Slide 1

Chapter 4
Probability

Slide 2

Definitions

- **Event**

  Any collection of results or outcomes of a procedure.

- **Simple Event**

  An outcome or an event that cannot be further broken down into simpler components.

- **Sample Space**

  Consists of all possible *simple* events. That is, the sample space consists of all outcomes that cannot be broken down any further.

Slide 3

Notation for Probabilities

P **- denotes a probability.**

A, B,* and *C **- denote specific events.**

***P* (*A*)** **- denotes the probability of event A occurring.**

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Basic Rules for Computing Probability

Rule 1: Relative Frequency Approximation of Probability

Conduct (or observe) a procedure a large number of times, and count the number of times event *A* actually occurs. Based on these actual results, *P(A)* is *estimated* as follows:

$$P(A) = \frac{\text{number of times A occurred}}{\text{number of times trial was repeated}}$$

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Basic Rules for Computing Probability

Rule 2: Classical Approach to Probability (Requires Equally Likely Outcomes)

Assume that a given procedure has *n* different simple events and that each of those simple events has an *equal chance* of occurring. If event *A* can occur in *s* of these *n* ways, then

$$P(A) = \frac{s}{n} = \frac{\text{number of ways } A \text{ can occur}}{\text{number of different simple events}}$$

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Basic Rules for Computing Probability

Rule 3: Subjective Probabilities

***P*(*A*), the probability of event *A*, is found by simply guessing or estimating its value based on knowledge of the relevant circumstances.**

Law of Large Numbers

Slide 7

As a procedure is repeated again and again, the relative frequency probability (from Rule 1) of an event tends to approach the actual probability.

Example

Slide 8

Roulette You plan to bet on number 13 on the next spin of a roulette wheel. What is the probability that you will lose?

Solution A roulette wheel has 38 different slots, only one of which is the number 13. A roulette wheel is designed so that the 38 slots are equally likely. Among these 38 slots, there are 37 that result in a loss. Because the sample space includes equally likely outcomes, we use the classical approach (Rule 2) to get

$$P(\text{loss}) = \frac{37}{38}$$

Probability Limits

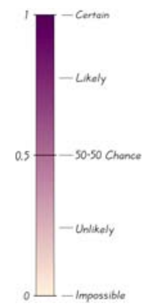
Slide 9

- The probability of an impossible event is 0.
- The probability of an event that is certain to occur is 1.
- $0 \le P(A) \le 1$ for any event *A*.

Possible Values for Probabilities

Slide 10

Figure 3-2



Definition

Slide 11

The complement of event *A*, denoted by $\bar{A}$, consists of all outcomes in which the event *A* does *not* occur.

Example

Slide 12

Birth Genders In reality, more boys are born than girls. In one typical group, there are 205 newborn babies, 105 of whom are boys. If one baby is randomly selected from the group, what is the probability that the baby is *not* a boy?

Solution Because 105 of the 205 babies are boys, it follows that 100 of them are girls, so

$$P(\text{not selecting a boy}) = P(\overline{\text{boy}}) = P(\text{girl}) = \frac{100}{205} = 0.488$$

Rounding Off Probabilities

Slide 13

When expressing the value of a probability, either give the *exact* fraction or decimal or round off final decimal results to three significant digits. (*Suggestion*: When the probability is not a simple fraction such as 2/3 or 5/9, express it as a decimal so that the number can be better understood.)

Definitions

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- **The actual odds against event *A* occurring are the ratio $P(\overline{A})/P(A)$, usually expressed in the form of *a:b* (or "*a* to *b*"), where *a* and *b* are integers having no common factors.**
- **The actual odds in favor event *A* occurring are the reciprocal of the actual odds against the event. If the odds against *A* are *a:b*, then the odds in favor of *A* are *b:a*.**
- **The payoff odds against event *A* represent the ratio of the net profit (if you win) to the amount bet.**

payoff odds against event A = (net profit) : (amount bet)

Definition

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Compound Event

Any event combining 2 or more simple events

Notation

***P*(*A* or *B*) = *P* (event *A* occurs or event *B* occurs or they both occur)**

General Rule for a Compound Event

Slide 16

When finding the probability that event *A* occurs or event *B* occurs, find the total number of ways *A* can occur and the number of ways *B* can occur, but *find the total in such a way that no outcome is counted more than once.*

Compound Event

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Formal Addition Rule

***P*(*A* or *B*) = *P*(*A*) + *P*(*B*) – *P*(*A* and *B*)**

where *P*(*A* and *B*) denotes the probability that *A* and *B* both occur at the same time as an outcome in a trial or procedure.

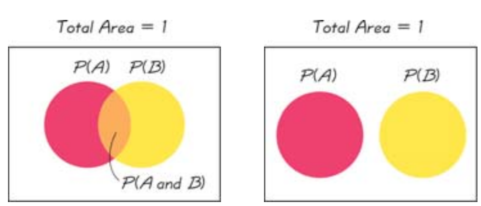
Intuitive Addition Rule

To find *P*(*A* or *B*), find the sum of the number of ways event *A* can occur and the number of ways event *B* can occur, *adding in such a way that every outcome is counted only once*. *P*(*A* or *B*) is equal to that sum, divided by the total number of outcomes. In the sample space.

Definition

Slide 18

Events *A* and *B* are disjoint (or mutually exclusive) if they cannot both occur together.



Figures 3-4 and 3-5

Applying the Addition Rule

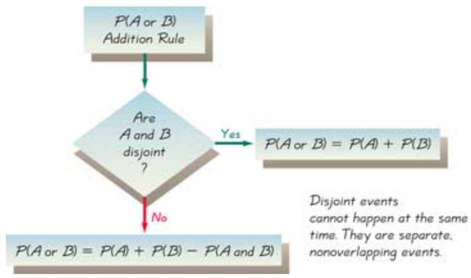


Figure 3-6

Example

Titanic Passengers

	Men	Women	Boys	Girls	Totals
Survived	332	318	29	27	706
Died	1360	104	35	18	1517
Total	1692	422	64	56	2223

Find the probability of randomly selecting a man or a boy.

Adapted from Exercises 9 thru 12

Example

	Men	Women	Boys	Girls	Totals
Survived	332	318	29	27	706
Died	1360	104	35	18	1517
Total	1692	422	64	56	2223

Find the probability of randomly selecting a man or a boy.

$$P(\text{man or boy}) = \frac{1692}{2223} + \frac{64}{2223} = \frac{1756}{2223} = 0.790$$

* Disjoint *

Adapted from Exercises 9 thru 12

Example

	Men	Women	Boys	Girls	Totals
Survived	332	318	29	27	706
Died	1360	104	35	18	1517
Total	1692	422	64	45	2223

Find the probability of randomly selecting a man or someone who survived.

Adapted from Exercises 9 thru 12

Example

	Men	Women	Boys	Girls	Totals
Survived	332	318	29	27	706
Died	1360	104	35	18	1517
Total	1692	422	64	45	2223

Find the probability of randomly selecting a man or someone who survived.

$$P(\text{man or survivor}) = \frac{1692}{2223} + \frac{706}{2223} - \frac{332}{2223} = \frac{2066}{2223}$$

$= 0.929$

* NOT Disjoint *

Adapted from Exercises 9 thru 12

Complementary Events

***P*(*A*) and P(*$\bar{A}$*) are mutually exclusive**

All simple events are either in *A* or *$\bar{A}$*.

Rules of Complementary Events

Slide 25

$$P(A) + P(\bar{A}) = 1$$

$$P(\bar{A}) = 1 - P(A)$$

$$P(A) = 1 - P(\bar{A})$$

Venn Diagram for the Complement of Event A

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Figure 3-7

Notation

Slide 27

P(*A* and *B*) =

P(event *A* occurs in a first trial and event *B* occurs in a second trial)

Tree Diagrams

Slide 28

A tree diagram is a picture of the possible outcomes of a procedure, shown as line segments emanating from one starting point. These diagrams are helpful in counting the number of possible outcomes if the number of possibilities is not too large.

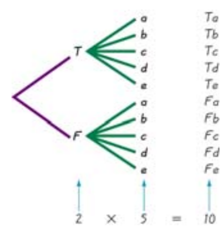

Figure 3-8 summarizes the possible outcomes for a true/false followed by a multiple choice question.

Note that there are 10 possible combinations.

Example

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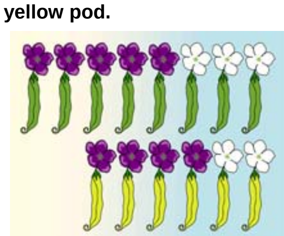
Genetics Experiment Mendel's famous hybridization experiments involved peas, like those shown in Figure 3-3 (below). If two of the peas shown in the figure are randomly selected ***without replacement***, find the probability that the first selection has a green pod and the second has a yellow pod.



Example - Solution

Slide 30

First selection: *P*(green pod) = 8/14 (14 peas, 8 of which have green pods)

Second selection: *P*(yellow pod) = 6/13 (13 peas remaining, 6 of which have yellow pods)

With *P*(first pea with green pod) = 8/14 and *P*(second pea with yellow pod) = 6/13, we have

P(First pea with green pod and second pea with yellow pod) =

$$\frac{8}{14} \bullet \frac{6}{13} \approx 0.264$$

Example
Important Principle

Slide 31

The preceding example illustrates the important principle that the ***probability for the second event B should take into account the fact that the first event A has already occurred*****.**

Notation for Conditional Probability

Slide 32

$P(B|A)$ represents the probability of event B occurring after it is assumed that event A has already occurred (read $B|A$ as "B given A.")

Definitions

Slide 33

Independent Events

Two events A and B are independent if the occurrence of one does not affect the probability of the occurrence of the other. (Several events are similarly independent if the occurrence of any does not affect the occurrence of the others.) If A and B are not independent, they are said to be dependent.

Formal Multiplication Rule

Slide 34

- $P(A \text{ and } B) = P(A) \bullet P(B|A)$
- **Note that if A and B are independent events, P(B|A) is really the same as P(B)**

Intuitive Multiplication Rule

Slide 35

When finding the probability that event A occurs in one trial and B occurs in the next trial, multiply the probability of event A by the probability of event B, but be sure that the probability of event B takes into account the previous occurrence of event A.

Applying the Multiplication Rule

Slide 36

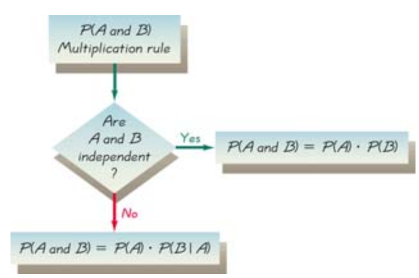


Figure 3-9

Small Samples from Large Populations

Slide 37

If a sample size is no more than 5% of the size of the population, treat the selections as being *independent* (even if the selections are made without replacement, so they are technically dependent).

Summary of Fundamentals

Slide 38

- In the addition rule, the word "or" on *P*(*A* or *B*) suggests addition. Add *P*(*A*) and *P*(*B*), being careful to add in such a way that every outcome is counted only once.
- In the multiplication rule, the word "and" in *P*(*A* and *B*) suggests multiplication. Multiply *P*(*A*) and *P*(*B*), but be sure that the probability of event *B* takes into account the previous occurrence of event *A*.

Complements: The Probability of "At Least One"

Slide 39

- "At least one" is equivalent to "one or more."
- The complement of getting at least one item of a particular type is that you get no items of that type.

Example

Slide 40

Gender of Children Find the probability of a couple having at least 1 girl among 3 children. Assume that boys and girls are equally likely and that the gender of a child is independent of the gender of any brothers or sisters.

Solution

Step 1: Use a symbol to represent the event desired. In this case, let *A* = at least 1 of the 3 children is a girl.

Example

Slide 41

Solution (cont)

Step 2: Identify the event that is the complement of *A*.

$\bar{A}$ = not getting at least 1 girl among 3 children

= all 3 children are boys

= boy and boy and boy

Step 3: Find the probability of the complement.

$P(\bar{A})$ = *P*(boy and boy and boy)

$$= \frac{1}{2} \bullet \frac{1}{2} \bullet \frac{1}{2} = \frac{1}{8}$$

Example

Slide 42

Solution (cont)

Step 4: Find P(A) by evaluating $1 - P(\bar{A})$.

$$P(A) = 1 - P(\bar{A}) = 1 - \frac{1}{8} = \frac{7}{8}$$

Interpretation There is a 7/8 probability that if a couple has 3 children, at least 1 of them is a girl.

Key Principle

Slide 43

To find the probability of ***at least one*** of something, calculate the probability of ***none***, then subtract that result from 1. That is,

$$P(\text{at least one}) = 1 - P(\text{none})$$

Definition

Slide 44

A **conditional probability** of an event is a probability obtained with the additional information that some other event has already occurred. $P(B \mid A)$ denotes the conditional probability of event B occurring, given that A has already occurred, and it can be found by dividing the probability of events A and B both occurring by the probability of event A:

$$P(B \mid A) = \frac{P(A \text{ and } B)}{P(A)}$$

Intuitive Approach to Conditional Probability

Slide 45

The conditional probability of B given A can be found by assuming that event A has occurred and, working under that assumption, calculating the probability that event B will occur.

Testing for Independence

Slide 46

In Section 3-4 we stated that events A and B are independent if the occurrence of one does not affect the probability of occurrence of the other. This suggests the following test for independence:

Two events A and B are *independent* if	Two events A and B are *dependent* if
$P(B \mid A) = P(B)$ or $P(A \text{ and } B) = P(A) \bullet P(B)$	$P(B \mid A) \neq P(B)$ or $P(A \text{ and } B) \neq P(A) \bullet P(B)$

Fundamental Counting Rule

Slide 47

For a sequence of two events in which the first event can occur m ways and the second event can occur n ways, the events together can occur a total of $m \bullet n$ ways.

Notation

Slide 48

The **factorial symbol !** Denotes the product of decreasing positive whole numbers. For example,

$$4! = 4 \bullet 3 \bullet 2 \bullet 1 = 24.$$

By special definition, $0! = 1$.

Factorial Rule

Slide 49

A collection of n different items can be arranged in order $n!$ different ways. (This factorial rule reflects the fact that the first item may be selected in n different ways, the second item may be selected in $n - 1$ ways, and so on.)

Permutations Rule
(when items are all different)

Slide 50

The number of permutations (or sequences) of r items selected from n available items (without replacement is

$$_nP_r = \frac{n!}{(n - r)!}$$

Permutation Rule: Conditions

Slide 51

- We must have a total of n different items available. (This rule does not apply if some items are identical to others.)
- We must select r of the n items (without replacement.)
- We must consider rearrangements of the same items to be different sequences.

Permutations Rule
(when some items are identical to others)

Slide 52

If there are n items with n_1 alike, n_2 alike, . . . n_k alike, the number of permutations of all n items is

$$\frac{n!}{n_1! \, . \, n_2! \, .. \, . \, . \, . \, . \, . \, . \, . \, n_k!}$$

Combinations Rule

Slide 53

The number of combinations of r items selected from n different items is

$$_nC_r = \frac{n!}{(n - r)! \, r!}$$

Combinations Rule: Conditions

Slide 54

- We must have a total of n different items available.
- We must select r of the n items (without replacement.)
- We must consider rearrangements of the same items to be the same. (The combination *ABC* is the same as *CBA*.)

Permutations versus Combinations

Slide 55

When different orderings of the same items are to be counted separately, we have a permutation problem, but when different orderings are not to be counted separately, we have a combination problem.