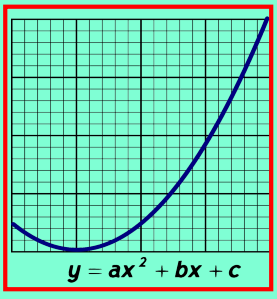
Math 125

Spring 2021

Lecture 21



Class QZ 16

1) Solve $\sqrt{2x-9} = 5$

Index = 2

$\left(\sqrt{2x-9}\right)^2 = (5)^2$

$2x - 9 = 25$

$2x = 34$

$x = 17$ ✓

{17}

Check $\sqrt{2(17)-9} = 5$

$\sqrt{34-9} = 5$ $\sqrt{25} = 5$ $5 = 5$ ✓

2) Solve $x = \sqrt{x^2 - 4x + 4}$

$(x)^2 = \left(\sqrt{x^2-4x+4}\right)^2$

$x^2 = x^2 - 4x + 4$

$0 = -4x + 4$

$4x = 4$

$x = 1$ ✓

{1}

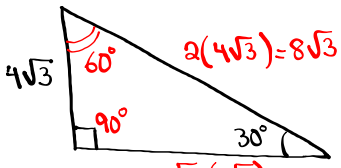
Check

$1 = \sqrt{1^2 - 4(1) + 4}$

$1 = \sqrt{1 - 4 + 4}$

$1 = \sqrt{1}$ $1 = 1$ ✓

Find the missing sides/angles:

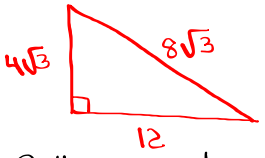


$2(4\sqrt{3}) = 8\sqrt{3}$

$4\sqrt{3}(\sqrt{3}) =$

$4\sqrt{9} =$

$4 \cdot 3 = 12$



Pythagorean Thm

$(4\sqrt{3})^2 + (12)^2 = (8\sqrt{3})^2$

$16 \cdot 3 + 144 = 64 \cdot 3$

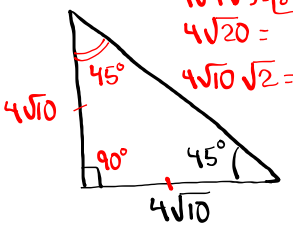
$48 + 144 = 192$

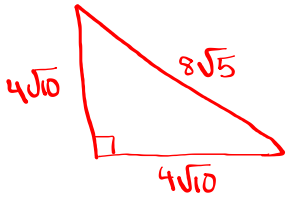
$192 = 192$ ✓

$4\sqrt{4}\sqrt{5} = \boxed{8\sqrt{5}}$

$4\sqrt{20} =$

$4\sqrt{10}\sqrt{2} =$





Pythagorean thm

$a^2 + b^2 = c^2$

$(4\sqrt{10})^2 + (4\sqrt{10})^2 = (8\sqrt{5})^2$

$16 \cdot 10 + 16 \cdot 10 = 64 \cdot 5$

$160 + 160 = 320$

$320 = 320$ ✓

Simplify

1) $\sqrt{50x} - \sqrt{32x} + 3\sqrt{8x} =$

$= \sqrt{25}\sqrt{2x} - \sqrt{16}\sqrt{2x} + 3\sqrt{4}\sqrt{2x} =$

$= 5\sqrt{2x} - 4\sqrt{2x} + 3 \cdot 2\sqrt{2x} = \sqrt{2x} + 6\sqrt{2x} = \boxed{7\sqrt{2x}}$

2) $-2\sqrt{5}(3\sqrt{5} - 1)$

$= -6\sqrt{25} + 2\sqrt{5}$

$= -6 \cdot 5 + 2\sqrt{5}$

$= \boxed{-30 + 2\sqrt{5}}$ ✓

3) $(5+\sqrt{2})(5-\sqrt{2})$

conjugates

$= 25 - 5\sqrt{2} + 5\sqrt{2} - \sqrt{4}$

$= 25 - 2 = \boxed{23}$

4) $(\sqrt{5} + 2\sqrt{2})^2$

$= (\sqrt{5} + 2\sqrt{2})(\sqrt{5} + 2\sqrt{2})$

$= \sqrt{25} + 2\sqrt{10} + 2\sqrt{10} + 4\sqrt{4}$

$= 5 + 4\sqrt{10} + 4 \cdot 2$

$= \boxed{13 + 4\sqrt{10}}$

5) $(\sqrt[3]{5} - \sqrt[3]{2})(\sqrt[3]{25} + \sqrt[3]{10} + \sqrt[3]{4})$

$= \sqrt[3]{125} + \sqrt[3]{50} + \sqrt[3]{20}$

$- \sqrt[3]{50} - \sqrt[3]{20} - \sqrt[3]{8}$

$= 5 - 2$

$= \boxed{3}$

Rationalize the denominator:

1) $\frac{3x}{\sqrt{6x}} = \frac{3x\sqrt{6x}}{\sqrt{6x}\sqrt{6x}}$

$= \frac{3x\sqrt{6x}}{\sqrt{36x^2}} = \frac{3x\sqrt{6x}}{6x} = \boxed{\frac{\sqrt{6x}}{2}}$

2) $\frac{\sqrt{3}}{\sqrt{3}-1} \cdot \frac{\sqrt{3}+1}{\sqrt{3}+1} =$

$= \frac{\sqrt{3}(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)}$

$= \frac{\sqrt{9}+\sqrt{3}}{\sqrt{9}+\sqrt{3}-\sqrt{3}-1}$

$= \frac{3+\sqrt{3}}{3-1} = \boxed{\frac{3+\sqrt{3}}{2}}$

3) $\frac{\sqrt{10}}{\sqrt{5}+\sqrt{2}}$

$= \frac{\sqrt{10}(\sqrt{5}-\sqrt{2})}{(\sqrt{5}+\sqrt{2})(\sqrt{5}-\sqrt{2})}$

$= \frac{\sqrt{50}-\sqrt{20}}{\sqrt{25}-\sqrt{10}+\sqrt{10}-\sqrt{4}} = \frac{\sqrt{25}\sqrt{2}-\sqrt{4}\sqrt{5}}{5-2}$

$= \boxed{\frac{5\sqrt{2}-2\sqrt{5}}{3}}$

Rationalize the numerator

1) $\frac{\sqrt{10x}}{5}$

$= \frac{\sqrt{10x}\cdot\sqrt{10x}}{5\cdot\sqrt{10x}}$

$= \frac{\sqrt{100x^2}}{5\sqrt{10x}} = \frac{10x}{5\sqrt{10x}}$

$= \boxed{\frac{2x}{\sqrt{10x}}}$

2) $\frac{\sqrt{3}+\sqrt{2}}{\sqrt{6}}$

$= \frac{(\sqrt{3}+\sqrt{2})(\sqrt{3}-\sqrt{2})}{\sqrt{6}(\sqrt{3}-\sqrt{2})}$

$= \frac{\sqrt{9}-\sqrt{6}+\sqrt{6}-\sqrt{4}}{\sqrt{18}-\sqrt{12}}$

$= \frac{3-2}{\sqrt{9}\sqrt{2}-\sqrt{4}\sqrt{3}} = \boxed{\frac{1}{3\sqrt{2}-2\sqrt{3}}}$

Solve $x - \sqrt{x-2} = 2$

$x - 2 = \sqrt{x-2}$

$(x-2)^2 = (\sqrt{x-2})^2$

$(x-2)(x-2) = x-2$

$x^2 - 2x - 2x + 4 \quad -x + 2 = 0$

$x^2 - 5x + 6 = 0$

$(x-2)(x-3) = 0$

$x-2=0 \qquad x-3=0$

$\boxed{x=2}\ \checkmark \qquad \boxed{x=3}\ \checkmark$

$\boxed{\{2,3\}}$

Check

$x - \sqrt{x-2} = 2$

$2 - \sqrt{2-2} = 2$

$2 - \sqrt{0} = 2$

$2 - 0 = 2$

$2 = 2\ \checkmark$

$x - \sqrt{x-2} = 2$

$3 - \sqrt{3-2} = 2$

$3 - \sqrt{1} = 2$

$3 - 1 = 2$

$2 = 2\ \checkmark$

Solve $2x + \sqrt{x-1} = 5$

$\sqrt{x-1} = 5 - 2x$

$(\sqrt{x-1})^2 = (5-2x)^2$

$x - 1 = (5-2x)(5-2x)$

$x - 1 = 25 - 10x - 10x + 4x^2$

$x - 1 = 25 - 20x + 4x^2$

$4x^2 - 20x + 25 - x + 1 = 0$

$4x^2 - 21x + 26 = 0$

$(x-2)(4x-13) = 0$

$-8x$

$-13x$

$x-2=0 \qquad 4x-13=0$

$\boxed{x=2} \qquad x = \frac{13}{4} = 3.25$

E.S.

$\boxed{\{2\}}$

Check

$2x + \sqrt{x-1} = 5$

$2(2) + \sqrt{2-1} = 5$

$4 + \sqrt{1} = 5$

$4 + 1 = 5\ \checkmark$

$2(3.25) + \sqrt{3.25-1} = 5$

$6.5 + \sqrt{2.25} = 5$

$6.5 + 1.5 = 5$

$8 = 5$ False

1) Write $\sqrt{64} - \sqrt{-16}$ in $a+bi$ form.

$= 8 - \sqrt{16}\sqrt{-1} = \boxed{8 - 4i}$

2) Simplify: $i^{250} - i^{85}$

$= (i^2)^{125} - i^{84} \cdot i$

$= (-1)^{125} - (i^2)^{42} \cdot i$

$= -1 - (-1)^{42} i = \boxed{-1 - i}$

3) Simplify: $2i(3-5i) - 4(1+3i)$

$= 6i - 10i^2 - 4 - 12i$

$= 6i - 10(-1) - 4 - 12i$

$= 6i + 10 - 4 - 12i$

$= \boxed{6 - 6i}$

4) Simplify: $(3+4i)(-2+3i)$

$= -6 + 9i - 8i + 12i^2$

$= -6 + i + 12(-1)$

$= \boxed{-18 + i}$

5) Divide $\frac{1+2i}{6+8i} = \frac{(1+2i)(6-8i)}{(6+8i)(6-8i)}$

$= \frac{6 - 8i + 12i - 16i^2}{36 - 48i + 48i - 64i^2}$ (+16)

$= \frac{22+4i}{36+64} = \frac{22+4i}{100}$

$= \frac{22}{100} + \frac{4}{100}i$

$= \frac{11}{50} + \frac{1}{25}i$

Class QZ 17

Divide $\dfrac{3+4i}{3-4i} = \dfrac{(3+4i)(3+4i)}{(3-4i)(3+4i)}$

$= \dfrac{9+12i+12i+16i^2}{9+12i-12i-16i^2} = \dfrac{9+24i-16}{9+16}$

$= \dfrac{-7+24i}{25} = \boxed{\dfrac{-7}{25} + \dfrac{24}{25}i}$

More equations:

Solve $5\sqrt{x} - \sqrt{10x+15} = 0$

$5\sqrt{x} = \sqrt{10x+15}$

$(5\sqrt{x})^2 = (\sqrt{10x+15})^2$

$25x = 10x + 15$

$15x = 15$

$\boxed{x = 1}$ $\boxed{\{1\}}$

Check

$5\sqrt{x} - \sqrt{10x+15} = 0$

$5\sqrt{1} - \sqrt{10(1)+15} = 0$

$5 \cdot 1 - \sqrt{10+15} = 0$

$5 - \sqrt{25} = 0$

$5 - 5 = 0$

$0 = 0$ ✓

Solve $3\sqrt{x-2} + 2 = x$

$3\sqrt{x-2} = x - 2$

$(3\sqrt{x-2})^2 = (x-2)^2$

$9(x-2) = (x-2)(x-2)$

$9x - 18 = x^2 - 2x - 2x + 4$

$9x - 18 = x^2 - 4x + 4$

$x^2 - 4x + 4 - 9x + 18 = 0$

$x^2 - 13x + 22 = 0$

$(x-2)(x-11) = 0$

$x - 2 = 0$ $\quad$ $x - 11 = 0$

$x = 2$ ✓ $\quad$ $x = 11$ ✓

Check both solutions in the original eqn.

$\{2, 11\}$

Solve $x - 9 = \sqrt{x} + 3$

$x - 9 - 3 = \sqrt{x}$

$x - 12 = \sqrt{x}$

$(x-12)^2 = (\sqrt{x})^2$

$(x-12)(x-12) = x$

$x^2 - 12x - 12x + 144 = x$

$x^2 - 24x + 144 - x = 0$

$x^2 - 25x + 144 = 0$

$(x-16)(x-9) = 0$

✓ $x = 16$ $\quad$ ~~$x = 9$~~

Check both

$\{16\}$

Solve $\sqrt{x} - 4 = \sqrt{x-32}$

$(\sqrt{x} - 4)^2 = (\sqrt{x-32})^2$

$(\sqrt{x} - 4)(\sqrt{x} - 4) = x - 32$

$\sqrt{x^2} - 4\sqrt{x} - 4\sqrt{x} + 16 = x - 32$

$x - 8\sqrt{x} + 16 = x - 32$

$-8\sqrt{x} = -32 - 16$

$-8\sqrt{x} = -48$

Divide by -8

$\sqrt{x} = \frac{-48}{-8}$

$\sqrt{x} = 6$

$(\sqrt{x})^2 = (6)^2$

$x = 36$ ✓

Make Sure to check

$\{36\}$

Circle $(x-h)^2 + (y-k)^2 = r^2$

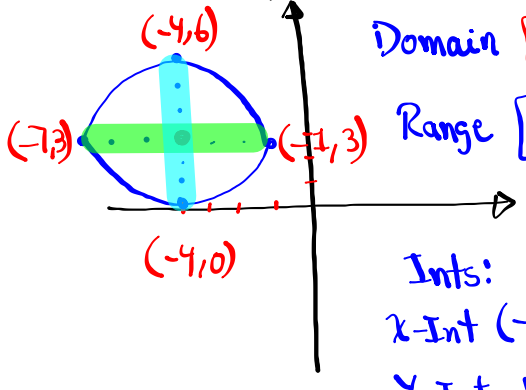
Given $(x+4)^2 + (y-3)^2 = 9$

$r^2 = 9$ $r = 3$

Center $(-4, 3)$

radius 3

Draw



Domain $[-7, -1]$

Range $[0, 6]$

Ints:

x-Int $(-4, 0)$

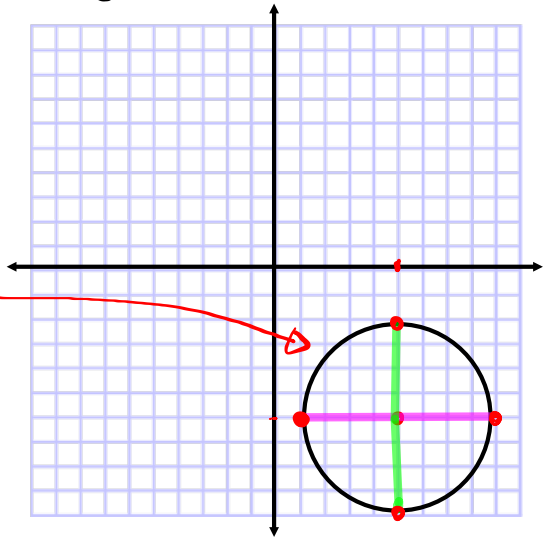
Y-Int None

Given $(x-5)^2+(y+6)^2=16$

Center $(5,-6)$

Radius 4

Draw



Domain $[1,9]$

Range $[-10,-2]$

Ints None

Ellipse $\dfrac{(x-h)^2}{a^2}+\dfrac{(y-k)^2}{b^2}=1$ $\quad a,b>0$

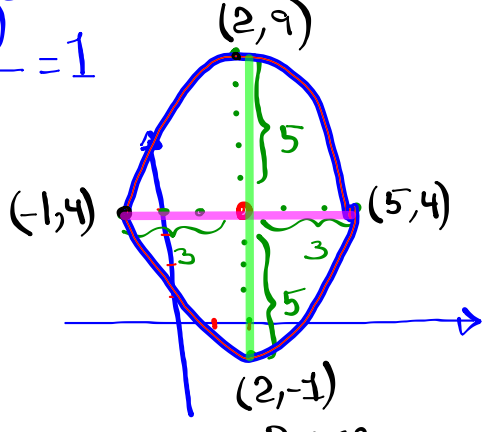
Center (h,k)

Ex: $\dfrac{(x-2)^2}{9}+\dfrac{(y-4)^2}{25}=1$

Center $(2,4)$

$a^2=9$ $\quad b^2=25$

$a=3$ $\quad b=5$



Domain $[-1,5]$ $\quad$ Range $[-1,9]$

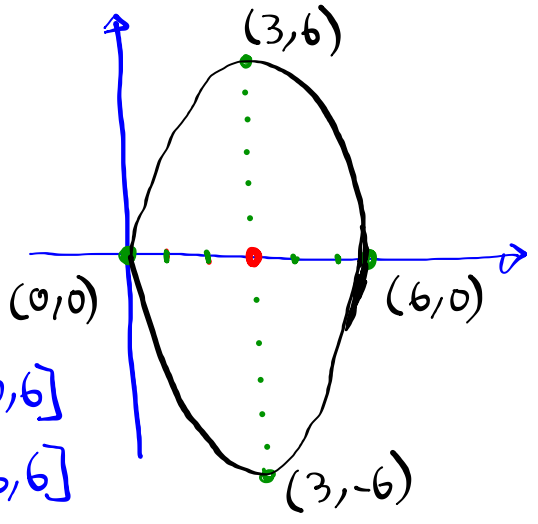
$\frac{(x-3)^2}{9} + \frac{y^2}{36} = 1$

$a^2 = 9 \quad b^2 = 36$

Center $(3, 0)$

$a = 3$

$b = 6$



Domain $[0, 6]$

Range $[-6, 6]$

Ints: x-Int $(0,0)$, $(6,0)$

Y-Int $(0,0)$

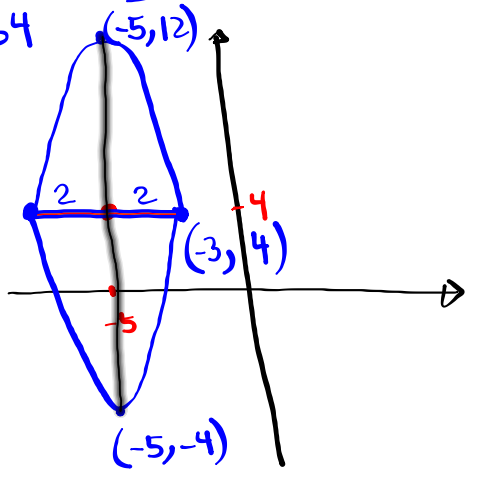
Given $\frac{(x+5)^2}{4} + \frac{(y-4)^2}{64} = 1$

Center $(-5, 4)$

$a = 2$

$b = 8$

Draw



Domain $[-7, -3]$ Range $[-4, 12]$

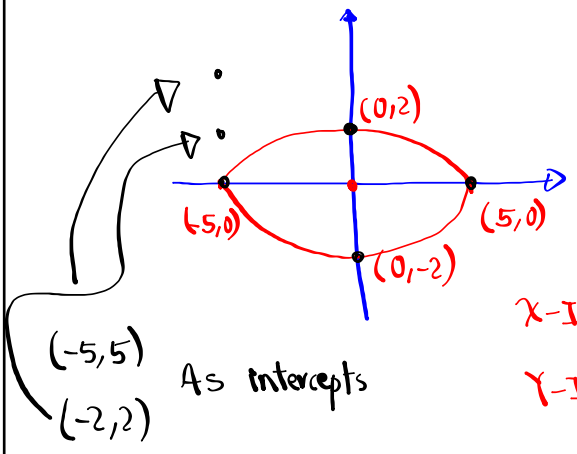
Hint: Make RHS 1

$4x^2 + 25y^2 = 100$

Divide by 100

$\frac{4x^2}{100} + \frac{25y^2}{100} = \frac{100}{100}$

$\frac{x^2}{25} + \frac{y^2}{4} = 1$



Center (0,0)

$a = 5$ $b = 2$

Domain: $[-5,5]$

Range: $[-2,2]$

x-Ints: $(-5,0), (5,0)$

y-Int: $(0,-2), (0,2)$

$4(x+2)^2 + 9(y+3)^2 = 36$

Hint: Make RHS = 1.
Divide by 36.

$\frac{4(x+2)^2}{36} + \frac{9(y+3)^2}{36} = \frac{36}{36}$

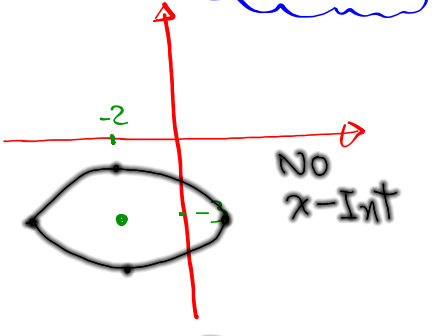
$\frac{(x+2)^2}{9} + \frac{(y+3)^2}{4} = 1$

Exam II:
Monday March 22, 2021

Center $(-2,-3)$

$a = 3$

$b = 2$



Zoom link for office hours.

Domain

Range