec x + 1)(\sec x - 1)$

$(A + B)(A - B)$

$A^2 - B^2$

$$(\sec x + 1)(\sec x - 1) = \sec^2 x - 1$$

$$= \cancel{1} + \tan^2 x \cancel{-1}$$

$$= \boxed{\tan^2 x}$$

Jan 8-10:04 AM

Simplify

$$(\underbrace{1+\tan^2 x}) (\underbrace{1-\cos^2 x})$$

$$= \sec^2 x \cdot \sin^2 x$$

$$= \frac{1}{\cos^2 x} \cdot \sin^2 x = \frac{\sin^2 x}{\cos^2 x} = \boxed{\tan^2 x}$$

Jan 8-10:07 AM

Factor $\cot^2 x - 7\csc x + 11$

$1 + 10$

Hint: $1 + \cot^2 x = \csc^2 x$

$$\cot^2 x - 7\csc x + 11 = \underbrace{1 + \cot^2 x} - 7\csc x + 10$$

$$= \csc^2 x - 7\csc x + 10$$

$$= \boxed{(\csc x - 5)(\csc x - 2)}$$

Jan 8-10:10 AM

Simplify

LCD = Cos x

$$\frac{\sin x \cdot \sec x}{\tan x} = \frac{\sin x \cdot \frac{1}{\cos x}}{\frac{\sin x}{\cos x}}$$

$$= \frac{\cancel{\cos x} \cdot \sin x \cdot \frac{1}{\cancel{\cos x}}}{\cancel{\cos x} \cdot \frac{\sin x}{\cancel{\cos x}}} = \frac{\sin x}{\sin x} = \boxed{1}$$

Jan 8-10:15 AM

Simplify

Hint:
LCD = $\sin x(\cos x - 1)$

$$\frac{\cos x - 1}{\sin x} - \frac{\sin x}{\cos x - 1}$$

$$= \frac{\cos x - 1}{\sin x} \cdot \frac{\cos x - 1}{\cos x - 1} - \frac{\sin x}{\cos x - 1} \cdot \frac{\sin x}{\sin x}$$

$$= \frac{(\cos x - 1)(\cos x - 1) - \sin x \cdot \sin x}{\sin x(\cos x - 1)}$$

$$= \frac{\cos^2 x - 2\cos x + 1 - \sin^2 x}{\sin x(\cos x - 1)}$$

$$= \frac{\cos^2 x - 2\cos x + \cos^2 x}{\sin x(\cos x - 1)} = \frac{2\cos^2 x - 2\cos x}{\sin x(\cos x - 1)}$$

$$= \frac{2\cos x\cancel{(\cos x - 1)}}{\sin x\cancel{(\cos x - 1)}} = \frac{2\cos x}{\sin x} = \boxed{2\cot x}$$

Jan 8-10:19 AM

Verify

$$(\cos x - \sec x)^2 = \tan^2 x - \sin^2 x$$

work on one Side To get the other Side.

Recall
$1 + \tan^2 x = \sec^2 x$
$\sin^2 x + \cos^2 x = 1$

$\tan^2 x - \sin^2 x =$

$\sec^2 x - 1 - (1 - \cos^2 x) =$

$\sec^2 x - 1 - 1 + \cos^2 x =$

$\cos^2 x - 2 \cdot 1 + \sec^2 x =$

$\cos^2 x - 2 \cdot \cos x \cdot \sec x + \sec^2 x =$

$\boxed{(\cos x - \sec x)^2}$

Recall
$A^2 - 2AB + B^2 = (A - B)^2$

Jan 8-10:29 AM

Verify

$$\frac{1 - \cos x}{\sin x} + \frac{\sin x}{1 - \cos x} = 2 \csc x \checkmark$$

$$\frac{1 - \cos x}{\sin x} + \frac{\sin x}{1 - \cos x} =$$

$$\frac{1 - \cos x}{\sin x} \cdot \frac{1 - \cos x}{1 - \cos x} + \frac{\sin x}{1 - \cos x} \cdot \frac{\sin x}{\sin x} =$$

$$\frac{(1 - \cos x)(1 - \cos x) + \sin x \cdot \sin x}{\sin x (1 - \cos x)} =$$

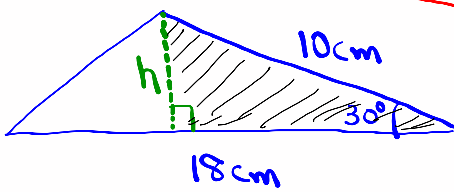
$$\frac{1 - 2\cos x + \underbrace{\cos^2 x + \sin^2 x}_{1}}{\sin x (1 - \cos x)} =$$

$$\frac{2 - 2\cos x}{\sin x (1 - \cos x)} = \frac{2\cancel{(1 - \cos x)}}{\sin x \cancel{(1 - \cos x)}} = \frac{2}{\sin x}$$

$$= 2 \cdot \frac{1}{\sin x} = \boxed{2 \csc x}$$

Jan 8-10:38 AM

Find the area of the triangle below:



$\text{Area} = \dfrac{bh}{2}$

$\sin 30° = \dfrac{h}{10}$

$\dfrac{1}{2} = \dfrac{h}{10}$ $\boxed{h = 5}$

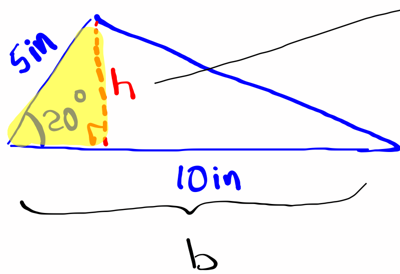
base = 18

height = ? = 5

$A = \dfrac{bh}{2} = \dfrac{18 \cdot 5}{2} = \boxed{45\ \text{cm}^2}$

Jan 8-10:45 AM

Find the area of the triangle below



$\sin 20° = \dfrac{h}{5}$ $h = 5\sin 20°$

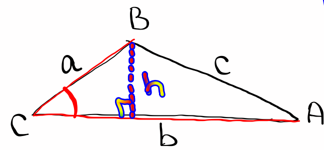
$\text{Area} = \dfrac{bh}{2} = \dfrac{\cancel{10}^{\,5} \cdot 5\sin 20°}{\cancel{2}}$

$= 25\sin 20°$

$\approx \boxed{8.55\ \text{in}^2}$

Jan 8-10:52 AM

Area of any triangle if two sides and angle between them are given.



$\text{Area} = \frac{1}{2} ab \sin C$

$\text{Area} = \frac{1}{2} ac \sin B$

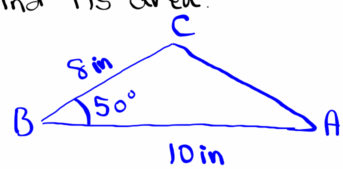
$\text{Area} = \frac{1}{2} bc \sin A$

$\sin C = \frac{h}{a}$

$h = a \sin C$

In triangle ABC, $a = 8$ in, $c = 10$ in, and $B = 50^\circ$

Find its area.



$\text{Area} = \frac{1}{2} ac \sin B$

$= \frac{1}{2} \cdot 8 \cdot 10 \cdot \sin 50^\circ$

$= 40 \cdot \sin 50^\circ \approx \boxed{30.64}$ in^2

Jan 8-10:56 AM

Angles in Standard position.

1) vertex at (0,0)

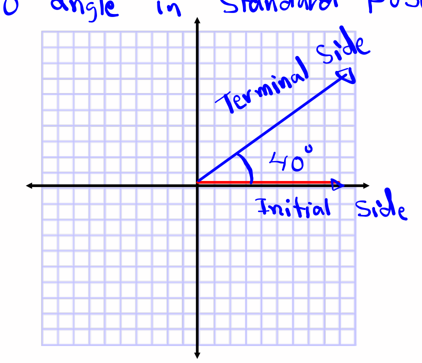
2) Initial Side on positive direction of x-axis.

3) Counterclockwise for $\alpha > 0$.

Clockwise for $\alpha < 0$

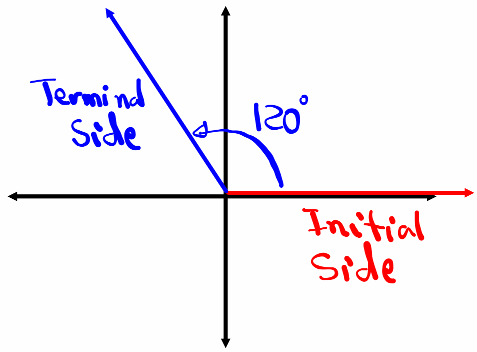
to get to terminal Side.

Draw 40° angle in standard position.

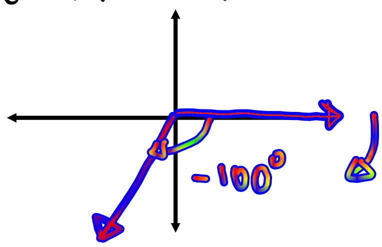


Jan 8-11:25 AM

Draw 120° in Standard Position.



Draw −100° in Standard Position

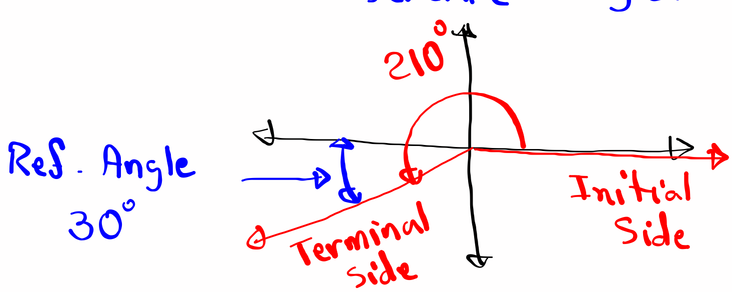


Jan 8-11:29 AM

Reference Angle:

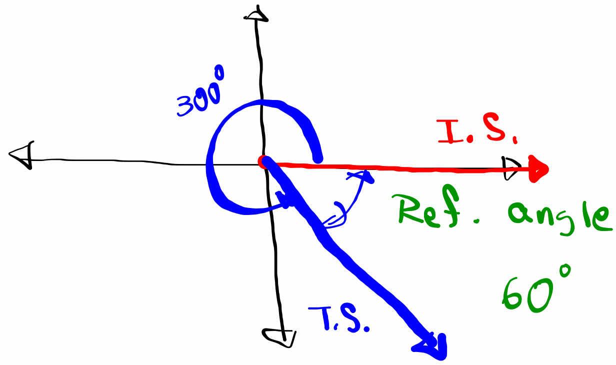
It is an angle between 0° & 90° that terminal Side makes with x-axis.

Draw 210° in standard Position and determine its reference angle.



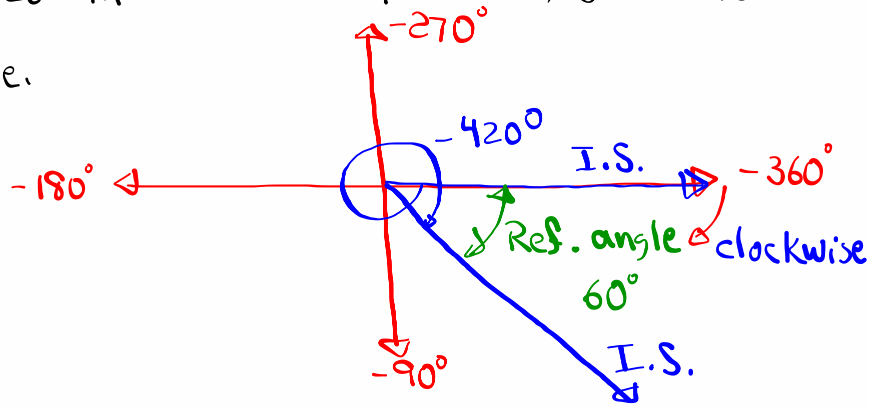
Jan 8-11:33 AM

Draw 300° in standard position, find its ref. angle.



Jan 8-11:37 AM

Draw −420° in standard position, find its ref. angle.



Jan 8-11:40 AM

Every $180°$ is called π radians.

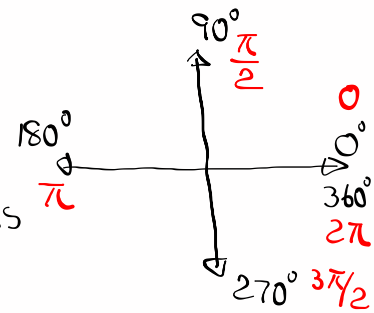
$$180° = \pi \text{ radians}$$

$$\boxed{1° = \frac{\pi}{180} \text{ radian}}$$

$$\boxed{\left(\frac{180}{\pi}\right)° = 1 \text{ radian}}$$

$$90° = 90 \cdot \frac{\pi}{180} = \frac{\pi}{2} \text{ Radians}$$

$$270° = 3 \cdot 90° = 3 \cdot \frac{\pi}{2} = \frac{3\pi}{2} \text{ Radians}$$



Jan 8-11:44 AM

Find

$$30° = 30 \cdot 1° = 30 \cdot \frac{\pi}{180} = \boxed{\frac{\pi}{6}}$$

$$45° = 45 \cdot 1° = 45 \cdot \frac{\pi}{180} = \boxed{\frac{\pi}{4}}$$

$$60° = 60 \cdot 1° = 60 \cdot \frac{\pi}{180} = \boxed{\frac{\pi}{3}}$$

in radians.

Convert $\frac{5\pi}{12}$ to degrees.

$$\frac{5\pi}{12} = \frac{5\pi}{12} \cdot 1 \text{ radian} = \frac{5\cancel{\pi}}{\cancel{12}\ \cancel{2}\ 1} \cdot \frac{\cancel{180}\ \cancel{30}\ 15}{\cancel{\pi}} \text{ degrees} = \boxed{75°}$$

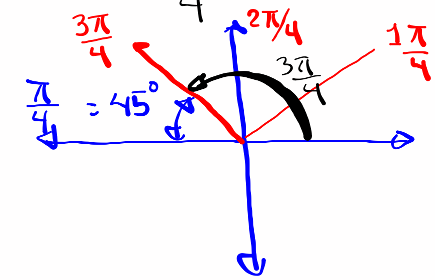
Jan 8-11:48 AM

Convert $\frac{11\pi}{9}$ from radian to degrees.

$\frac{11\pi}{9}$ radian $= \frac{11\cancel{\pi}}{\cancel{9}} \cdot \frac{\overset{20}{\cancel{180}}}{\cancel{\pi}}$ degrees $= 220$ degrees

$= \boxed{220^\circ}$

Draw $\frac{3\pi}{4}$ in standard position, find its ref. angle.



$\frac{\pi}{4} = 45^\circ$

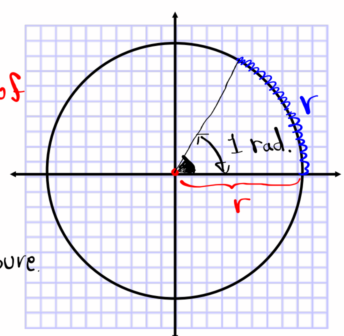
Jan 8-11:54 AM

What is a radian?

Draw a circle, centered at (0,0) with radius r.

Measure a portion of circle equal to r.

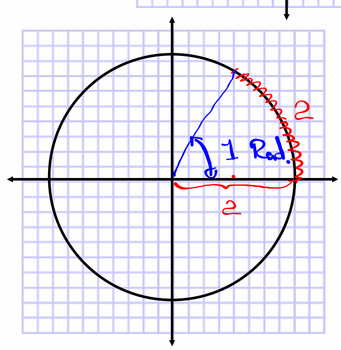
Central angle has a 1 radian measure.



$x^2+y^2=4$

Center (0,0)

radius 2



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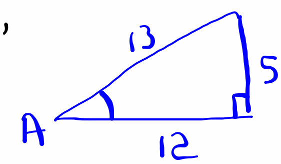
Class QZ 3

In right triangle ABC, $\sin A = \frac{5}{13}$,

Complete the chart below



$\sin A = 5/13$	$\csc A = 13/5$
$\cos A = 12/13$	$\sec A = 13/12$
$\tan A = 5/12$	$\cot A = 12/5$

Jan 8-12:06 PM