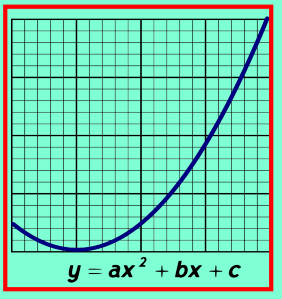
Math 125
Spring 2021
Lecture 26



1) Solve: $(2x+5)^2 = -32$

use S.R.M.

$2x+5 = \pm\sqrt{-32}$

$2x = -5 \pm \sqrt{16}\sqrt{2}\sqrt{-1}$

$2x = -5 \pm 4\sqrt{2}\,i$

$x = \frac{-5}{2} \pm \frac{4\sqrt{2}}{2}\,i$

$x = \frac{-5}{2} \pm 2\sqrt{2}\,i$

$\left\{\frac{-5}{2} \pm 2\sqrt{2}\,i\right\}^2$

2) Make a perfect-square: $x^2 - 12x + (-6)^2 = (x-6)^2$

$\frac{1}{2}\cdot(-12) = -6$

$x^2 - 12x + 36 = (x-6)^2$

3) Solve by Completing the Square method:

$x^2 + 20x + 104 = 0$

$x^2 + 20x + 10^2 = -104 + 10^2$

$\frac{1}{2} \cdot 20 = 10$

$x^2 + 20x + 100 = -4$

$(x+10)^2 = -4$

SRM

$x + 10 = \pm\sqrt{-4}$

$x = -10 \pm 2i$

$\{-10 \pm 2i\}$

4) Solve by using the quadratic formula:

$2x^2 + 5x - 7 = 0$

$a = 2 \quad b = 5 \quad c = -7$

$b^2 - 4ac = 5^2 - 4(2)(-7)$

$= 25 + 56$

$= 81$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-5 \pm \sqrt{81}}{2(2)}$

$= \frac{-5 \pm 9}{4}$

$x = \frac{-5+9}{4} = \frac{4}{4} = 1$

$x = \frac{-5-9}{4} = \frac{-14}{4} = \frac{-7}{2}$

$\{-\frac{7}{2}, 1\}$

5) Solve $2x^2 + 5x - 7 = 0$ by Completing the square method. Hint: Make Lead. Coef. 1. ⇒ Divide by 2

$\frac{2}{2}x^2 + \frac{5}{2}x - \frac{7}{2} = 0$

$x^2 + \frac{5}{2}x + \frac{25}{16} = \frac{7}{2} + \frac{25}{16}$

$\frac{1}{2} \cdot \frac{5}{2} = \frac{5}{4}, \left(\frac{5}{4}\right)^2 = \frac{25}{16}$

$\left(x + \frac{5}{4}\right)^2 = \frac{81}{16}$

SRM

$x + \frac{5}{4} = \pm\frac{9}{4}$

$x = -\frac{5}{4} \pm \frac{9}{4}$

$x = 1, -\frac{7}{2}$

$\{-\frac{7}{2}, 1\}$

6) Solve $(3x-2)(2x+3) = 5$ by using the quadratic formula. Hint: Foil, Simplify, write in form $ax^2 + bx + c = 0$

$6x^2 + 9x - 4x - 6 - 5 = 0$

$6x^2 + 5x - 11 = 0$

$a = 6 \quad b = 5 \quad c = -11$

$b^2 - 4ac = 5^2 - 4(6)(-11)$

$= 289$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$= \frac{-5 \pm \sqrt{289}}{12} = \frac{-5 \pm 17}{12}$

$x = \frac{-5+17}{12} = \frac{12}{12} = 1$

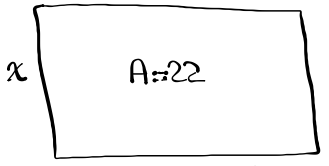
$x = \frac{-5-17}{12} = \frac{-11}{6}$

$\{-\frac{11}{6}, 1\}$

Area of a rectangle is $22\ m^2$.

The length is 5m longer than 3 times its width.

Find its dimensions.



$A = LW$

$x(3x+5) = 22$

$3x^2 + 5x - 22 = 0$

$a=3 \quad b=5 \quad c=-22$

$b^2 - 4ac = 5^2 - 4(3)(-22) = 289$

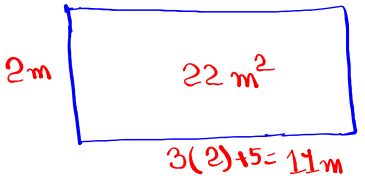
$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$x = \frac{-5 \pm \sqrt{289}}{2(3)}$

$x = \frac{-5 \pm 17}{6}$

$x = \frac{-5+17}{6} = \frac{12}{6} = 2$

$x = \frac{-5-17}{6} = \frac{-22}{6} = -\frac{11}{3}$ ✗



Discuss the type of Solutions:

$3x^2 - 2x + 10 = 0$

$a=3 \quad b=-2 \quad c=10$

$b^2 - 4ac = (-2)^2 - 4(3)(10)$

$= 4 - 120 = -116 < 0$

Two imaginary Solutions

Discuss the type of Solutions:

$9x^2 - 12x = -4$

Hint: $ax^2+bx+c=0$

$9x^2 - 12x + 4 = 0$

$a=9 \quad b=-12 \quad c=4$

$b^2-4ac=(-12)^2-4(9)(4)=144-144=0$

↳ one repeated real Solution

Find a quadratic equation in $ax^2+bx+c=0$ form with Solutions -3 and 5.

$x=-3$ $\qquad$ $x=5$

$x+3=0$ $\qquad$ $x-5=0$

$(x+3)(x-5)=0$

$x^2-5x+3x-15=0$

$\boxed{x^2-2x-15=0}$

Find a quadratic equation in $ax^2+bx+c=0$ Form with Solutions $-3 \pm 2\sqrt{5}$.

$x = -3 + 2\sqrt{5}$ $\qquad$ $x = -3 - 2\sqrt{5}$

$x + 3 - 2\sqrt{5} = 0$ $\qquad$ $x + 3 + 2\sqrt{5} = 0$

$(x + 3 - 2\sqrt{5})(x + 3 + 2\sqrt{5}) = 0$

Conjugates

$(x+3)^2 - (2\sqrt{5})^2 = 0$

$x^2 + 6x + 9 - 4 \cdot 5 = 0$

$x^2 + 6x + 9 - 20 = 0$

$x^2 + 6x - 11 = 0$ ✓ ✓

Find a quadratic equation in $ax^2+bx+c=0$ Form with Solutions $\frac{2}{3} \pm \frac{5}{3}i$.

$x = \frac{2}{3} + \frac{5}{3}i$ $\qquad$ $x = \frac{2}{3} - \frac{5}{3}i$

Use LCD = 3 to clear fractions

$3x = 2 + 5i$ $\qquad$ $3x = 2 - 5i$

$3x - 2 - 5i = 0$ $\qquad$ $3x - 2 + 5i = 0$

$(3x - 2 - 5i)(3x - 2 + 5i) = 0$

Conjugates

$(3x-2)^2 - (5i)^2 = 0$

$i^2 = -1$

$9x^2 - 12x + 4 - 25i^2 = 0$

$9x^2 - 12x + 4 - 25(-1) = 0$

** $9x^2 - 12x + 29 = 0$ **

Final Ans.

Solving quadratic equations in form:

$x^4 - 5x^2 - 36 = 0$

Notice $x^4 = (x^2)^2$

Let $u = x^2$ Equation becomes

$u^2 - 5u - 36 = 0$

$(u-9)(u+4) = 0$

$u = 9 \quad u = -4$

$u = 9$

$x^2 = 9$

$x = \pm\sqrt{9}$

$\boxed{x = \pm 3}$

$u = -4$

$x^2 = -4$

$x = \pm\sqrt{-4}$

$\boxed{x = \pm 2i}$

$\{\pm 2i, \pm 3\}$

Solve $x^4 - 12x^2 + 32 = 0$

Hint: $x^4 = (x^2)^2$

$(x^2)^2 - 12x^2 + 32 = 0$

Let $u = x^2 \Rightarrow u^2 - 12u + 32 = 0$

$(u-4)(u-8) = 0$

$u = 4 \quad u = 8$

$u = 4$

$x^2 = 4$

$\boxed{x = \pm 2}$

$u = 8$

$x^2 = 8$

$x = \pm\sqrt{8}$

$\boxed{x = \pm 2\sqrt{2}}$

$\{\pm 2, \pm 2\sqrt{2}\}$

Solve $(\sqrt{x}-1)^2 - 3(\sqrt{x}-1) + 2 = 0$

by making Proper Subs.

Let $u = \sqrt{x} - 1$

Equation becomes $u^2 - 3u + 2 = 0$

$(u-2)(u-1) = 0$

$u = 2 \quad u = 1$

$u = 1$	$u = 2$
$\sqrt{x} - 1 = 1$	$\sqrt{x} - 1 = 2$
$\sqrt{x} = 2$	$\sqrt{x} = 3$
$x = 4$	$x = 9$

$\{4, 9\}$

Solve $x^{2/5} - x^{1/5} - 2 = 0$

Hint: $x^{2/5} = \left[x^{1/5}\right]^2$

Rewrite the equation $\left[x^{\frac{1}{5}}\right]^2 - \left[x^{1/5}\right] - 2 = 0$

Let $u = x^{1/5} \Rightarrow$ $u^2 - u - 2 = 0$

$(u-2)(u+1) = 0$

$u = 2 \quad u = -1$

$u = 2$	$u = -1$
$x^{1/5} = 2$	$x^{\frac{1}{5}} = -1$
$\sqrt[5]{x} = 2$	$\sqrt[5]{x} = -1$
$(\sqrt[5]{x})^5 = (2)^5$	$(\sqrt[5]{x})^5 = (-1)^5$
$x = 32$	$x = -1$

$\{-1, 32\}$

Solve $x^{\frac{1}{2}} + 2x^{\frac{1}{4}} - 3 = 0$ by making proper subs.

Hint: $x^{\frac{1}{2}} = \left(x^{\frac{1}{4}}\right)^2$ $\quad 2 \cdot \frac{1}{4} = \frac{1}{2}$

Now equation can be written as follow

$$\left[x^{\frac{1}{4}}\right]^2 + 2\left[x^{\frac{1}{4}}\right] - 3 = 0$$

Let $u = x^{1/4} \Rightarrow u^2 + 2u - 3 = 0$

$(u+3)(u-1) = 0$

$u = -3 \quad u = 1$

$u = -3$	$u = 1$
$x^{\frac{1}{4}} = -3$	$x^{1/4} = 1$
$\sqrt[4]{x} = -3$	$\sqrt[4]{x} = 1$
False even index	$\left(\sqrt[4]{x}\right)^4 = 1^4$
	$x = 1$

$\{1\}$

$6\sqrt{x} \neq \sqrt[6]{x}$

Solve $\left(\sqrt[6]{x} + 10\right)^2 - 7\left(\sqrt[6]{x} + 10\right) + 12 = 0$

Let $u = \sqrt[6]{x} + 10 \Rightarrow u^2 - 7u + 12 = 0$

$(u-3)(u-4) = 0$

$u = 3 \quad u = 4$

$u = 3$	$u = 4$
$\sqrt[6]{x} + 10 = 3$	$\sqrt[6]{x} + 10 = 4$
$\sqrt[6]{x} = -7$	$\sqrt[6]{x} = -6$
NO Solution.	NO Solution

$\emptyset$

$f(x) = \frac{1}{2}(x-2)^2 - 2$

$F(x) = a(x-h)^2 + k$

$a = \frac{1}{2}$ $h = 2$ $K = -2$

opens upward, $a > 0$

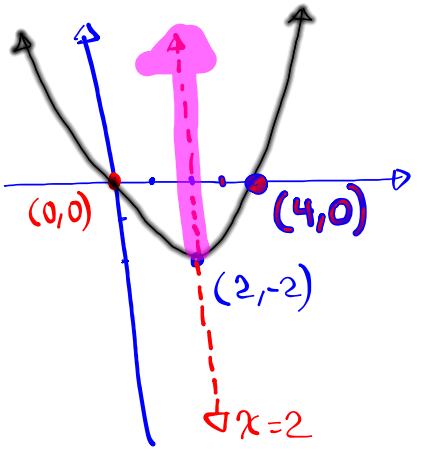
Vertex $(2, -2)$

A.O.S. $x = 2$

Y-Int $(0, 0)$

$\frac{1}{2}(0-2)^2 - 2 = 0$

x-Ints $(0,0)$, $(4,0)$



Domain: $(-\infty, \infty)$

Range: $[-2, \infty)$

$f(x) = -(x+3)^2 + 4$

$f(x) = a(x-h)^2 + k$

$a = -1$ $h = -3$ $K = 4$

opens downward $a < 0$

Vertex $(-3, 4)$

A.O.S. $x = -3$

Y-Int $(0, -5)$

x-Int $y = 0$

$-(x+3)^2 + 4 = 0$

$-(x+3)^2 = -4$

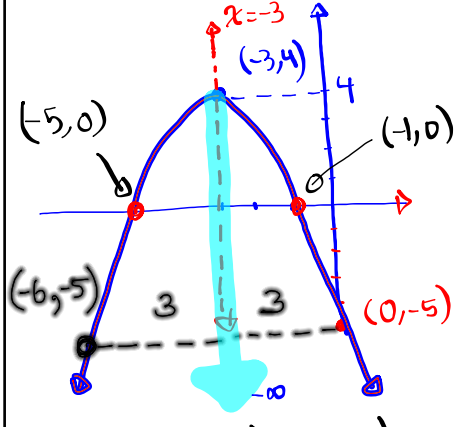
$(x+3)^2 = 4$

S.R.M.

$x + 3 = \pm\sqrt{4}$

$x = -3 \pm 2$

$x = -1$ $x = -5$



x-Int $(-5, 0)$, $(-1, 0)$

Domain : $(-\infty, \infty)$

Range : $(-\infty, 4]$

$f(x) = x^2 + 2x + 3$

$f(x) = ax^2 + bx + c$

$a = 1 \quad b = 2 \quad c = 3$

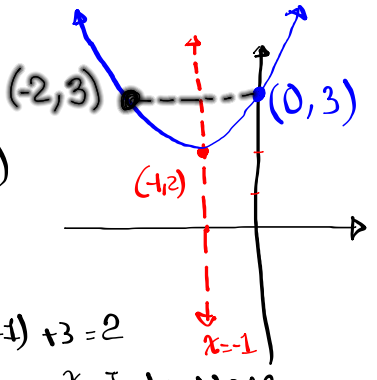
Opens upwards, $a > 0$

Vertex $(h, k) = (-1, 2)$

$h = \frac{-b}{2a} = \frac{-2}{2(1)} = \frac{-2}{2} = -1$

$K = f(h) = f(-1) = (-1)^2 + 2(-1) + 3 = 2$

A.O.S. $x = h \quad x = -1$



x-Int: None

y-Int: (0,3)

Domain $\Rightarrow (-\infty, \infty)$

Range $\Rightarrow [2, \infty)$

$f(x) = -x^2 + 4x$

$f(x) = ax^2 + bx + c$

$a = -1 \quad b = 4 \quad c = 0$

Opens downward $a < 0$

Vertex $(h, k) = (2, 4)$

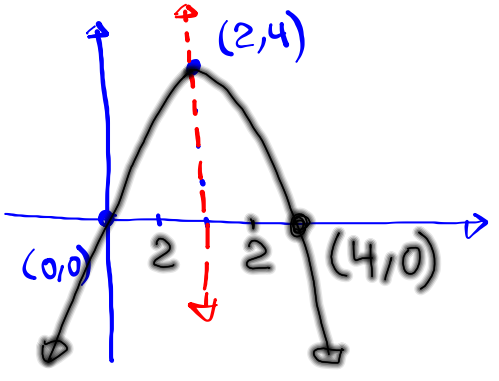
A.O.S. $x = h \quad x = 2$

$h = \frac{-b}{2a} = \frac{-4}{2(-1)} = \frac{-4}{-2} = 2$

y-Int $(0, 0)$

$K = f(h) = -(2)^2 + 4(2)$

$= -4 + 8 = 4$



x-Ints: $(0,0), (4,0)$

Domain: $(-\infty, \infty)$

Range: $(-\infty, 4]$

$f(x) = x^2 - 2x + 1$ $\quad a=1 \quad b=-2 \quad c=1$

$f(x) = ax^2 + bx + c$ $\quad$ opens upward, $a>0$

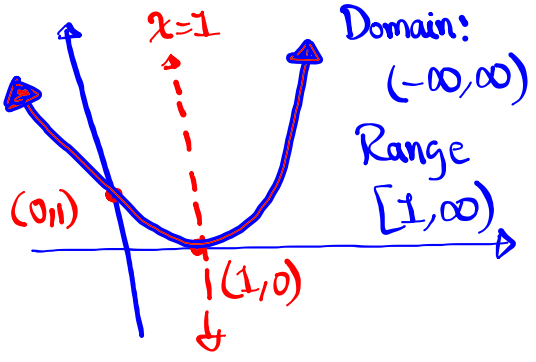
$h = \frac{-b}{2a} = \frac{-(-2)}{2(1)} = 1$ $\quad k = f(h) = f(1) = 1^2 - 2(1) + 1 = 0$

Vertex $(1, 0)$

A.O.S. $x = 1$

Y-Int $(0, 1)$

x-Int $(1, 0)$



Class QZ 19

1) Solve $(x-2)^2 = -9$

2) Solve $4x^2 + 9 = 12x$ by the quadratic formula.