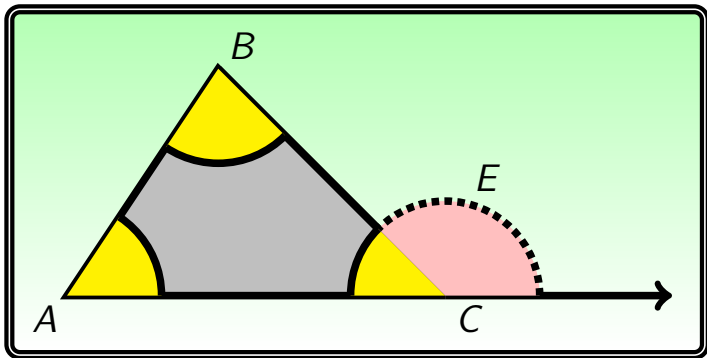
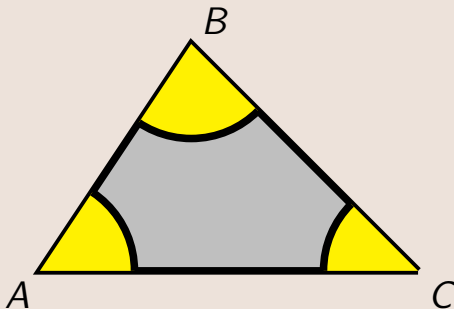


Trigonometry DLA Series



Angles of Triangle

In this DLA, we are going to study the relationship between interior and exterior angles in any triangle.



First, the sum of all three interior angles is equal to 180° , that is

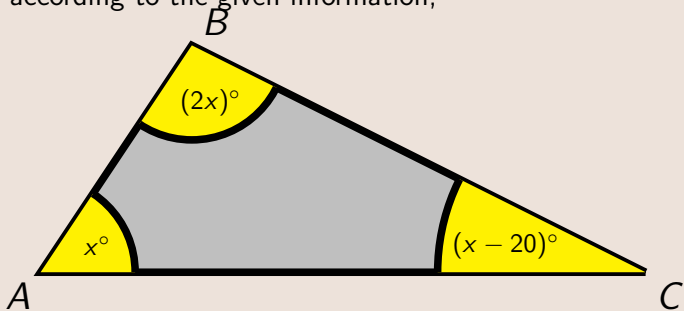
$$m\angle A + m\angle B + m\angle C = 180^\circ$$

Example:

In triangle ABC , angle B is twice angle A , and angle C is 20° less than angle A . Find the measure of all three angles.

Solution:

Let x be the measure of angle A , draw a triangle, and label each angle according to the given information,



Solution(continued):

$$m\angle A + m\angle B + m\angle C = 180^\circ \quad (\text{Given Fact})$$

$$x + 2x + x - 20 = 180^\circ \quad (\text{Substitution})$$

$$4x - 20 = 180 \quad (\text{Simplify})$$

$$4x - 20 + 20 = 180 + 20 \quad (\text{Addition Property})$$

$$4x + 0 = 200 \quad (\text{Inverse \& Simplify})$$

$$4x = 200 \quad (\text{Identity})$$

$$x = 50 \quad (\text{Division Property})$$

So angle A is 50° , angle B is 100° , and angle C is 30° .

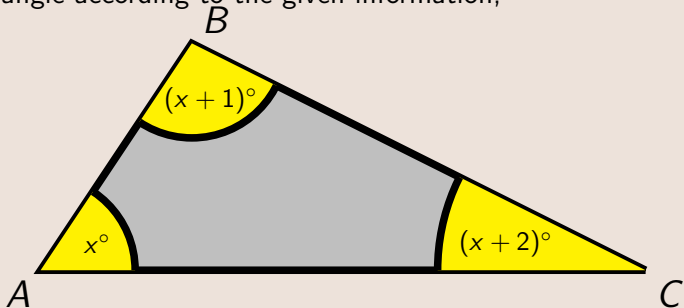
$$m\angle A = 50^\circ, m\angle B = 100^\circ, m\angle C = 30^\circ$$

Example:

The measure of all three angles in triangle ABC are three consecutive integers. Find the measure of all three angles.

Solution:

Let x be the measure of angle A , then draw a triangle, and label each angle according to the given information,



Solution(continued):

$$m\angle A + m\angle B + m\angle C = 180^\circ \quad (\text{Given Fact})$$

$$x + x + 1 + x + 2 = 180^\circ \quad (\text{Substitution})$$

$$3x + 3 = 180 \quad (\text{Simplify})$$

$$3x + 3 - 3 = 180 - 3 \quad (\text{Subtraction Property})$$

$$3x + 0 = 177 \quad (\text{Inverse \& Simplify})$$

$$3x = 177 \quad (\text{Identity})$$

$$x = 59 \quad (\text{Division Property})$$

So angle A is 59° , angle B is 60° , and angle C is 61° .

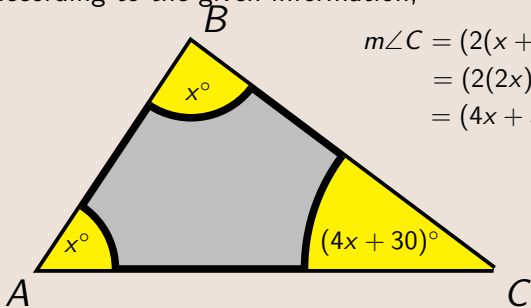
$$m\angle A = 59^\circ, m\angle B = 60^\circ, m\angle C = 61^\circ$$

Example:

In triangle ABC , angle B is equal to angle A , and angle C is 30° more than twice the sum of angles A and B . Find the measure of all three angles.

Solution:

Let x be the measure of angle A , draw a triangle, and label each angle according to the given information,



$$\begin{aligned} m\angle C &= (2(x + x) + 30)^\circ \\ &= (2(2x) + 30)^\circ \\ &= (4x + 30)^\circ \end{aligned}$$

Solution(continued):

$$m\angle A + m\angle B + m\angle C = 180^\circ \quad (\text{Given Fact})$$

$$x + x + 4x + 30 = 180^\circ \quad (\text{Substitution})$$

$$6x + 30 = 180 \quad (\text{Simplify})$$

$$6x + 30 - 30 = 180 - 30 \quad (\text{Subtraction Property})$$

$$6x + 0 = 150 \quad (\text{Inverse \& Simplify})$$

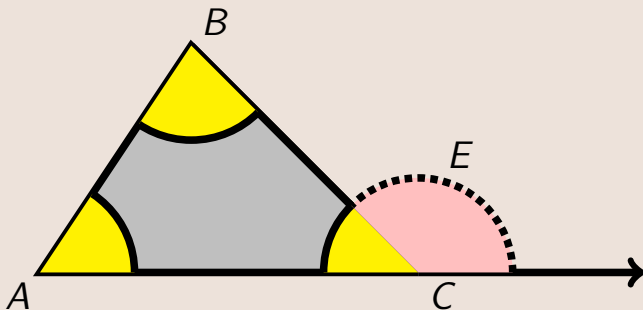
$$6x = 150 \quad (\text{Identity})$$

$$x = 25 \quad (\text{Division Property})$$

So angle A is 25° , angle B is 25° , and angle C is 130° .

$$m\angle A = 25^\circ, m\angle B = 25^\circ, m\angle C = 130^\circ$$

The Exterior Angle is the angle between any side of a triangle, and a line extended from the other side of that angle.

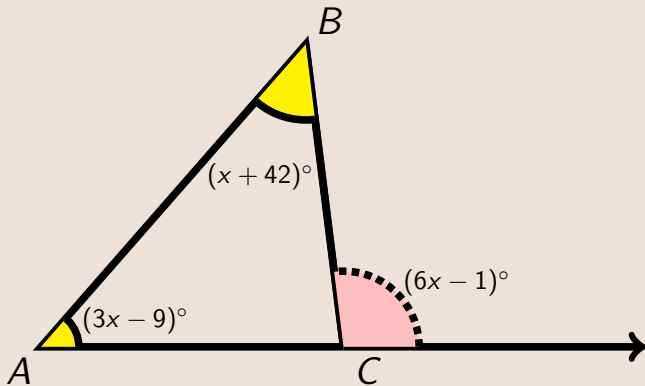


The sum of any two interior angles in any triangle is equal to the exterior angle of the other interior angle, that is

$$m\angle A + m\angle B = m\angle E$$

Example:

Use the drawing below to find the measure of all three interior angles of the triangle ABC .



Solution:

We are given the measure of one exterior angle in the form of an expression $(6x - 1)^\circ$ as well as the measure of two opposite interior angles with measurement $(3x - 9)^\circ$ and $(x + 42)^\circ$.

$$m\angle E = m\angle A + m\angle B \quad (\text{Exterior \& Interior Angles})$$

$$6x - 1 = 3x - 9 + x + 42 \quad (\text{Making Substitution})$$

$$6x - 1 = 4x + 33 \quad (\text{Simplify})$$

$$6x - 4x = 33 + 1 \quad (\text{Subtraction \& Addition})$$

$$2x = 34 \quad (\text{Simplify})$$

$$x = 17 \quad (\text{Division})$$

Now that we have the value for x , we can compute the measure of each angle.

Solution(continued):

For angle A ,

$$m\angle A = 3x - 9 \quad \text{(Given Information)}$$

$$m\angle A = 3(17) - 9 \quad \text{(Making Substitution)}$$

$$= 51 - 9 \quad \text{(Simplify)}$$

$$= 42 \quad \text{(Simplify More)}$$

For angle B ,

$$m\angle B = x + 42 \quad \text{(Given Information)}$$

$$m\angle B = 17 + 42 \quad \text{(Making Substitution)}$$

$$= 59 \quad \text{(Simplify)}$$

Solution(continued):

Now we know the measure of angles A and B , we can use the fact that the sum of all three interior angles in any triangle is 180° to find the measure of angle C .

$$m\angle A + m\angle B + m\angle C = 180^\circ \quad (\text{Given Fact})$$

$$42 + 59 + m\angle C = 180^\circ \quad (\text{Substitution})$$

$$101 + m\angle C = 180 \quad (\text{Simplify})$$

$$m\angle C = 180 - 101 \quad (\text{Subtraction Property})$$

$$m\angle C = 79 \quad (\text{Simplify})$$

So angle A is 42° , angle B is 59° , and angle C is 79° .

$$m\angle A = 42^\circ, m\angle B = 59^\circ, m\angle C = 79^\circ$$

Example:

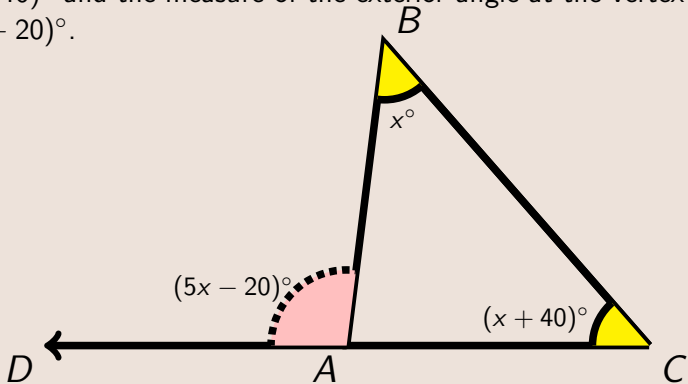
In triangle ABC , the measure of angle C is 40° more than the measure of angle B .

The measure of the exterior angle at the vertex A is 20° less than five times the measure of angle B .

- ▶ Draw and clearly label using given information.
- ▶ Set up the equation using geometry facts discussed here.
- ▶ Solve the equation.
- ▶ Find the measure of all three interior angles of the triangle ABC .

Solution:

Let x be the measure of angle B , then the measure of angle C is $(x + 40)^\circ$ and the measure of the exterior angle at the vertex A is $(5x - 20)^\circ$.



Solution(continued):

We now the measure of one exterior angle in the form of an expression $(5x - 20)^\circ$ as well as the measure of two opposite interior angles with measurement x° and $(x + 40)^\circ$.

$$m\angle E = m\angle B + m\angle C \quad (\text{Exterior \& Interior Angles})$$

$$5x - 20 = x + x + 40 \quad (\text{Making Substitution})$$

$$5x - 20 = 2x + 40 \quad (\text{Simplify})$$

$$5x - 2x = 40 + 20 \quad (\text{Subtraction \& Addition})$$

$$3x = 60 \quad (\text{Simplify})$$

$$x = 20 \quad (\text{Division})$$

Now that we have the value for x , we can compute the measure of each angle.

Solution(continued):

For angle B ,

$$m\angle B = x \quad (\text{Given Information})$$

$$m\angle B = 20 \quad (\text{Making Substitution})$$

For angle C ,

$$m\angle C = x + 40 \quad (\text{Given Information})$$

$$m\angle C = 20 + 40 \quad (\text{Making Substitution})$$

$$= 60 \quad (\text{Simplify})$$

Solution(continued):

Now we know the measure of angles B and C , we can use the fact that the sum of all three interior angles in any triangle is 180° to find the measure of angle A .

$$m\angle A + m\angle B + m\angle C = 180^\circ \quad (\text{Given Fact})$$

$$m\angle A + 20 + 60 = 180^\circ \quad (\text{Substitution})$$

$$m\angle A + 80 = 180 \quad (\text{Simplify})$$

$$m\angle A = 180 - 80 \quad (\text{Subtraction Property})$$

$$m\angle A = 100 \quad (\text{Simplify})$$

So angle A is 100° , angle B is 20° , and angle C is 60° .

$$m\angle A = 100^\circ, m\angle B = 20^\circ, m\angle C = 60^\circ$$



Start at ELAC, Go Anywhere