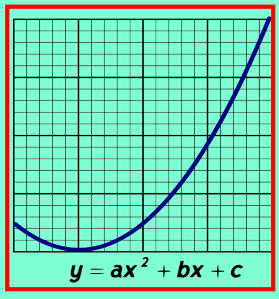
Math 125

Spring 2021

Lecture 23



Class QZ 18

Solve & Check: $\sqrt{4x+1} + 1 = x$

Isolate the radical $\sqrt{4x+1} = x - 1$

Square both Sides $(\sqrt{4x+1})^2 = (x-1)^2$ → Foil & Simplify

$4x + 1 = x^2 - 2x + 1$

$x^2 - 2x - 4x = 0$

$x^2 - 6x = 0$

$x(x-6) = 0$

$x = 0$ $\quad x = 6$

$\{6\}$

Check $x = 0$

$\sqrt{4(0)+1} + 1 = 0$

$\sqrt{1} + 1 = 0$

$1 + 1 = 0$

false

Check $x = 6$

$\sqrt{4(6)+1} + 1 = 6$

$\sqrt{25} + 1 = 6$

$5 + 1 = 6$

$6 = 6$ ✓

Solve $\sqrt{2x+1} + \sqrt{x} = 5$

$\sqrt{2x+1} = 5 - \sqrt{x}$, $\left(\sqrt{2x+1}\right)^2 = (5-\sqrt{x})^2$

$2x+1 = (5-\sqrt{x})(5-\sqrt{x})$

$2x+1 = 25 - 5\sqrt{x} - 5\sqrt{x} + (\sqrt{x})^2$

$2x+1 = 25 - 10\sqrt{x} + x$

$2x+1-25-x = -10\sqrt{x}$

$x - 24 = -10\sqrt{x}$

Square both Sides

$(x-24)^2 = (-10\sqrt{x})^2$

$(x-24)(x-24) = 100x$

$x^2 - 24x - 24x + 576 = 100x$

$x^2 - 48x + 576 - 100x = 0$

$x^2 - 148x + 576 = 0$

$(x-4)(x-144) = 0$

$x-4=0$ $\quad$ $x-144=0$

$\checkmark\ x=4$ $\quad$ $x=144$ E.S.

$\{4\}$

Check $x=4$

$\sqrt{2(4)+1} + \sqrt{4} = 5$

$\sqrt{9} + \sqrt{4} = 5$

$3+2=5\checkmark$

Check $x=144$

$\sqrt{2(144)+1} + \sqrt{144} = 5$

$\sqrt{289} + 12 = 5$

$13 + 12 = 5$ False

Solve

$\sqrt{x+5} + \sqrt{x-3} = 4$

$\sqrt{x+5} = 4 - \sqrt{x-3}$

$\left(\sqrt{x+5}\right)^2 = \left(4 - \sqrt{x-3}\right)^2$

$x+5 = (4-\sqrt{x-3})(4-\sqrt{x-3})$

$x+5 = 16 - 4\sqrt{x-3} - 4\sqrt{x-3} + (\sqrt{x-3})^2$

$x+5 = 16 - 8\sqrt{x-3} + x - 3$

$5 - 13 = -8\sqrt{x-3}$

$-8 = -8\sqrt{x-3}$ Divide by -8

$1 = \sqrt{x-3}$

$(1)^2 = (\sqrt{x-3})^2$

$1 = x-3$ $\checkmark\ x=4$

Check

$\sqrt{x+5} + \sqrt{x-3} = 4$

$\sqrt{4+5} + \sqrt{4-3} = 4$

$\sqrt{9} + \sqrt{1} = 4$

$3+1=4\checkmark$

$\{4\}$

Solve $\sqrt{2x-3} - \sqrt{x-2} = 1$

$\sqrt{2x-3} = 1 + \sqrt{x-2}$

$(\sqrt{2x-3})^2 = (1+\sqrt{x-2})^2$

$2x-3 = (1+\sqrt{x-2})(1+\sqrt{x-2})$

$2x-3 = 1 + \sqrt{x-2} + \sqrt{x-2} + (\sqrt{x-2})^2$

$2x-3 = 1 + 2\sqrt{x-2} + x-2$

$2x-3-x+1 = 2\sqrt{x-2}$

$x-2 = 2\sqrt{x-2}$

Solve. Simplify $(x-2)^2 = (2\sqrt{x-2})^2$

$x^2-4x+4 = 4(x-2)$

$x^2-4x+4 = 4x-8$

$x^2-4x+4-4x+8=0$

$x^2-8x+12=0$

$(x-2)(x-6)=0$

$x=2$ $\quad$ $x=6$

Check $x=2$: $\sqrt{2(2)-3} - \sqrt{2-2} = 1$; $\sqrt{4-3} - \sqrt{0} = 1$; $\sqrt{1} - 0 = 1$

Check $x=6$: $\sqrt{2(6)-3} - \sqrt{6-2} = 1$; $\sqrt{9} - \sqrt{4} = 1$; $3-2=1$ ✓

$\{2, 6\}$

Rationalize the deno:

1) $\frac{2}{\sqrt{x}} \cdot \frac{\sqrt{x}}{\sqrt{x}} = \frac{2\sqrt{x}}{\sqrt{x^2}} = \boxed{\frac{2\sqrt{x}}{x}}$

2) $\frac{2}{\sqrt[3]{x}} \cdot \frac{\sqrt[3]{x^2}}{\sqrt[3]{x^2}}$

$= \frac{2\sqrt[3]{x^2}}{\sqrt[3]{x^3}} = \boxed{\frac{2\sqrt[3]{x^2}}{x}}$

3) $\frac{\sqrt{3}}{\sqrt{3}+1} \cdot \frac{\sqrt{3}-1}{\sqrt{3}-1}$

$= \frac{\sqrt{3}(\sqrt{3}-1)}{(\sqrt{3}+1)(\sqrt{3}-1)} = \frac{\sqrt{9} - \sqrt{3}}{\sqrt{9} - \sqrt{3} + \sqrt{3} - 1}$

$= \frac{3-\sqrt{3}}{3-1} = \boxed{\frac{3-\sqrt{3}}{2}}$

Rationalize the deno:

1) $\frac{-3x}{\sqrt{3x}} \cdot \frac{\sqrt{3x}}{\sqrt{3x}}$

$= \frac{-3x\sqrt{3x}}{\sqrt{3^2x^2}} = \frac{-3x\sqrt{3x}}{3x}$

$= -\sqrt{3x}$

2) $\frac{-3x}{\sqrt[4]{3^1x^2}} \cdot \frac{\sqrt[4]{3^3x^2}}{\sqrt[4]{3^3x^2}}$

$= \frac{-3x\sqrt[4]{27x^2}}{\sqrt[4]{3^4x^4}}$

$= \frac{-3x\sqrt[4]{27x^2}}{3x}$

$= -\sqrt[4]{27x^2}$

3) $\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}$

$= \frac{(\sqrt{5}+\sqrt{3})(\sqrt{5}+\sqrt{3})}{(\sqrt{5}-\sqrt{3})(\sqrt{5}+\sqrt{3})} = \frac{\sqrt{25}+\sqrt{15}+\sqrt{15}+\sqrt{9}}{\sqrt{25}+\sqrt{15}-\sqrt{15}-\sqrt{9}}$

$= \frac{5+2\sqrt{15}+3}{5-3} = \frac{8+2\sqrt{15}}{2}$

$= \frac{8}{2} + \frac{2\sqrt{15}}{2}$

$= 4+\sqrt{15}$

Simplify

1) $(2-3i)(3-5i) = 6-10i-9i+15i^2$

$= 6-19i-15$

$= -9-19i$

2) $\frac{2+5i}{3-2i}$

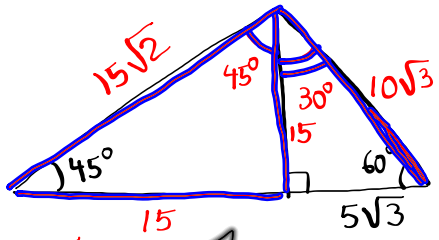
$= \frac{(2+5i)(3+2i)}{(3-2i)(3+2i)} = \frac{6+4i+15i+10i^2}{9+6i-6i-4i^2}$

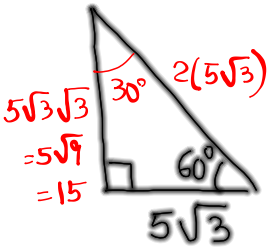
$= \frac{6+19i-10}{9-4(-1)} = \frac{-4+19i}{13}$

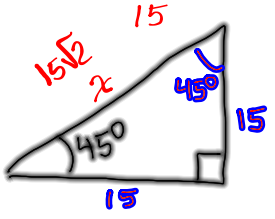
$\rightarrow \frac{-4}{13} + \frac{19}{13}i$

March 18, 2021

Find missing Sides / Angles:







$x^2 = 15^2 + 15^2$

$x^2 = 225 + 225$

$= 450$

$x = \sqrt{225}\sqrt{2}$

$= 15\sqrt{2}$

$450 = 9 \cdot 50$

$= 9 \cdot 25 \cdot 2$

$\sqrt{450} = \sqrt{9}\sqrt{25}\sqrt{2} = 3 \cdot 5 \cdot \sqrt{2} = 15\sqrt{2}$

Consider $x^2 - 8x + 16 + y^2 + 10y + 25 = 9$

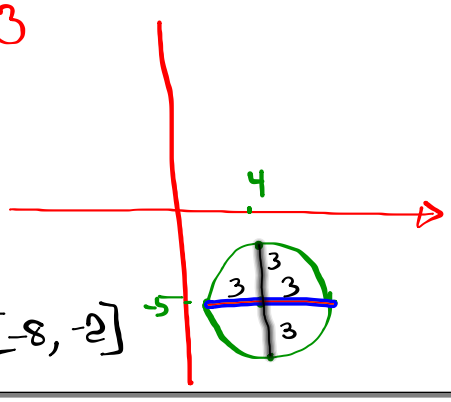
1) write in $(x-h)^2 + (y-k)^2 = r^2$

$(x-4)^2 + (y+5)^2 = 3^2$

2) Center $(4,-5)$ Radius 3

3) Intercepts None

4) Domain $[1,7]$, Range $[-8,-2]$



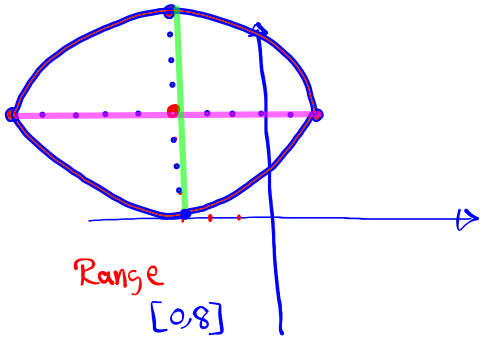
Consider $16(x+3)^2 + 25(y-4)^2 = 400$

1) Write in $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$

Divide by 400 $\Rightarrow$ $\frac{16(x+3)^2}{400} + \frac{25(y-4)^2}{400} = \frac{400}{400}$

$\frac{(x+3)^2}{25} + \frac{(y-4)^2}{16} = 1$

2) Center $(-3,4)$ $a=5$ $b=4$



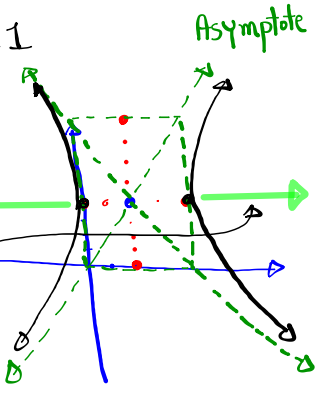
3) Domain $[-8,2]$ Range $[0,8]$

Hyperbolas:

$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$) (center (h,k)

$\frac{(y-k)^2}{b^2} - \frac{(x-h)^2}{a^2} = 1$ ∪ ∩ Center (h,k)

Graph $\frac{(x-2)^2}{4} - \frac{(y-4)^2}{16} = 1$



Center $(2,4)$

$a^2 = 4 \Rightarrow a = 2$

$b^2 = 16 \Rightarrow b = 4$

Domain $(-\infty, 0] \cup [4, \infty)$

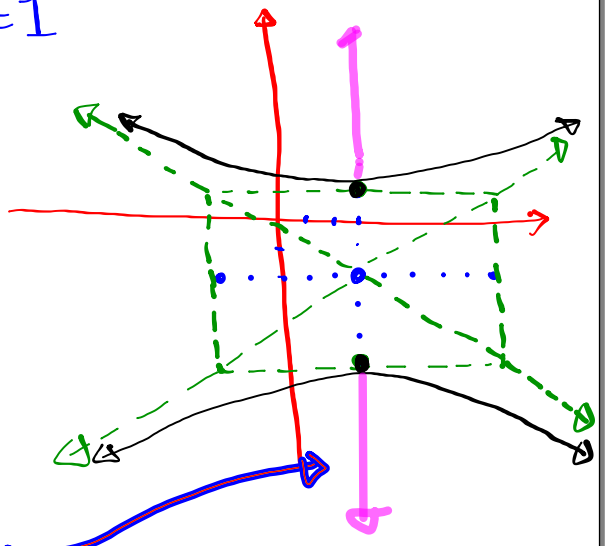
Range $(-\infty, \infty)$

$$\frac{(y+2)^2}{9} - \frac{(x-3)^2}{25} = 1$$

Center $(3, -2)$

$a^2 = 25 \quad a = 5$

$b^2 = 9 \quad b = 3$



Domain: $(-\infty, \infty)$

Range: $(-\infty, -5] \cup [1, \infty)$

$$(x+2)^2 - (y-4)^2 = 9$$

Hyperbola

Make RHS = 1

Divide by 9

$$\frac{(x+2)^2}{9} - \frac{(y-4)^2}{9} = 1$$

Center $(-2, 4)$

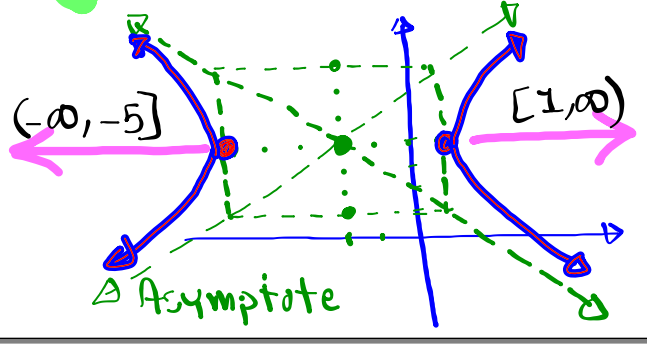
$a^2 = 9 \quad a = 3$

$b^2 = 9 \quad b = 3$

Domain:

$(-\infty, -5] \cup [1, \infty)$

Range : $(-\infty, \infty)$



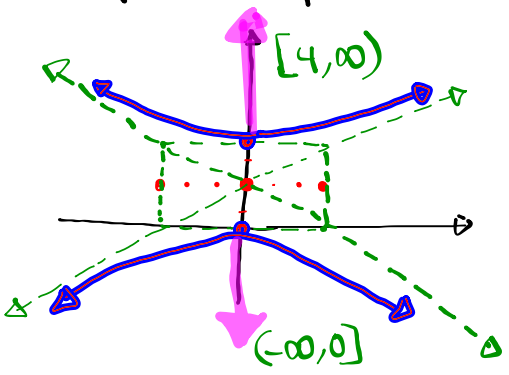
$4x^2 - 9(y-2)^2 = -36$

Make RHS = 1 by dividing everything by -36.

$\frac{4x^2}{-36} - \frac{9(y-2)^2}{-36} = \frac{-36}{-36}$

$-\frac{x^2}{9} + \frac{(y-2)^2}{4} = 1$

$\frac{(y-2)^2}{4} - \frac{x^2}{9} = 1$



Center $(0,2)$

$a^2 = 9 \quad a = 3$

$b^2 = 4 \quad b = 2$

Domain: $(-\infty, \infty)$

Range: $(-\infty, 0] \cup [4, \infty)$

$4y^2 - 25(x-2)^2 = -100$

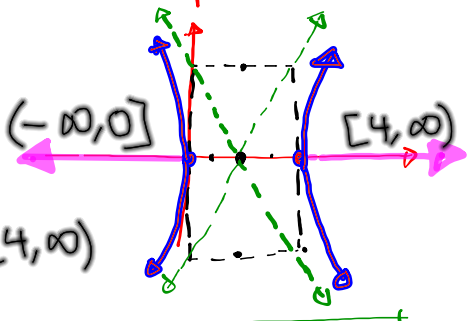
Divide by -100, and Simplify

$-\frac{y^2}{25} + \frac{(x-2)^2}{4} = 1 \Rightarrow \frac{(x-2)^2}{4} - \frac{y^2}{25} = 1$

Center $(2,0)$

$a^2 = 4 \quad a = 2$

$b^2 = 25 \quad b = 5$



D: $(-\infty, 0] \cup [4, \infty)$

R: $(-\infty, \infty)$

SG 21 & SG 22 work on them

Exam 2: Monday start as early as 6:45 AM