Class Quiz

1) Solve by Square-Root Method: \((3x - 5)^2 = -4\)
   \[3x - 5 = \pm \sqrt{-4}\]
   \[3x - 5 = \pm 2i\]
   \[x = \frac{5}{3} \pm \frac{2}{3}i\]

2) Solve by Completing the Square: \(x^2 - 8x - 4 = 0\)
   \[x^2 - 8x + 16 = 4 + 16\]
   \[(x - 4)^2 = 20\]
   \[x - 4 = \pm \sqrt{20}\]
   \[x = 4 \pm 2\sqrt{5}\]

3) Solve: \(\sqrt{x - 2} + x = 8\)
   \[\sqrt{x - 2} = 8 - x\]
   \[(\sqrt{x - 2})^2 = (8 - x)^2\]
   \[x^2 - 17x + 66 = 0\]
   \[(x - 11)(x - 6) = 0\]
   \[x = 11, x = 6\]
   \[\{6\}\]
Graph: \((x - 4)^2 + (y + 3)^2 = 25\)
Center \((4, -3)\)
Radius 5
Domain \([-1, 9]\)
Range \([-8, 2]\)

Graph: \(36(x + 2)^2 + 4(y - 3)^2 = 144\)
Divide by 144
\(a = 2\) \((x+2)^2 + \frac{(y-3)^2}{36} = 1\)
\(b = 6\)
Domain: \([-4, 0]\)
Range: \([-3, 9]\)

Hint: RHS = 1
Graph: \( \frac{(x+4)^2}{9} - \frac{(y-2)^2}{4} = 1 \)

Hyperbola open Sideways

Center (-4,2)

\( a = 3 \)

\( b = 2 \)

Draw rectangle & Diagonals

Domain & range

\( D: (-\infty, -7] \cup [-1, \infty) \)

\( R: (-\infty, \infty) \)

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Graph: \( 4y^2 - 25(x-2)^2 = 100 \)

RHS = 1

1) Center (2,0)

\( \frac{y^2}{25} - \frac{(x-2)^2}{4} = 1 \)

2) \( a = 2 \), \( b = 5 \)

3) Domain (\(-\infty, \infty\))

4) Range (\(-\infty, -5] \cup [5, \infty)\))
When working with quadratic eqn \( a\chi^2 + b\chi + c = 0 \),
Expression \( b^2 - 4ac \) is called the discriminant.

- \( b^2 - 4ac > 0 \) \quad \text{Two real Solutions}
- \( b^2 - 4ac = 0 \) \quad \text{One repeated real Soln.}
- \( b^2 - 4ac < 0 \) \quad \text{Two Complex number Solutions}

Identify the type of solutions for \( 4\chi^2 - 3\chi + 5 = 0 \).

\[ a = 4 \quad b = -3 \quad c = 5 \]
\[ b^2 - 4ac = (-3)^2 - 4(4)(5) = 9 - 80 = -71 \]
Since \( b^2 - 4ac < 0 \), we have two complex Solutions.
\[(3x - 1)(2x + 4) = 40\]

Identify the type of solutions by using the discriminant.

1) Put in \(ax^2 + bx + c = 0\) Form.

\[6x^2 + 12x - 2x - 4 - 40 = 0\]

\[6x^2 + 10x - 44 = 0 \Rightarrow 3x^2 + 5x - 22 = 0\]

2) Find \(b^2 - 4ac\) \(\Rightarrow b^2 - 4ac = (5)^2 - 4 \cdot 3 \cdot (-22)\)

\[= 25 + 264 = 289\]

Since \(b^2 - 4ac > 0\),

Two Real Solns.

Find the value of the discriminant, then discuss the type of solutions:

\[4x^2 = 20x - 29\]

\[4x^2 - 20x + 29 = 0\]

\[b^2 - 4ac = (-20)^2 - 4 \cdot 4 \cdot (29) = -64\]

Since \(b^2 - 4ac < 0 \Rightarrow we\ will\ have\ two\ Complex\ Solutions.\]
Find the value of the discriminant, and then discuss the type of solutions.

\[ \chi(\chi - 20) = -100 \]
\[ \chi^2 - 20\chi + 100 = 0 \]
\[ b^2 - 4ac = (-20)^2 - 4(1)(-100) = 0 \]
Since \( b^2 - 4ac = 0 \), we will have one repeated real solution.

How to find the equation when solutions are given:

Find a quadratic eqn in the form of \( a\chi^2 + b\chi + c = 0 \) with solutions \( 4 \) and \(-5\).

\[ \chi^2 + \chi - 20 = 0 \] Eqn.
\[ (\chi - 4)(\chi + 5) = 0 \] Product of factors
\[ \chi - 4 = 0 \quad \chi + 5 = 0 \] Factors
\[ \chi = 4 \quad \chi = -5 \] Solutions
Find a quadratic eqn with solutions $\frac{2}{3}$ and $-\frac{3}{5}$ in the form of $ax^2 + bx + c = 0$.

$x = \frac{2}{3} \quad x = -\frac{3}{5}

3x - 2 = 0 \quad 5x - 3 = 0

(3x - 2)(5x + 3) = 0

Foil & Simplify

$15x^2 + 9x - 10x - 6 = 0$

Solutions: $3 \pm \sqrt{5}$, Equation 2?

$x = 3 + \sqrt{5} \quad x = 3 - \sqrt{5}

x - 3 - \sqrt{5} = 0 \quad x - 3 + \sqrt{5} = 0

(x - 3 - \sqrt{5})(x - 3 + \sqrt{5}) = 0

Conjugates

$(x - 3)^2 - (\sqrt{5})^2 = 0

x^2 - 6x + 9 - 5 = 0$
1) Find the discriminant, then discuss Solutions
\[5x^2 - 8x = -3\]
\[b^2 - 4ac = (-8)^2 - 4(5)(3) = 64 - 60 = 4\]
\[b^2 - 4ac > 0 \Rightarrow \text{Two Real Solns.}\]

2) Find a quadratic eqn in Stand. Form with Solutions
\[\frac{-3}{7} \& \frac{3}{7}\]
\[y = \frac{-3}{7}\]
\[y = \frac{3}{7}\]
\[7x = -3\]
\[7x = 3\]
\[-7x + 3 = 0\]
\[-7x - 3 = 0\]
\[(7x + 3)(7x - 3) = 0\]
\[49x^2 - 9 = 0\]

1) Find the discriminant, then discuss Solutions
\[25x^2 = 10x - 2\]
\[b^2 - 4ac = (-10)^2 - 4(25)(2) = 100 - 200 = -100\]
\[b^2 - 4ac < 0 \Rightarrow \text{Two Complex Solns.}\]

2) Find a quadratic eqn in Stand. Form with Solutions
\[-2 \pm 3\sqrt{2}\]
\[x = -2 + 3\sqrt{2}\]
\[x = -2 - 3\sqrt{2}\]
\[x + 2 - 3\sqrt{2} = 0\]
\[x + 2 + 3\sqrt{2} = 0\]
\[(x + 2 - 3\sqrt{2})(x + 2 + 3\sqrt{2}) = 0\]
\[(x + 2)^2 - (3\sqrt{2})^2 = 0\]
\[x^2 + 4x + 14 = 0\]
Solutions: $\pm 6i$, find eqn.

$s \equiv 6i$  \hspace{1em} $s \equiv -6i$

$s - 6i = 0$  \hspace{1em} $s + 6i = 0$

$(s - 6i)(s + 6i) = 0$

Conjugates

$s^2 - (6i)^2 = 0$

$s^2 - 36(-1) = 0$

$x^2 + 36 = 0$

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Solutions: $2 \pm 5i$, find eqn in

$a x^2 + bx + c = 0$.

$s = 2 + 5i$  \hspace{1em} $s = 2 - 5i$

$s - 2 - 5i = 0$  \hspace{1em} $s - 2 + 5i = 0$

$(s - 2 - 5i)(s - 2 + 5i) = 0$

$(s - 2)^2 - (5i)^2 = 0$

$s^2 - 4s + 4 - 25(-1) = 0$

$x^2 - 4x + 29 = 0$
Solutions: $-\frac{3}{5} \pm \frac{4}{5}i$, find eqn in $ax^2 + bx + c = 0$.

$x = -\frac{3}{5} + \frac{4}{5}i$  $x = -\frac{3}{5} - \frac{4}{5}i$

$5x = -3 + 4i$  $5x = -3 - 4i$

$5x + 3 - 4i = 0$  $5x + 3 + 4i = 0$

$(5x + 3 - 4i)(5x + 3 + 4i) = 0$

$25x^2 + 30x + 9 - 16(-1) = 0 
\Rightarrow 25x^2 + 30x + 25 = 0 
\Rightarrow 5x^2 + 6x + 5 = 0$

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Graphing Parabolas

- $a > 0$: $f(x) = a(x-h)^2 + k$
- $a < 0$: $f(x) = ax^2 + bx + c$
- $a \neq 0$: Not Functions
- $a > 0$: Functions

- $x = a(y-k)^2 + h$
- $x = ay^2 + by + c$
\[ f(x) = a(x-h)^2 + k \]  
**Parabola**

- opens up if \( a > 0 \)
- opens down if \( a < 0 \)

**Vertex** \((h, k)\)

**Axis of Symmetry** \(x = h\)

**Y-Int** \((0, ?)\)

**X-Int** \((?, 0)\)

**Example:** Graph \( f(x) = 2(x-1)^2 + 3 \)

- \( a = 2 \)  
- \( h = 1 \)  
- \( k = 3 \)

- opens upward
- Vertex \((1, 3)\)

**Axis of Symmetry** \(x = h\)  
\(x = 1\)

**Y-Int** \((0, \_)\)

- \( f(x) = 2(x-1)^2 + 3 \)

- \( y = 2(0-1)^2 + 3 \)

- \( y = 5 \)  
**Y-Int** \((0, 5)\)

**X-Int:** None
\[ f(x) = -(x+3)^2 - 1 \]

\[ f(x) = a(x-h)^2 + k \]

\[ a = -1, \ h = -3, \ k = -1 \]

opens downward
vertex \((-3, -1)\)
A.O.S. \(x = -3\)

\[ f(x) = \frac{1}{2}(x-4)^2 - 2 \]

\[ f(x) = a(x-h)^2 + k \]

\[ a = \frac{1}{2}, \ h = 4, \ k = -2 \]

opens up
vertex \((4, -2)\)
A.O.S. \(x = 4\)

\(y\)-Int \((0, 6)\)
\(x\)-Int \((4, 0)\)
\[ f(x) = - (x + 3)^2 + 4 \]

\[ f(x) = a(x-h)^2 + k \]

\[ a = -1 \quad h = -3 \quad k = 4 \]

Opens downward

Vertex \((-3, 4)\)

A.O.S. \(x = -3\)

Y-Int \((0, -5)\)

\[ \begin{align*}
\chi - \text{Int}^+ & \\
\gamma = 0 & \\
f(x) = 0 & \\
-(x+3)^2 + 4 = 0 & \\
\end{align*} \]

\[ (x+3)^2 = 4 \]

\[ x + 3 = \pm \sqrt{4} \]

\[ x + 3 = \pm 2 \]

\[ x = -1 \quad x = -5 \]

\[ \begin{align*}
(0, 5) & \\
(4, -2) & \\
\end{align*} \]

\[ \begin{align*}
\frac{f(x)}{2} & = (x-2)^2 \]

\[ a = -\frac{1}{2} \quad h = 2 \quad k = 0 \]

Opens downward

Vertex \((2, 0)\)

A.O.S. \(x = 2\)

\[ \begin{align*}
\chi - \text{Int}^+ & \\
(y, 2) & \\
(2, 0) & \\
(0, -2) & \\
(4, -2) & \\
\end{align*} \]
\[ f(x) = ax^2 + bx + c \]

- \( a > 0 \): opens up \[ h = \frac{-b}{2a} \]
- \( a < 0 \): opens down \[ k = f(h) \]
  
  **A.O.S.** \( x = h \)

Y-Int \((0, ?)\) \( x\)-Int \((?, 0)\)

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\[ f(x) = x^2 - 2x - 8 \]

\[ f(x) = ax^2 + bx + c \]

- \( a = 1 \)
- \( b = -2 \)
- \( c = -8 \)

opens upward

Y-Int \((0, -8)\)

\[ h = \frac{-b}{2a} = \frac{-(-2)}{2(1)} = \frac{2}{2} = 1 \]

\[ k = f(h) = f(1) = 1^2 - 2(1) - 8 = 1 - 2 - 8 = -9 \]

**A.O.S.** \( x = h \)

\[ x = 1 \]

\[ f(x) = -x^2 - 4x + 5 \]

\[ a = -1 \quad b = -4 \quad c = 5 \]

Opens downward

Vertex \((-2, 9)\)

\[ h = \frac{-b}{2a} = \frac{-(-4)}{2(-1)} = \frac{4}{-2} = -2 \]

\[ k = f(h) = f(-2) = -(-2)^2 - 4(-2) + 5 = 9 \]

A.O.S. \(x = h \quad x = -2\)

Y-Int \((0, 5)\)
\[ \chi + 1 = 0 \]
\[ \chi = -1 \]
\[ f(x) = 0 \]
\[ -\chi^2 - 4\chi + 5 = 0 \]
\[ \chi^2 + 4\chi - 5 = 0 \]
\[ \chi^2 + 4\chi + 4 = 5 + 4 \]
\[ (\chi + 2)^2 = 9 \]
\[ \chi + 2 = \pm \sqrt{9} \]
\[ \chi = -2 \pm 3 \]
\[ \chi = 1, \chi = -5 \]

\[ f(x) = \frac{1}{2} \chi^2 + 4\chi \]
\[ a = \frac{1}{2} \]
\[ b = 4 \]
\[ c = 0 \]

Opens up

Vertex \((-4, -8)\)  \[ A.O.S. \quad \chi = -4 \]
\[ h = \frac{-b}{2a} = \frac{-4}{2(\frac{1}{2})} = -4 \]
\[ k = f(h) = f(-4) = \frac{1}{2}(-4)^2 + 4(-4) = -8 \]

Y-Int \((0, 0)\)
Agenda for tomorrow:

1) Exam 3 @ 6:00.
   - All radicals
   - Complex numbers
   - S.R.M., Comp. the Sqr method
   - Quadratic Formula
   - Circle, Ellipse, Hyperbola
   - Review Exam 1 & 2.
2) Exam Ends @ 7:30
3) Due: SG 21, 22, Project 3
4) Cont. with Ch. 11

FOCUS