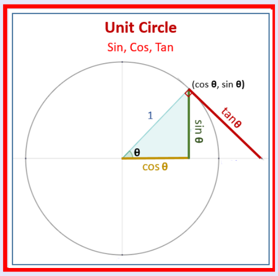
Math 241

Winter 2023

Lecture 14



Feb 19-8:47 AM

Solve $\quad 2\sin(x-30^\circ) - 1 = 0.$

$$\sin(x-30^\circ) = \frac{1}{2}$$

R.A. 30°

QI $\quad x - 30^\circ = 30^\circ + n\cdot 360^\circ \qquad \boxed{x = 60^\circ + n\cdot 360^\circ}$

QII $\quad x - \cancel{30^\circ} = 180^\circ - \cancel{30^\circ} + n\cdot 360^\circ \qquad \boxed{x = 180^\circ + n\cdot 360^\circ}$

Solutions in $[0^\circ, 360^\circ)$ $\quad n = 0 \quad 60^\circ, 180^\circ$

$n = 1 \quad$ Not in $[0^\circ, 360^\circ)$

$$\{60^\circ, 180^\circ\}$$

Jan 26-7:01 AM

Solve $\sqrt{2}\cos\left(2x+\frac{\pi}{2}\right)+1=0$

$\cos\left(2x+\frac{\pi}{2}\right)=\frac{-1}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}}$

$\cos\left(2x+\frac{\pi}{2}\right)=-\frac{\sqrt{2}}{2}$

R.A. $\frac{\pi}{4}$

QII $2x+\frac{\pi}{2}=\pi-\frac{\pi}{4}+n\cdot 2\pi$

LCD=4

$8x+2\pi=\underbrace{4\pi-\pi}_{3\pi}+n\cdot 8\pi$

$8x=\pi+n\cdot 8\pi$

$\boxed{x=\frac{\pi}{8}+n\cdot\pi}$

QIII $2x+\frac{\pi}{2}=\pi+\frac{\pi}{4}+n\cdot 2\pi$

LCD=4

$8x+2\pi=\underbrace{4\pi+\pi}_{5\pi}+n\cdot 8\pi$

$8x=3\pi+n\cdot 8\pi$

$\boxed{x=\frac{3\pi}{8}+n\cdot\pi}$

Solutions in $[0,2\pi)$

$n=0$ $\frac{\pi}{8},\frac{3\pi}{8}$

$n=1$ $\frac{9\pi}{8},\frac{11\pi}{8}$

$n=2$ Not in $[0,2\pi)$

$\left\{\frac{\pi}{8},\frac{3\pi}{8},\frac{9\pi}{8},\frac{11\pi}{8}\right\}$

Jan 26-7:06 AM

Solve $3\sin\frac{\theta}{4}-2=7\sin\frac{\theta}{4}-1$

$3\sin\frac{\theta}{4}-7\sin\frac{\theta}{4}=-1+2$

$-4\sin\frac{\theta}{4}=1$

$\sin\frac{\theta}{4}=\frac{-1}{4}$

R.A. $\sin^{-1}\left(\frac{1}{4}\right)\approx 14^\circ$

Q III $\frac{\theta}{4}=180^\circ+14^\circ+n\cdot 360^\circ$

$\frac{\theta}{4}=194^\circ+n\cdot 360^\circ$

$\boxed{\theta=776^\circ+n\cdot 1440^\circ}$

Q IV $\frac{\theta}{4}=360^\circ-14^\circ+n\cdot 360^\circ$

$\frac{\theta}{4}=346^\circ+n\cdot 360^\circ$

$\boxed{\theta=1384^\circ+n\cdot 1440^\circ}$

Solutions in $[0^\circ,360^\circ)$

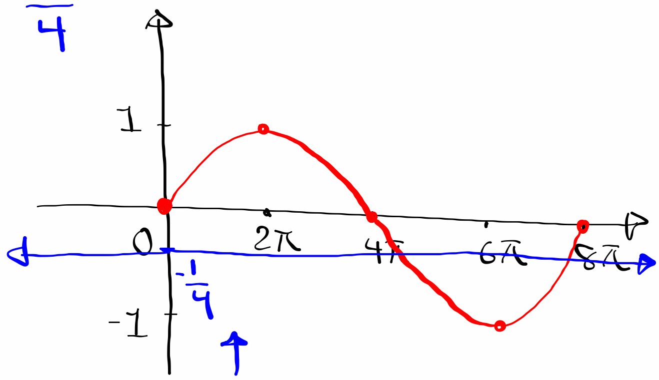
No Solution

Jan 26-7:16 AM

Graph $y = \sin\frac{\theta}{4}$

$0 \le \frac{\theta}{4} \le 2\pi$

$0 \le \theta \le 8\pi$



No Solutions $[0, 2\pi)$

Jan 26-7:23 AM

Solve $2x^2 - 9x = 5$

$2x^2 - 9x - 5 = 0$

$(2x + 1)(x - 5) = 0$

Zero-Product Rule

$2x + 1 = 0 \qquad x - 5 = 0$

$x = \frac{-1}{2} \qquad x = 5$

Solve $2\cos^2\theta - 9\cos\theta = 5$

$2\cos^2\theta - 9\cos\theta - 5 = 0$

$(2\cos\theta + 1)(\cos\theta - 5) = 0$

$2\cos\theta + 1 = 0 \qquad \cos\theta - 5 = 0$

$\cos\theta = \frac{-1}{2} \qquad \cos\theta = 5 \quad \emptyset$

R.A. 60°

QII

$\theta = 180^\circ - 60^\circ + n\cdot 360^\circ$

$\boxed{\theta = 120^\circ + n\cdot 360^\circ}$

QIII

$\theta = 180^\circ + 60^\circ + n\cdot 360^\circ$

$\theta = 240^\circ + n\cdot 360^\circ$

$n = 0 \rightarrow 120^\circ, 240^\circ$

$n = 1 \rightarrow$ Not in $[0^\circ, 360^\circ)$

$\{120^\circ, 240^\circ\}$

Jan 26-7:27 AM

Solve $2\tan^2\theta + 2\tan\theta - 1 = 0$

$a\,x^2 + b\,x + c = 0$

$a=2,\ b=2,\ c=-1$

$b^2-4ac = (2)^2-4(2)(-1) = 4+8 = 12$

$\tan\theta = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a} = \dfrac{-2 \pm \sqrt{12}}{2(2)}$

$\tan\theta = \dfrac{-2+\sqrt{12}}{4} \approx .366$

$\boxed{\tan\theta = .366}$

R.A. $\tan^{-1}(.366) \approx 20^\circ$

QI, QIII

QI $\boxed{\theta = 20^\circ + n\cdot 180^\circ}$

QIII $\theta = 180^\circ + 20^\circ + n\cdot 180^\circ$

$\boxed{\theta = 200^\circ + n\cdot 180^\circ}$

$\tan\theta = \dfrac{-2-\sqrt{12}}{4} \approx -1.366$

$\tan\theta = -1.366$

R.A. $\tan^{-1}(1.366) \approx 54^\circ$

QII, QIV

QII $\theta = 180^\circ - 54^\circ + n\cdot 180^\circ$

$\boxed{\theta = 126^\circ + n\cdot 180^\circ}$

QIV $\theta = 360^\circ - 54^\circ + n\cdot 180^\circ$

$\boxed{\theta = 306^\circ + n\cdot 180^\circ}$

what about solutions in $[0^\circ, 360^\circ)$.

$n=0 \rightarrow 20^\circ, 200^\circ, 126^\circ, 306^\circ$

$n=1 \rightarrow 200^\circ, 306^\circ$

$\{20^\circ, 126^\circ, 200^\circ, 306^\circ\}$

Jan 26-7:36 AM

Solve $\sqrt{3}\tan 2\theta - 2\sin\theta\tan 2\theta = 0$

Hint: Factor $\tan 2\theta \cdot [\sqrt{3} - 2\sin\theta] = 0$

$\tan 2\theta = 0$ OR $\sqrt{3} - 2\sin\theta = 0$

$\tan 2\theta = 0$ OR $\sin\theta = \dfrac{\sqrt{3}}{2}$

RA $0^\circ, 180^\circ$

$2\theta = 0^\circ + n\cdot 180^\circ \rightarrow \theta = n\cdot 90^\circ$

$2\theta = 180^\circ + n\cdot 180^\circ \rightarrow \theta = 90^\circ + n\cdot 90^\circ$

$[0^\circ, 360^\circ)$

$n=0$ $n=1$ $n=2$

$\boxed{0^\circ, 90^\circ, 180^\circ, 270^\circ}$

$\sin\theta = \dfrac{\sqrt{3}}{2}$

R.A. $\rightarrow 60^\circ$

QI, QII

$\theta = 60^\circ + n\cdot 360^\circ$

$\theta = 180^\circ - 60^\circ + n\cdot 360^\circ$

$\theta = 120^\circ + n\cdot 360^\circ$

$n=0$

$\boxed{60^\circ, 120^\circ,}$

$\{60^\circ, 120^\circ, 0^\circ, 90^\circ, 180^\circ, 270^\circ\}$

Jan 26-7:48 AM

Solve $\sin 2\theta + \sqrt{2}\cos\theta = 0$

Hint: what is $\sin 2\theta$?

$2\sin\theta\cos\theta + \sqrt{2}\cos\theta = 0$

$\cos\theta(2\sin\theta + \sqrt{2}) = 0$

$\cos\theta = 0$ OR $\sin\theta = -\frac{\sqrt{2}}{2}$

$90^\circ, 270^\circ$

R.A. 45°

QIII $\theta = 180^\circ + 45^\circ$

QIV $\theta = 360^\circ - 45^\circ$

$\theta = 90^\circ + n\cdot 360^\circ$

$\theta = 270^\circ + n\cdot 360^\circ$

$\theta = 225^\circ + n\cdot 360^\circ$ $\quad n=0 \quad 90^\circ, 270^\circ, 225^\circ, 315^\circ$

$\theta = 315^\circ + n\cdot 360^\circ$ $\quad n=1 \quad$ Not in $[0^\circ, 360^\circ)$

$\{90^\circ, 270^\circ, 225^\circ, 315^\circ\}$

Jan 26-7:57 AM

Solve $4\cos^2 x + 4\sin x - 5 = 0$

Hint:

Express $\cos^2 x$ in terms of $\sin x$

$\sin^2 x + \cos^2 x = 1 \Rightarrow \cos^2 x = 1 - \sin^2 x$

$4(1 - \sin^2 x) + 4\sin x - 5 = 0$

$4 - 4\sin^2 x + 4\sin x - 5 = 0$

$-4\sin^2 x + 4\sin x - 1 = 0$

$4\sin^2 x - 4\sin x + 1 = 0$

$a=4 \quad b=-4 \quad c=1$

$b^2 - 4ac = (-4)^2 - 4(4)(1) = 16 - 16 = 0$

$\sin x = \frac{-b \pm\sqrt{b^2-4ac}}{2a} = \frac{-(-4)\pm\sqrt{0}}{2(4)} = \frac{4\pm 0}{8} = \frac{4}{8} = \frac{1}{2}$

Solve $\sin x = \frac{1}{2}$ $\quad \boxed{x = 30^\circ + n\cdot 360^\circ}$

R.A. 30° $\quad x = 180^\circ - 30^\circ + n\cdot 360^\circ$

QI, QII $\quad \boxed{x = 150^\circ + n\cdot 360^\circ}$

If angle is greek letter → do degrees

If " " $x, t, \ldots$ → do radians

Jan 26-8:03 AM

Solve $\sin\theta - \cos\theta = 1$

false $2 = -2$

square $4 = 4$ True

Hint: Square both sides

$$(\sin\theta - \cos\theta)^2 = 1^2$$

$$\sin^2\theta - 2\sin\theta\cos\theta + \cos^2\theta = 1$$

$$1 - 2\sin\theta\cos\theta = 1$$

$$-\sin 2\theta = 1 - 1$$

$\sin 2\theta = 0$

$0°, 180°$

$2\theta = 0° + n \cdot 360° \to \theta = n \cdot 180°$

$2\theta = 180° + n \cdot 360°$

$\theta = 90° + n \cdot 180°$

Solutions for $[0°, 360°)$

$n = 0 \to \theta = \cancel{0°}, \boxed{90°}$

$n = 1 \to \theta = \boxed{180°}, \cancel{270°}$

Check $\theta = 90°$

$\sin 90° - \cos 90° = 1$

$1 - 0 = 1$

$1 = 1$ ✓

Check $\theta = 180°$

$\sin 180° - \cos 180° \stackrel{?}{=} 1$

$0 - (-1) = 1$

$1 = 1$ ✓

$\sin\theta - \cos\theta = 1$

Check $\theta = 0°$

$\sin 0° - \cos 0° \stackrel{?}{=} 1$

$0 - 1 = 1$

$-1 = 1$

false

Check $\theta = 270°$

$\sin 270° - \cos 270° \stackrel{?}{=} 1$

$-1 - 0 \stackrel{?}{=} 1$

$-1 = 1$ false

Jan 26-8:35 AM

Solve $\sin\frac{\theta}{2} + \cos\theta = 0$ for $0° \le \theta < 360°$

Hint: Write $\sin\frac{\theta}{2}$ as $\cos\theta$.

$$\sin\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{2}}$$

$$\pm\sqrt{\frac{1-\cos\theta}{2}} + \cos\theta = 0$$

$$\pm\sqrt{\frac{1-\cos\theta}{2}} = -\cos\theta$$

Square both sides to remove the radical.

$$\left(\pm\sqrt{\frac{1-\cos\theta}{2}}\right)^2 = (-\cos\theta)^2$$

$$\frac{1-\cos\theta}{2} = \cos^2\theta$$

Cross-Multiply

$$2\cos^2\theta = 1 - \cos\theta$$

$$2\cos^2\theta + \cos\theta - 1 = 0$$

$$(2\cos\theta - 1)(\cos\theta + 1) = 0$$

$2\cos\theta - 1 = 0$

$\cos\theta = \frac{1}{2}$

R.A. $60°$

QI $\cancel{\theta = 60°}$

QIV $\cancel{\theta = 300°}$

$\cos\theta + 1 = 0$

$\cos\theta = -1$

$\boxed{\theta = 180°}$

$\theta = 180°$

$\sin\frac{180°}{2} + \cos 180° \stackrel{?}{=} 0$

$1 + (-1) = 0$ ✓

$\theta = 60°$

$\sin 30° + \cos 60°$

$\frac{1}{2} + \frac{1}{2} = 0$

$1 = 0$

$\theta = 300°$

$\sin 150° + \cos 300° = 0$

$(+) + (+) = 0$

false

$\{180°\}$

Jan 26-8:45 AM

Solve $\underbrace{\cos 2x \cos x - \sin 2x \sin x}_{\text{use known identities to rewrite}} = \frac{\sqrt{2}}{2}$

$\cos(2x + x) = \frac{\sqrt{2}}{2}$

$\cos 3x = \frac{\sqrt{2}}{2}$

R.A. 45°

QI, QIV

QI $\Rightarrow$ $3x = 45^\circ + n\cdot 360^\circ$

$\boxed{x = 15^\circ + n\cdot 120^\circ}$

QIV $\Rightarrow$ $3x = 360^\circ - 45^\circ + n\cdot 360^\circ$

$3x = 315^\circ + n\cdot 360^\circ$

$\boxed{x = 105^\circ + n\cdot 120^\circ}$

Jan 26-8:58 AM

Solve $\sqrt{3}\sin x + 1\cos x = 2$

Divide everything by $\sqrt{(\sqrt{3})^2 + 1^2} = 2$

$\boxed{\frac{\sqrt{3}}{2}}\sin x + \boxed{\frac{1}{2}}\cos x = \frac{2}{2}$

think of an angle α Such that

$\cos\alpha = \frac{\sqrt{3}}{2}$ & $\sin\alpha = \frac{1}{2}$ $\Rightarrow \alpha = 30^\circ$

$\boxed{\cos 30^\circ}\sin x + \boxed{\sin 30^\circ}\cos x = 1$

$\sin(\underbrace{x + 30^\circ}_{90^\circ}) = 1$

$x + 30^\circ = 90^\circ + n\cdot 360^\circ$ $\Rightarrow x = 60^\circ + n\cdot 360^\circ$

$n = 0$

$\boxed{x = 60^\circ}$

Verify:

$\sqrt{3}\sin 60^\circ + 1\cdot\cos 60^\circ = 2$ ✓

$\sqrt{3}\cdot\frac{\sqrt{3}}{2} + \frac{1}{2} = 2$ $\quad \frac{3}{2} + \frac{1}{2} = \frac{4}{2} = 2$ ✓

Jan 26-9:05 AM

Solve: $\sin x - \cos x = -1$

Divide everything by $\sqrt{(1)^2+(-1)^2} = \sqrt{2}$

$$\frac{1}{\sqrt{2}}\sin x - \frac{1}{\sqrt{2}}\cos x = \frac{-1}{\sqrt{2}}$$

$$\frac{\sqrt{2}}{2}\sin x - \frac{\sqrt{2}}{2}\cos x = -\frac{\sqrt{2}}{2}$$

Think of angle α such that

$\cos\alpha = \frac{\sqrt{2}}{2}$ and $\sin\alpha = -\frac{\sqrt{2}}{2}$

$$\alpha = 315^\circ$$

$$\cos(315^\circ)\sin x - \sin(315^\circ)\cos x = -\frac{\sqrt{2}}{2}$$

$$\cos 315^\circ \sin x + \sin 315^\circ \cos x = -\frac{\sqrt{2}}{2}$$

$$\sin x \cos 315^\circ + \cos x \sin 135^\circ = -\frac{\sqrt{2}}{2}$$

$$\sin(x + 315^\circ) = -\frac{\sqrt{2}}{2}$$

RA 45°

QIII $x + 315^\circ = 180^\circ + 45^\circ + n \cdot 360^\circ$

$x = 225^\circ - 315^\circ + n \cdot 360^\circ \rightarrow x = -90^\circ + n \cdot 360^\circ$

QIV $x + 315^\circ = 360^\circ - 45^\circ + n \cdot 360^\circ$

$x + \cancel{315^\circ} = \cancel{315^\circ} + n \cdot 360^\circ$

$x = n \cdot 360^\circ$

$n = 0$ $x = 0$ $x = -90^\circ = 270^\circ$

$n = 1$ $x = 270$

$$\{0^\circ, 270^\circ\}$$

Jan 26-9:14 AM