Product-to-Sum Formulas

$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$

$$\cos(A-B) = \cos A \cos B + \sin A \sin B$$

$$\cos(A+B) + \cos(A-B) = 2\cos A \cos B$$

$$\cos A \cos B = \frac{1}{2}\left[\cos(A-B) + \cos(A+B)\right]$$

$$\cos(A-B) - \cos(A+B) = 2\sin A \sin B$$

$$\sin A \sin B = \frac{1}{2}\left[\cos(A-B) - \cos(A+B)\right]$$

$\sin(A+B) = \sin A \cos B + \cos A \sin B$

$\sin(A-B) = \sin A \cos B - \cos A \sin B$

$\sin(A+B) + \sin(A-B) = 2 \sin A \cos B$

$\sin A \cos B = \frac{1}{2}\left[\sin(A+B) + \sin(A-B)\right]$

$\sin A \sin B = \frac{1}{2}\left[\cos(A-B) - \cos(A+B)\right]$

$\cos A \cos B = \frac{1}{2}\left[\cos(A-B) + \cos(A+B)\right]$

$\sin A \cos B = \frac{1}{2}\left[\sin(A+B) + \sin(A-B)\right]$

Product - to - Sum formulas

find exact value for $\cos 45^\circ \sin 15^\circ$

$\cos 45^\circ \sin 15^\circ = \frac{1}{2}\left[\sin(45^\circ + 15^\circ) + \sin(45^\circ - 15^\circ)\right]$

$= \frac{1}{2}\left[\sin 60^\circ + \sin 30^\circ\right]$

$= \frac{1}{2}\left[\frac{\sqrt{3}}{2} + \frac{1}{2}\right] = \frac{1}{2} \cdot \frac{1}{2}\left[\sqrt{3} + 1\right]$

$\boxed{= \frac{1}{4}(\sqrt{3} + 1)}$

Find exact value for $4\sin\left(x+\frac{\pi}{4}\right)\sin\left(x-\frac{\pi}{4}\right)$

$$4\sin\left(x+\frac{\pi}{4}\right)\sin\left(x-\frac{\pi}{4}\right) = 4\cdot\frac{1}{2}\left[\cos(\underbrace{\quad - \quad}_{\frac{\pi}{2}}) - \cos(\underbrace{\quad + \quad}_{2x})\right]$$

$$x+\frac{\pi}{4}-\left(x-\frac{\pi}{4}\right)=\frac{\pi}{2}$$

$$x+\frac{\pi}{4}+x-\frac{\pi}{4}=2x$$

$$\cos\frac{\pi}{2}=\cos 90^\circ = 0$$

$$= 2\left[\cos\frac{\pi}{2} - \cos 2x\right]$$

$$= 2\left[0 - \cos 2x\right]$$

$$= \boxed{-2\cos 2x}$$

Evaluate $\cos\frac{5\pi}{12}\cos\frac{\pi}{12}$

$$\cos\frac{5\pi}{12}\cos\frac{\pi}{12} = \frac{1}{2}\left[\cos(\underbrace{\quad - \quad}_{\pi/3}) + \cos(\underbrace{\quad + \quad}_{\frac{\pi}{2}})\right]$$

$$\frac{5\pi}{12}-\frac{\pi}{12}=\frac{4\pi}{12}=\frac{\pi}{3}$$

$$\frac{5\pi}{12}+\frac{\pi}{12}=\frac{6\pi}{12}=\frac{\pi}{2}$$

$$= \frac{1}{2}\left[\cos\frac{\pi}{3} + \cos\frac{\pi}{2}\right]$$

$$= \frac{1}{2}\left[\frac{1}{2} + 0\right]$$

$$= \boxed{\frac{1}{4}}$$