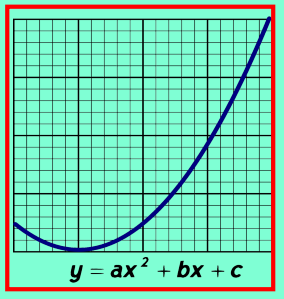
Math 125
Spring 2021
Lecture 6



Class Qiz 5

① Simplify: $(3x-7)(3x+7) = 9x^2 + 21x - 21x - 49$

$= \boxed{9x^2 - 49}$

② Simplify: $\dfrac{(x^3)^3}{(x^4)^2}$

$= \dfrac{x^{3\cdot 3}}{x^{4\cdot 2}} = \dfrac{x^9}{x^8} = x^{9-8} = x^1$

$= \boxed{x}$

③ Solve:

$(x-8)(x+6) = 0$

Zero-Product Rule

$x-8=0$ OR $x+6=0$

$\boxed{x=8}$ $\boxed{x=-6}$

$\{-6, 8\}$

$f(x) = x^2 + 5x$

Find

1) $f(0) = 0^2 + 5(0)$

$= 0 + 0$

$= \boxed{0}$

2) $f(-5) = (-5)^2 + 5(-5)$

$= 25 - 25$

$= \boxed{0}$

3) $f(2x) = (2x)^2 + 5(2x)$

$= \boxed{4x^2 + 10x}$

4) $f(x-2)$

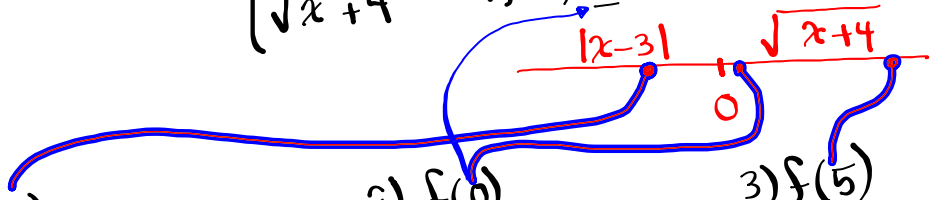
$= (x-2)^2 + 5(x-2)$

$= (x-2)(x-2) + 5(x-2)$

$= x^2 - 2x - 2x + 4 + 5x - 10$

$= \boxed{x^2 + x - 6}$

Given $f(x) = \begin{cases} |x-3| & \text{if } x < 0 \\ \sqrt{x+4} & \text{if } x \geq 0 \end{cases}$



Find

1) $f(-3)$

$= |-3-3| = |-6|$

$= \boxed{6}$

2) $f(0)$

$= \sqrt{0+4}$

$= \sqrt{4} = \boxed{2}$

3) $f(5)$

$= \sqrt{5+4}$

$= \sqrt{9}$

$= \boxed{3}$

$f(x) = 2x + 5$ $\quad g(x) = x - 5$

Find

1) $f(0) = 2(0) + 5$
$= \boxed{5}$

2) $g(0) = 0 - 5$
$= \boxed{-5}$

3) $(f+g)(x) = f(x) + g(x)$
$= 2x + 5 + x - 5$
$= \boxed{3x}$

4) $(f-g)(x)$
$= f(x) - g(x)$
$= 2x + 5 - (x - 5)$
$= 2x + 5 - x + 5$
$= \boxed{x + 10}$

5) $(f \cdot g)(x) = f(x) \cdot g(x)$
$= (2x + 5)(x - 5)$
$= 2x^2 - 10x + 5x - 25$
$= \boxed{2x^2 - 5x - 25}$

6) $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$; $g(x) \neq 0$
$= \boxed{\frac{2x+5}{x-5}}$; $x - 5 \neq 0$, $x \neq 5$

7) $\left(\frac{g}{f}\right)(x)$
$= \frac{g(x)}{f(x)}$; $f(x) \neq 0$
$= \boxed{\frac{x-5}{2x+5}}$; $2x + 5 \neq 0$, $x \neq -\frac{5}{2}$

8) $(f \circ g)(x) = f(g(x))$

Composition

$= 2(g(x)) + 5$
$= 2(x - 5) + 5 = 2x - 10 + 5$
$= \boxed{2x - 5}$

Find the domain

1) $f(x) = \frac{x}{x-4}$ $\quad x-4 \neq 0$, $x \neq 4$

All reals except 4

2) $g(x) = \frac{x-1}{x+3}$ $\quad x+3 \neq 0$, $x \neq -3$

All reals except -3

3) $h(x) = \frac{4}{x^2-100}$

$x^2 - 100 \neq 0$

$x^2 \neq 100$

$x \neq \pm 10$

All reals except ± 10

Factor Completely:

1) $x^2 - 5x$

$= x(x-5)$

2) $3x^2 + 15x$

$= 3x(x+5)$

3) $x^2 - 5x - 24$

$(x+3)(x-8)$

1, 24
2, 12
3, 8
4, 6

4) $2x^2 + 7x + 5$

$(2x+5)(x+1)$

1, 5
verify by foiling.

Simplify

$x^m \cdot x^n = x^{m+n}$

$\frac{x^m}{x^n} = x^{m-n}$

$(x^m)^n = x^{mn}$

$(xy)^n = x^n y^n$

1) $x^5 \cdot x^3 = x^{5+3} = \boxed{x^8}$

2) $\frac{x^5}{x^3} = x^{5-3} = \boxed{x^2}$

3) $(x^5)^3 = x^{5 \cdot 3} = \boxed{x^{15}}$

4) $(-2x^3)^5 = (-2)^5 \cdot (x^3)^5 = \boxed{-32x^{15}}$

Solve $2(x-1) - 8 < 4x - 30$

$2x - 2 - 8 < 4x - 30$

$2x - 10 < 4x - 30$

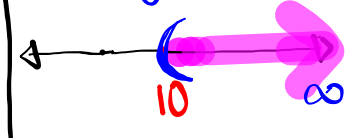
$2x - 4x < -30 + 10$

$-2x < -20$

Divide by -2

$\frac{-2}{-2}x > \frac{-20}{-2}$

$x > 10$



Set-builder notation

$\{x \mid x > 10\}$

Interval Notation $\boxed{(10, \infty)}$

Solve $\frac{2}{3}x - \frac{1}{6} \geq \frac{3}{4}x + \frac{1}{2}$

Hint: Use LCD to Clear Fractions

LCD = 12

Multiply by LCD

$12 \cdot \frac{2}{3}x - 12 \cdot \frac{1}{6} \geq 12 \cdot \frac{3}{4}x + 12 \cdot \frac{1}{2}$

$8x - 2 \geq 9x + 6$

$8x - 9x \geq 6 + 2$

$-x \geq 8$

Divide by -1

$\frac{-x}{-1} \leq \frac{8}{-1}$

$x \leq -8$

S.B.N. $\{x \mid x \leq -8\}$

I.N. $(-\infty, -8]$

Solve $-5 \leq 2x + 1 < 17$

Subtract 1

$-5 - 1 \leq 2x + 1 - 1 < 17 - 1$

$-6 \leq 2x < 16$

Divide by 2

$-3 \leq x < 8$

S.B.N. $\{x \mid -3 \leq x < 8\}$

I.N. $[-3, 8)$

Solve

$-8 \leq -3x - 2 < 7$

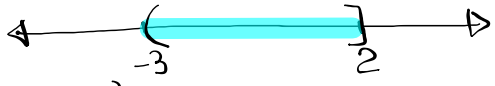
Add 2

$-6 \leq -3x < 9$

Divide by -3

$\frac{-6}{-3} \geq \frac{-3}{-3}x > \frac{9}{-3}$

$2 \geq x > -3 \Rightarrow \boxed{-3 < x \leq 2}$

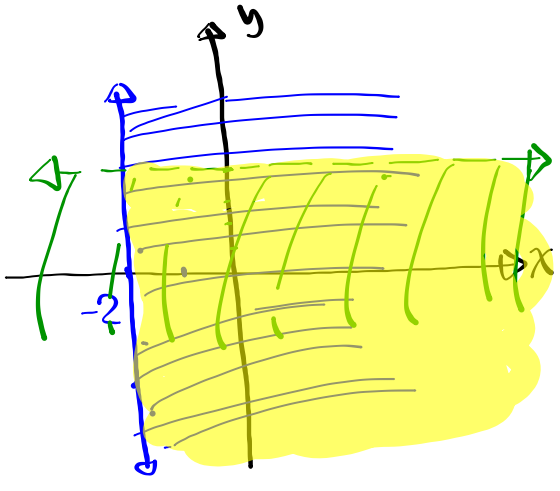


S.B.N. $\{x \mid -3 < x \leq 2\}$ I.N. $(-3, 2]$

Graph & Shade on rectangular Coordinate System:

$\begin{cases} x \geq -2 & \text{Vertical} \\ y < 4 & \text{Horizontal} \end{cases}$

$\{(x,y) \mid x \geq -2 \text{ and } y < 4\}$

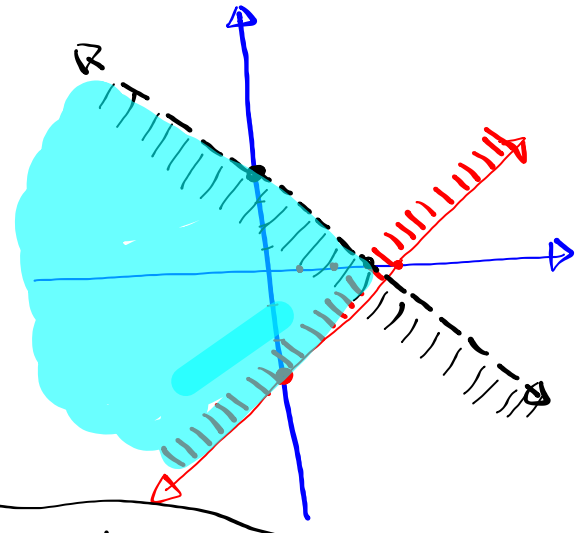


Graph & Shade

$$\begin{cases} y \geq \frac{3}{4}x - 3 \\ y < \frac{-4}{3}x + 4 \end{cases}$$

Solid line

dashed line



SG 4 ✓

attend office hrs for help

Look for math tutoring lab on elac.edu

Absolute Value Equations:

Type I: $|ax + b| = K$

1) If $K < 0 \Rightarrow$ NO Solution

2) If $K \geq 0 \Rightarrow$ Solve $ax+b=K$ OR $ax+b=-K$

Ex: Solve $|2x-1| = -5 \Rightarrow$ NO Solution $\emptyset$

Ex: Solve $|2x-1| = 5$

$2x-1=5$ OR $2x-1=-5$

$2x=6$ $2x=-4$

$x=3$ $x=-2$

$\{-2, 3\}$

Solve $|3x+2|=-7$ ← K

$\emptyset$

Solve $|3x+2|=7$ ← K

$3x+2=7$ OR $3x+2=-7$

$3x=5$ $3x=-9$

$x=\frac{5}{3}$ $x=\frac{-9}{3}=-3$

$\left\{-3, \frac{5}{3}\right\}$

Type II: $|ax+b|=|cx+d|$

Solve $ax+b=cx+d$ OR $ax+b=-(cx+d)$

Ex: Solve $|2x-7|=|x-8|$

$2x-7=x-8$ OR $2x-7=-(x-8)$

$2x-x=-8+7$

$x=-1$

$2x-7=-x+8$

$2x+x=8+7$

$3x=15$

$x=5$

$\{-1, 5\}$

Solve $|x+9| = |x-9|$

$x+9 = x-9$ OR $x+9 = -(x-9)$

$x - x = -9 - 9$

$0 = -18$

false

$\{0\}$

$x+9 = -x+9$

$x + x = 9 - 9$

$2x = 0$

$x = \frac{0}{2}$ $\boxed{x=0}$

---

1) Solve $|2x+3| + 5 = 0$

$|2x+3| = -5$

$\emptyset$

$\{\emptyset\}$

Hint: Always isolate Abs. Value

2) Solve $|3x-2| - 4 = 3$

$|3x-2| = 7$

$\{-\frac{5}{3}, 3\}$

$3x-2=7$ OR $3x-2=-7$

$3x=9$ $3x=-5$

$\boxed{x=3}$ $\boxed{x=\frac{-5}{3}}$

3) Solve $|3x+7| = |x-9|$

$3x+7 = x-9$ OR $3x+7 = -(x-9)$

$3x-x = -9-7$ $\quad$ $3x+7 = -x+9$

$2x = -16$ $\quad$ $3x+x = 9-7$

$x = -8$ $\quad$ $4x = 2$

$\{-8, \frac{1}{2}\}$ $\quad$ $x = \frac{2}{4}$ $\quad$ $\boxed{x = \frac{1}{2}}$

Simplify

1) $\dfrac{x^2+5x}{x^2-25} = \dfrac{x(x+5)}{(x+5)(x-5)} = \boxed{\dfrac{x}{x-5}}$

$A^2 - B^2 = (A+B)(A-B)$

2) $\dfrac{8}{x^2-4x+3} \div \dfrac{2}{x^2-9} = \dfrac{8}{x^2-4x+3} \cdot \dfrac{x^2-9}{2}$

$= \dfrac{4\,8}{(x-3)(x-1)} \cdot \dfrac{(x+3)(x-3)}{2} = \boxed{\dfrac{4(x+3)}{x-1}}$

3) $\frac{3}{4} - \frac{1}{5} = \frac{3 \cdot 5}{4 \cdot 5} - \frac{1 \cdot 4}{5 \cdot 4}$

LCD = 4·5

$= \frac{15}{20} - \frac{4}{20} = \frac{15-4}{20}$

$= \boxed{\frac{11}{20}}$

4) $\frac{3}{x-5} - \frac{1}{x+4} = \frac{3(x+4)}{(x-5)(x+4)} - \frac{1(x-5)}{(x+4)(x-5)}$

LCD = $(x-5)(x+4)$

$= \frac{3(x+4) - 1(x-5)}{(x-5)(x+4)}$

$= \frac{3x+12-x+5}{(x-5)(x+4)}$

$= \boxed{\frac{2x+17}{(x-5)(x+4)}}$

Graph & Shade

$5x - 3y + 2 \geq x + 4y - 5$

$-3y - 4y \geq x - 5 - 5x - 2$

$-7y \geq -4x - 7$

Divide by -7

$\frac{-7}{-7}y \leq \frac{-4}{-7}x - \frac{7}{-7}$

$y \leq \frac{4}{7}x + 1$

$m = \frac{4}{7}$

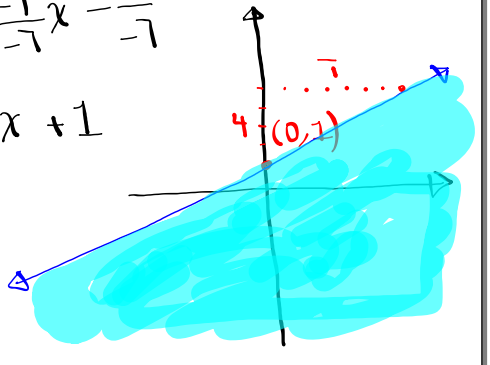
Shade below

Hint: write in Slope-Int form

$y = mx + b$

$y \geq mx + b$

$y \leq mx + b$



Class Qz 6

$f(x) = 4x - 3$ $\quad g(x) = x + 5$

1) $(f+g)(x)$

2) $(f-g)(x)$

3) $(f \cdot g)(x)$

4) $\left(\frac{f}{g}\right)(x)$