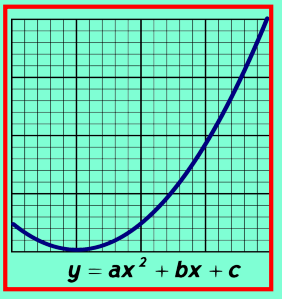
Math 125

Spring 2021

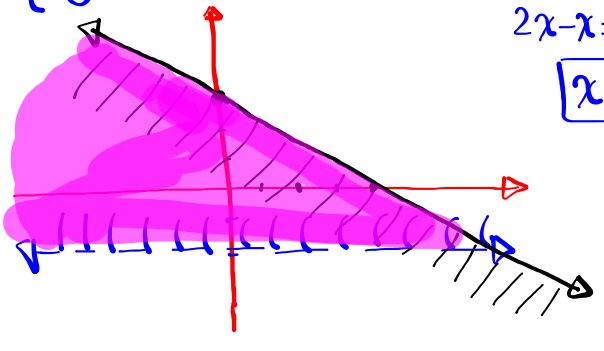
Lecture 8



Class QZ 7

① Graph and Shade

$$\begin{cases} y > -2 \\ y \le -\frac{3}{4}x + 3 \end{cases}$$



② Solve

$|2x-7| = |x-8|$

$2x-7 = x-8$ OR $2x-7 = -(x-8)$

$2x-x = -8+7$ $\quad 2x-7 = -x+8$

$\boxed{x=-1}$ OR $2x+x = 8+7$

$3x = 15$

$\boxed{x=5}$

$\boxed{\{-1, 5\}}$

Simplify

1) $\dfrac{x^2+7x+12}{x^2+x-12} = \dfrac{(x+3)(x+4)}{(x+4)(x-3)} = \boxed{\dfrac{x+3}{x-3}}$

$(A+B)(A-B) = A^2-B^2$

2) $\dfrac{x^2+4x}{x^2-5x} \div \dfrac{x^2-16}{x^2-25} = \dfrac{x^2+4x}{x^2-5x} \cdot \dfrac{x^2-25}{x^2-16}$

3) $\dfrac{3}{x^2-7x+10} - \dfrac{2}{x^2-25}$

$= \dfrac{x(x+4)}{x(x-5)} \cdot \dfrac{(x+5)(x-5)}{(x+4)(x-4)}$

$= \boxed{\dfrac{x+5}{x-4}}$

$= \dfrac{3(x+5)}{(x-5)(x-2)(x+5)} - \dfrac{2(x-2)}{(x+5)(x-5)(x-2)}$

$= \dfrac{3(x+5)-2(x-2)}{(x-5)(x-2)(x+5)} = \dfrac{3x+15-2x+4}{(x-5)(x-2)(x+5)} = \boxed{\dfrac{x+19}{(x-5)(x-2)(x+5)}}$

Solve $2|3x+1|-3=7$

Isolate abs. value first.

$2|3x+1| = 7+3$

$2|3x+1| = 10$

$|3x+1| = \dfrac{10}{2} \Rightarrow |3x+1| = 5$

Now $3x+1=5$ OR $3x+1=-5$

$3x=4$ $3x=-5-1$

$\boxed{x=\dfrac{4}{3}}$ $3x=-6$

$\boxed{x=-2}$

$\left\{-2, \dfrac{4}{3}\right\}$

Solve

1) $|4x-1|+8=3$

$|4x-1|=3-8$

$|4x-1|=-5$

False $\Rightarrow \boxed{\phi}$

$\{\phi\}$ Not empty set. NO Solution $\boxed{\{\}}$

$\{$No Solution$\}$

2) $|x-8|=|x+8|$

$x-8=x+8$ OR $x-8=-(x+8)$

$x-x=8+8$ $\quad x-8=-x-8$

$0=16$ $\quad x+x=-8+8$

False $\quad 2x=0$

$x=\frac{0}{2}$

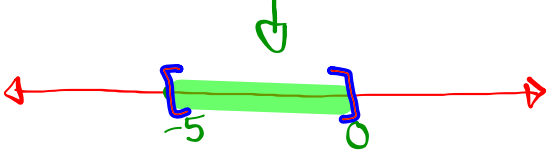
$\boxed{\{0\}}$ $\quad \boxed{x=0}$

Solve $|2x+5|\leq 5$

Solve $|2x+5|=5$ $\quad 2x+5=5$ OR $2x+5=-5$

$2x=0$ $\quad 2x=-10$

$\boxed{x=0}$ $\quad \boxed{x=-5}$



S.B.N.

$\{x \mid -5\leq x\leq 0\}$

I.N.

$[-5, 0]$

Solve $-3|x-4|-2<-11$

Always isolate abs. value.

$-3|x-4|<-11+2$

$-3|x-4|<-9$

$\frac{-3}{-3}|x-4| > \frac{-9}{-3}$

$|x-4|>3$

Solve $|x-4|=3$

$x-4=3$ OR $x-4=-3$

$x=7$ $\quad x=1$

S.B.N. $\{x \mid x<1 \text{ or } x>7\}$

I.N. $(-\infty, 1) \cup (7, \infty)$

1) Solve: $|2x+7|+9>3$

$|2x+7|>3-9$

$|2x+7|>-6$

0 or +

$(-\infty, \infty)$ $\quad \mathbb{R}$

2) $-2|3x-5|+1 \leq 1$

$-2|3x-5| \leq 0$

$|3x-5| \geq \frac{0}{-2}$

$|3x-5| \geq 0$

$(-\infty, \infty)$, $\mathbb{R}$

3) $|7x+1|+8<2$

$|7x+1|<2-8$

$|7x+1|<-6$

0 or +

No Solution

$\{\}$, $\emptyset$

Solve $-3x + 2 < 11$ OR $2x - 7 \leq -13$

Solve & graph on the same number line system.

$-3x < 11 - 2$

$-3x < 9$

$x > -3$

OR

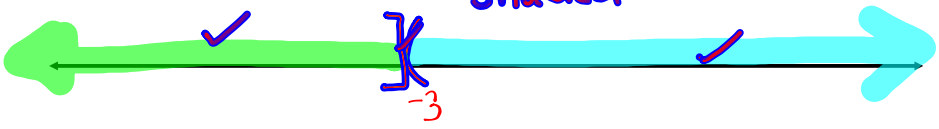
↓ take whatever shaded

$2x - 7 \leq -13$

$2x \leq -13 + 7$

$2x \leq -6$

$x \leq -3$



$(-\infty, \infty)$ $\{x \mid x \text{ is a real number}\}$ $\mathbb{R}$

Solve

$\frac{1}{2}x - \frac{2}{3} \leq \frac{5}{6}$

LCD = 6

$\overset{3}{\cancel{6}} \cdot \frac{1}{\cancel{2}}x - \overset{2}{\cancel{6}} \cdot \frac{2}{\cancel{3}} \leq \cancel{6} \cdot \frac{5}{\cancel{6}}$

$3x - 4 \leq 5$

$3x \leq 9$

$x \leq 3$

AND $-5x + 3 \leq -47$

↓ Take the overlapping shaded segment.

$-5x \leq -47 - 3$

$-5x \leq -50$

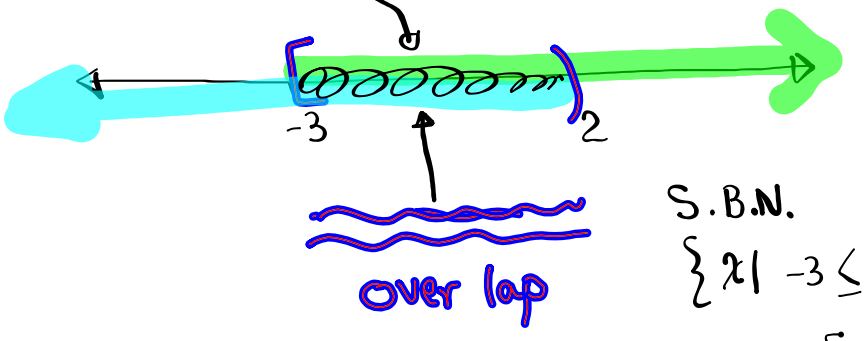
$\frac{-5}{-5}x \geq \frac{-50}{-5}$

$x \geq 10$

No overlapping segment.

$\Rightarrow \emptyset, \{\}$, NO Solution

$x \geq -3$ AND $x < 2$



S.B.N.

$\{x \mid -3 \leq x < 2\}$

I.N. $[-3, 2)$

Graph & Shade

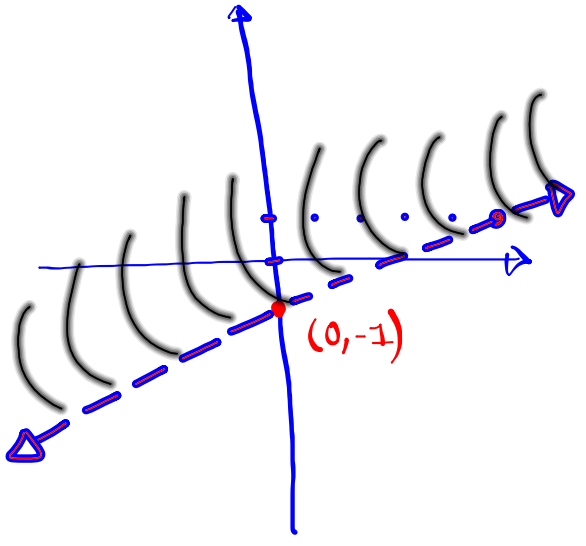
$f(x) > \frac{2}{5}x - 1$

Y-Int $(0, -1)$

$m = \frac{2}{5}$

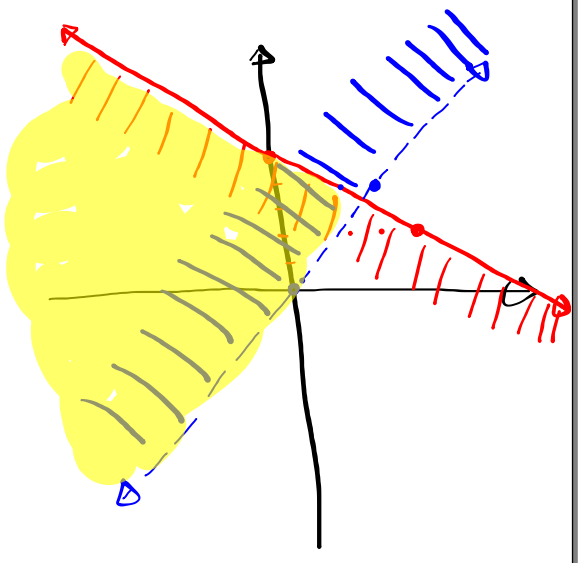
Dashed line

Shade above



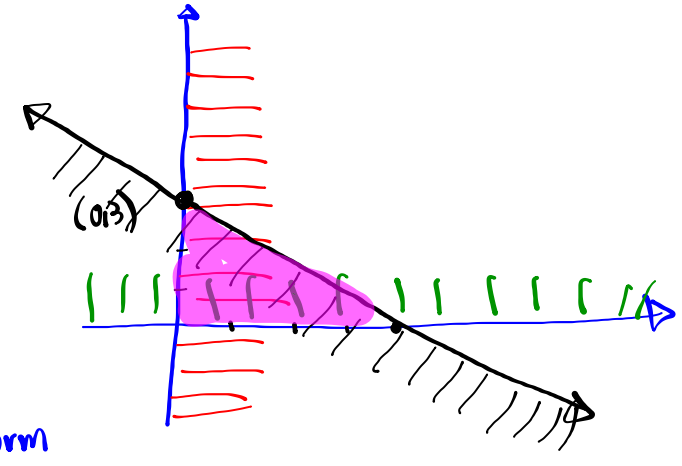
Graph & shade

$$\begin{cases} f(x) \leq \frac{-3}{4}x + 5 \\ g(x) > \frac{4}{3}x \end{cases}$$



Graph & shade

$$\begin{cases} x \geq 0 \\ y \geq 0 \\ 3x + 4y \leq 12 \end{cases}$$

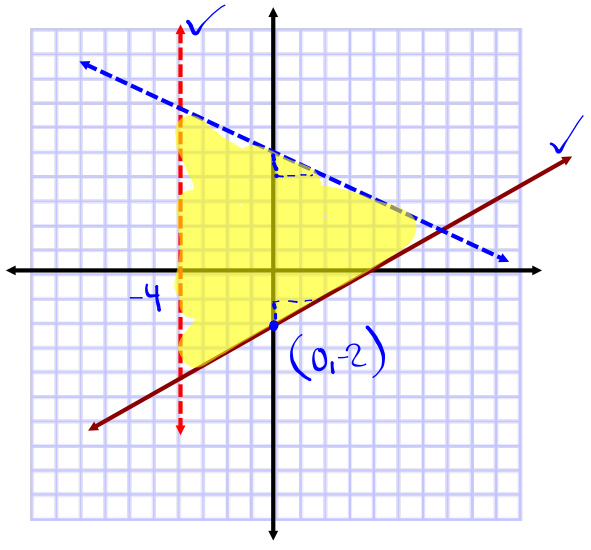


Hint:
write in
Slope-Int. Form

$$4y \leq -3x + 12$$

$$y \leq \frac{-3}{4}x + 3$$

Consider the shaded region below



Find system of linear inqualities for shaded region.

$$\begin{cases} x > -4 \\ y \geq \frac{1}{2}x - 2 \\ y < \frac{-1}{2}x + 5 \end{cases}$$

Give domain for $f(x) = \dfrac{8}{x^2+2x-8}$

in interval notation.

$x^2+2x-8 \neq 0$

$(x-2)(x+4) \neq 0$

$x-2 \neq 0 \quad x+4 \neq 0$

$x \neq 2 \quad x \neq -4$

$(-\infty,-4) \cup (-4,2) \cup (2,\infty)$

$f(x) = |3x - 5|$

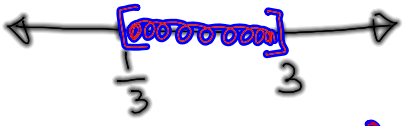
1) Solve $f(x) = 4$ $\quad |3x-5| = 4$

$3x - 5 = 4$ $\quad 3x - 5 = -4$

$x = 3$ $\quad x = \frac{1}{3}$

$\{\frac{1}{3}, 3\}$

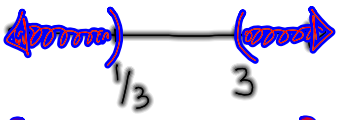
2) Solve $f(x) \leq 4$



S.B.N. $\{x \mid \frac{1}{3} \leq x \leq 3\}$

I.N. $[\frac{1}{3}, 3]$

3) Solve $f(x) > 4$



$\{x \mid x < \frac{1}{3} \text{ or } x > 3\}$

$(-\infty, \frac{1}{3}) \cup (3, \infty)$

Review From Algebra

Solve $\frac{4}{x+3} - \frac{2}{x-7} = \frac{2}{x^2 - 4x - 21}$

Hint: Use LCD to clear all denominators.

To find LCD, make sure all denominators are completely factored.

$\frac{4}{x+3} - \frac{2}{x-7} = \frac{2}{(x+3)(x-7)}$

LCD $= (x+3)(x-7)$

$x \neq -3, x \neq 7$

Excluded Values

$(x+3)(x-7) \cdot \frac{4}{x+3} - (x+3)(x-7) \cdot \frac{2}{x-7} = (x+3)(x-7) \cdot \frac{2}{(x+3)(x-7)}$

$4(x-7) - 2(x+3) = 2$

$4x - 28 - 2x - 6 = 2$

$2x - 34 = 2$

$2x = 2 + 34$

$2x = 36$

$x = 18$

$\{18\}$

Solve $\frac{2}{x-3} - \frac{1}{x+3} = \frac{12}{x^2-9}$

LCD $= (x-3)(x+3)$

E.N. $x-3 \neq 0$ $x+3 \neq 0$

$x \neq 3$ $x \neq -3$

$(x-3)(x+3)\cdot\frac{2}{x-3} - (x-3)(x+3)\cdot\frac{1}{x+3} = (x-3)(x+3)\cdot\frac{12}{x^2-9}$

$2(x+3) - 1(x-3) = 12$

$2x + 6 - x + 3 = 12$

$x + 9 = 12$

$x = 12 - 9$

$x = 3$

SE 5 ✓

SE 6 ✓

We start SE 7

3 is an excluded value.

No solution

$\{\}$, $\emptyset$

System of linear equations

$$\begin{cases} 2x + 3y = 8 \\ x - y = -1 \end{cases}$$

A solution must satisfy both eqns.

Is $(1,2)$ a solution?

$2x + 3y = 8$

$2(1) + 3(2) = 8$

$2 + 6 = 8$

$8 = 8$ ✓

$x - y = -1$

$1 - 2 = -1$

$-1 = -1$ ✓

Yes

Is $(-2, 3)$ a Solution of ← ordered-pair

$$\begin{cases} 4x + y = -5 \\ x - y = 1 \end{cases} ?$$

NO

$4x + y = -5$

$4(-2) + 3 = -5$

$-8 + 3 = -5$

$-5 = -5$ ✓

$x - y = 1$

$-2 - 3 = 1$

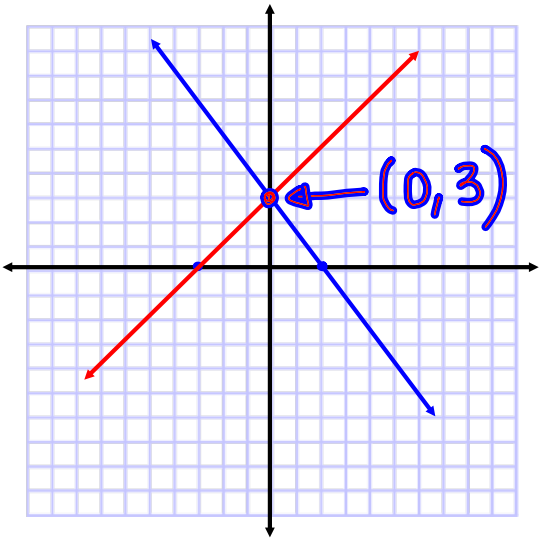
$-5 = 1$

False

Solve $\begin{cases} 3x + 2y = 6 \\ x - y = -3 \end{cases}$ by graphing.

x	y
0	3
2	0

x	y
0	3
-3	0



Solve by graphing

$$\begin{cases} y = \frac{2}{3}x - 2 \\ y = \frac{2}{3}x + 3 \end{cases}$$

$\emptyset$