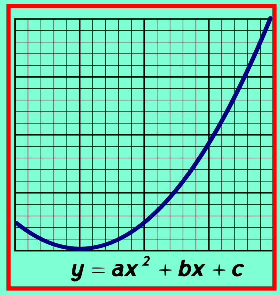
Math 125

Spring 2021

Lecture 18



Class QZ 14

Hint: Use $\underbrace{\text{Subs. Method}}$

Solve

$$\begin{cases} x^2 + y^2 = 50 \\ y - x = 0 \end{cases}$$

$y = x$

$x^2 + x^2 = 50$

$2x^2 = 50$

$x^2 = 25$

$x = \pm 5$

$x = 5 \rightarrow y = 5$

$x = -5 \rightarrow y = -5$

$$\boxed{\{(5,5), (-5,-5)\}}$$

The Sum of two numbers is 10. → x & y

Their Product is 24.

Find all such numbers.

$$\begin{cases} x+y=10 \\ xy=24 \end{cases}$$

Solve

$$\begin{cases} x+y=10 \\ xy=24 \end{cases}$$

$y=10-x$

$x(10-x)=24$

$10x-x^2=24$

$-x^2+10x-24=0$

Multiply by -1

$x^2-10x+24=0$

$(x-6)(x-4)=0$

$x-6=0$ $\quad$ $x-4=0$

$\boxed{x=6}$ $\quad$ $\boxed{x=4}$

$y=10-x$

$x=6 \Rightarrow y=10-6=4$

$x=4 \Rightarrow y=10-4=6$

4 & 6

One number is 1 more than twice another number.

Sum of their squares is 29.

Find all such numbers

x & y

$x=2y+1$

$x^2+y^2=29$

Solve $\begin{cases} x=2y+1 \\ x^2+y^2=29 \end{cases}$

Use Sub. method

$(2y+1)^2+y^2=29$

$(2y+1)(2y+1)+y^2=29$

$4y^2+2y+2y+1+y^2-29=0$

$5y^2+4y-28=0$

$ax^2+bx+c=0$ $\quad$ $a\neq 0$

Quadratic eqn

$a=5$ $\quad$ $b=4$ $\quad$ $c=-28$

$x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

Quadratic Formula

$b^2-4ac=4^2-4(5)(-28)$

$=4^2+4(5)(28)$

$=576$

$y=\dfrac{-4\pm\sqrt{576}}{2(5)}$

$y=\dfrac{-4\pm 24}{10}$

$y=\dfrac{-4+24}{10}$ $\quad$ $y=\dfrac{-4-24}{10}$

$y=\dfrac{20}{10}$ $\quad$ $y=\dfrac{-28}{10}$

$\boxed{y=2}$ $\quad$ $\boxed{y=-2.8}$

$x=2y+1$

$y=2 \Rightarrow x=2(2)+1=5$

$y=-2.8 \Rightarrow x=2(-2.8)+1=-4.6$

5 & 2 OR −4.6 & −2.8

y varies directly as x^4

y is 162 when x is 3

Find y when x is 5.

$y = k \cdot x^4$

$162 = k(3)^4$

$162 = k \cdot 81$

$\boxed{k = 2}$

$y = 2x^4$

$y = 2(5)^4$

$= 2 \cdot 625$

$\boxed{y = 1250}$

y varies inversely as square root of x.

y is 5 when x is 25.

Find y when x is 100

$y = \frac{k}{\sqrt{x}}$

$5 = \frac{k}{\sqrt{25}}$

$5 = \frac{k}{5}$

$\boxed{k = 25}$

$y = \frac{25}{\sqrt{x}}$

$y = \frac{25}{\sqrt{100}} = \frac{25}{10}$

$\boxed{y = 2.5}$

SG 11 Distance directly Sqr root of speed

11

$D = K\sqrt{S}$

SG 11 Intensity inversely square of distance

9

$I = \frac{K}{D^2}$

SG 11 ✓

Simplify $\sqrt{40x^9}$ Index = 2

Radicand = $40x^9$

$40 = 4 \cdot 10$

$x^9 = x^8 \cdot x$

$= \sqrt[2]{4x^8}\sqrt{10x}$

$= \boxed{2x^4\sqrt{10x}}$

Simplify $\sqrt[3]{40x^5y^8z^{10}} =$ Index = 3

Radicand = $40x^5y^8z^{10}$

$40 = 8 \cdot 5$

$= 2^3 \cdot 5$

$\sqrt[\cancel{3}]{2^{\cancel{3}}x^{\cancel{3}}y^{\cancel{6}}z^{\cancel{9}}}\sqrt[3]{5x^2y^2z}$

$x^5 = x^3 \cdot x^2$

$y^8 = y^6 \cdot y^2$, $z^{10} = z^9 \cdot z$

$= \boxed{2xy^2z^3\sqrt[3]{5x^2y^2z}}$

Solving Simple radical Equation:

1) I Solate the radical

2) Raise both Sides to index power. $\left(\sqrt[n]{x}\right)^n = x$

3) Simplify

4) Solve the new equation.

5) Always check every Solution in the original equation as they may not work.

Solve $\sqrt{x+2} - 3 = 1$

Isolate the radical

$\sqrt{x+2} = 4$

Index $= 2 \Rightarrow$ Square both Sides

$\left(\sqrt{x+2}\right)^2 = 4^2$

$x + 2 = 16$

$\boxed{x = 14}$ ✓

$\{14\}$

Check

$\sqrt{x+2} - 3 = 1$

$\sqrt{14+2} - 3 = 1$

$\sqrt{16} - 3 = 1$

$4 - 3 = 1$

$1 = 1$ ✓

$\cancel{\{x = 14\}}$

$\cancel{\{x = 1, y = 2\}}$ $\{(1,2)\}$

Solve $\sqrt[3]{2x-1} = 5$

Radical is already isolated, Index = 3

Raise both sides to the 3rd power.

$$\left(\sqrt[3]{2x-1}\right)^3 = (5)^3$$

$$2x - 1 = 125$$

$$2x = 125 + 1$$

$$2x = 126$$

$$\boxed{x = 63}$$

Check: $\sqrt[3]{2x-1} = 5$

$\sqrt[3]{2(63)-1} = 5$

$\sqrt[3]{126-1} = 5$

$\sqrt[3]{125} = 5$

$5 = 5$ ✓

$\{63\}$

Solve $2x - \sqrt{3x^2 - x + 6} = 0$

Isolate the radical

$$2x = \sqrt{3x^2 - x + 6}$$

Index = 2 ⇒ square both sides

$$(2x)^2 = \left(\sqrt[2]{3x^2 - x + 6}\right)^2$$

$4x^2 = 3x^2 - x + 6 \Rightarrow 4x^2 - 3x^2 + x - 6 = 0$

$x^2 + x - 6 = 0$

$(x+3)(x-2) = 0$

$x + 3 = 0 \quad x - 2 = 0$

$x = -3 \quad x = 2$

Check $x = -3$

$2x - \sqrt{3x^2 - x + 6} = 0$

$2(-3) - \sqrt{3(-3)^2 - (-3) + 6} = 0$

$-6 - \sqrt{27 + 3 + 6} = 0$

$-6 - \sqrt{36} = 0$

$-6 - 6 = 0$

$-12 = 0$

False

Check $x = 2$

$2x - \sqrt{3x^2 - x + 6} = 0$

$2(2) - \sqrt{3(2)^2 - 2 + 6} = 0$

$4 - \sqrt{12 - 2 + 6} = 0$

$4 - \sqrt{16} = 0$

$4 - 4 = 0$

$0 = 0$ ✓

$\boxed{\{2\}}$

$x = -3$ is an extraneous solution

Simplify $\dfrac{\sqrt[4]{x^3}}{\sqrt[5]{x^2}} = \dfrac{x^{\frac{3}{4}}}{x^{2/5}} = x^{\frac{3}{4}-\frac{2}{5}}$

$= x^{\frac{7}{20}}$

Final Answer in a Single radical.

$\dfrac{3\cdot5}{4\cdot5} - \dfrac{2\cdot4}{5\cdot4} = \dfrac{15}{20} - \dfrac{8}{20} = \boxed{\dfrac{7}{20}}$

$= \sqrt[20]{x^7}$

Index $= 20$

Radicand $= x^7$

Simplify $-3\sqrt{5}\,(2\sqrt{5} \;\; -1) \;\; +30$

$= -6\sqrt{25} \quad +3\sqrt{5} \quad +30$

$= \cancel{-6\cdot5} \quad +3\sqrt{5} \; \cancel{+30} \quad = \boxed{3\sqrt{5}}$

Simplify $(2\sqrt{3} + \sqrt{5})^2 - (\sqrt{3} - 1)^2$

$= (2\sqrt{3} + \sqrt{5})(2\sqrt{3} + \sqrt{5}) - (\sqrt{3} - 1)(\sqrt{3} - 1)$

$= [4\sqrt{9} + 2\sqrt{15} + 2\sqrt{15} + \sqrt{25}] - [\sqrt{9} - \sqrt{3} - \sqrt{3} + 1]$

$= [4\cdot3 + 4\sqrt{15} + 5] - [3 - 2\sqrt{3} + 1]$

$= 17 \;\; + 4\sqrt{15} \;\; - (4 - 2\sqrt{3})$

$= 17 \;\; + 4\sqrt{15} \;\; - 4 + 2\sqrt{3}$

$= \boxed{13 \;\; + 4\sqrt{15} \;\; + 2\sqrt{3}}$

March 10, 2021

$f(x) = \sqrt{-7 - 2x}$

Find $f(-4) = \sqrt{-7-2(-4)} = \sqrt{-7+8} = \sqrt{1} = \boxed{1}$

Find its domain in interval notation.

no index ↓

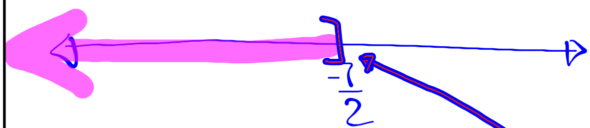
Index $= 2 \Rightarrow$ even index $\Rightarrow$ Radicand ≥ 0

$-7-2x \geq 0$

$-2x \geq 7$

$\frac{-2}{-2}x \leq \frac{7}{-2}$

$x \leq -\frac{7}{2}$



$\left(-\infty, -\frac{7}{2}\right]$

SG 12✓

working with Complex numbers:

$a + bi$

Real Part

Imaginary Part

$i = \sqrt{-1}$

$i^2 = -1$

ex: $3 - 4i$

Re.: 3

Im.: -4

$-\frac{4}{5} + \frac{3}{5}i$

Re. $-\frac{4}{5}$ Im. $\frac{3}{5}$

$-12i$

Re. : 0

Im. : -12

Standard Form

$a + bi$

Write in Complex Form

$$\sqrt{18} - \sqrt{-16} = \sqrt{9}\sqrt{2} - \sqrt{16}\sqrt{-1}$$

$$= 3\sqrt{2} - 4i$$

Re. $= 3\sqrt{2}$ Im. $= -4$

Write in Standard Form of a Complex #.

$$-\sqrt{20} + \sqrt{-50} = -\sqrt{4}\sqrt{5} + \sqrt{25}\sqrt{2}\sqrt{-1}$$

$$= -2\sqrt{5} + 5\sqrt{2}\,i$$

$$= \boxed{-2\sqrt{5} + 5i\sqrt{2}}$$

Re. $= -2\sqrt{5}$

Im. $= 5\sqrt{2}$

Operations with Complex numbers:

Simplify $2(3 - 2i) + 5(1 + 2i)$

$$= 6 - 4i + 5 + 10i$$

$$= \boxed{11 + 6i}$$

Re $= 11$

Im. $= 6$

Simplify: $3(1 - 4i) - 2(-3 + 5i)$

$$= 3 - 12i + 6 - 10i$$

$$= \boxed{9 - 22i}$$

Re. $= 9$

Im. $= -22$

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$i = \sqrt{-1}$

$i^2 = -1$

Simplify $(4+3i)(2-5i)$

"FOIL & Simplify"

$= 8 - 20i + 6i - 15i^2$

$= 8 - 14i - 15(-1)$

$= 8 - 14i + 15$

$= \boxed{23 - 14i}$

Re. = 23

Im. = −14

Simplify $(3-4i)^2 = (3-4i)(3-4i)$

$= 9 - 12i - 12i + 16i^2$

$= 9 - 24i + 16(-1)$

$= 9 - 24i - 16$

$= \boxed{-7 - 24i}$

Re. = −7

Im. = −24

$a + bi$ & $a - bi$ are Complex Conjugates.

Simplify $(6-8i)(6+8i)$

$= 36 + \cancel{48i} - \cancel{48i} - 64i^2$

$= 36 - 64(-1) = 36 + 64 = \boxed{100}$

Multiply $-5+2i$ by its Conjugate, then Simplify.

$(-5+2i)(-5-2i) = 25 + \cancel{10i} - \cancel{10i} - 4i^2$

$= 25 - 4(-1) = \boxed{29}$

How to divide complex numbers:
Multiply both numerator and denominator by complex conjugate of the denominator.
Simplify, final answer in $a+bi$ form.

$$\frac{5i}{2-i} = \frac{5i(2+i)}{(2-i)(2+i)}$$

$$= \frac{10i + 5i^2}{4 \cancel{+2i} \cancel{-2i} - i^2} = \frac{10i + 5(-1)}{4-(-1)} = \frac{-5+10i}{5}$$

$$= \frac{-5}{5} + \frac{10}{5}i$$

$$= \boxed{-1+2i}$$

$$\frac{3+2i}{3+4i} = \frac{(3+2i)(3-4i)}{(3+4i)(3-4i)} = \frac{9-12i+6i-8i^2}{9 \cancel{-12i} \cancel{+12i} - 16i^2}$$

$$= \frac{9-6i-8(-1)}{9-16(-1)}$$

$$= \frac{17-6i}{25}$$

$$= \boxed{\frac{17}{25} - \frac{6}{25}i}$$

working with power of i:

Even powers: $i^{50} = (i^2)^{25} = (-1)^{25} = \boxed{-1}$

$(-)^{odd} = -$

$i^{200} = (i^2)^{100} = (-1)^{100} = \boxed{1}$

$(-)^{even} = +$

Odd Powers: $i^{75} = i^{74} \cdot i = (i^2)^{37} i = (-1)^{37} i$

$= \boxed{-i}$

$i^{101} = i^{100} \cdot i = (i^2)^{50} \cdot i = (-1)^{50} \cdot i = +i$

$= \boxed{i}$

Simplify $3i^{36} - 5i^{91}$

even Power

odd power

$= 3(i^2)^{18} - 5\, i^{90}\, i$

$= 3(i^2)^{18} - 5(i^2)^{45} \cdot i$

$(-)^{odd} = -$

$(-)^{even} = +$

$= 3(-1)^{18} - 5(-1)^{45} i$

$= 3 \cdot 1 - 5 \cdot (-1) i$

$= \boxed{3 + 5i}$

SG 15