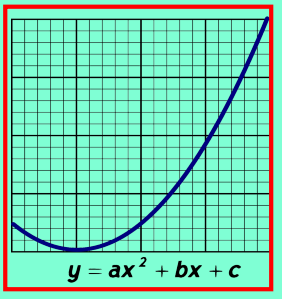
Math 125

Spring 2021

Lecture 13



Introduction to matrix:

Matrix is like a table. It has rows and Columns.

$$A = \begin{bmatrix} 1 & 2 & 5 \\ 3 & -1 & 4 \end{bmatrix}$$

2×3 Matrix

Rows Columns

$$B = \begin{bmatrix} 3 \\ -2 \\ 6 \end{bmatrix}$$

3×1

Rows Column

$$C = \begin{bmatrix} 2 & 0 \\ 0 & -3 \end{bmatrix}$$

2×2

Rows Columns

When # rows = # Columns ⇒ Square Matrix

$$D = \begin{bmatrix} 1 & -2 & 5 \\ 0 & 3 & 4 \\ 0 & 2 & 0 \end{bmatrix}$$

3×3 Square matrix

We can use matrices to work with system of linear equations

$$\begin{cases} x + 2y = 5 \\ 3x - y = 1 \end{cases} \Rightarrow \left[\begin{array}{cc|c} 1 & 2 & 5 \\ 3 & -1 & 1 \end{array}\right]$$

$$\begin{cases} x + 2y + 3z = 6 \\ 2x \quad\quad - z = 1 \\ \quad y + 2z = 3 \end{cases} \Rightarrow \left[\begin{array}{ccc|c} 1 & 2 & 3 & 6 \\ 2 & 0 & -1 & 1 \\ 0 & 1 & 2 & 3 \end{array}\right]$$

Augmented Matrix

There is a number associated with any square matrix and it is called its determinant.

$A=\begin{bmatrix}1 & 3\\ -2 & 4\end{bmatrix}$ Matrix

$\det(A)=\begin{vmatrix}1 & 3\\ -2 & 4\end{vmatrix}$ Det.

$\begin{vmatrix}a & b\\ c & d\end{vmatrix} = ad - bc$

$\begin{vmatrix}1 & 3\\ -2 & 4\end{vmatrix} = 1(4)-(-2)(3)$

$= 4 + 6 = \boxed{10}$

Evaluate

$\begin{vmatrix}-2 & 1\\ 3 & 5\end{vmatrix} = -2(5) - 3(1) = -10 - 3 = \boxed{-13}$

Evaluate:

$\begin{vmatrix}3 & 2\\ -6 & -4\end{vmatrix} = 3(-4) - (-6)(2) = -12 - (-12)$

$= -12 + 12$

$= \boxed{0}$

Cramer's Rule

$$\begin{cases} 2x - 3y = 5 \\ x + 2y = -1 \end{cases}$$

D Det. of Coef.

$D=\begin{vmatrix}2 & -3\\ 1 & 2\end{vmatrix} = 2(2) - 1(-3) = 7$

D_x det of Coef. after replacing x Coef. with RHS numbers

$D_x=\begin{vmatrix}5 & -3\\ -1 & 2\end{vmatrix} = 5(2) - (-1)(-3) = 7$

D_y det. of Coef. after replacing y Coef. with RHS numbers

$D_y=\begin{vmatrix}2 & 5\\ 1 & -1\end{vmatrix} = 2(-1) - 1(5) = -7$

Cramer's Rule

$x = \frac{D_x}{D}$, $y = \frac{D_y}{D}$ when $D \neq 0$

$x = \frac{7}{7} = 1$ $\quad y = \frac{-7}{7} = -1$

$\Rightarrow$ Final Solution $(1, -1)$

Solve $\begin{cases} 3x - 2y = 7 \\ 2x + 5y = -7 \end{cases}$ by Cramer's rule.

$D = \begin{vmatrix} 3 & -2 \\ 2 & 5 \end{vmatrix}$ $= 3(5) - 2(-2)$ $= \boxed{19}$

$D_x = \begin{vmatrix} 7 & -2 \\ -7 & 5 \end{vmatrix}$ $= 7(5) - (-7)(-2)$ $= \boxed{21}$

$D_y = \begin{vmatrix} 3 & 7 \\ 2 & -7 \end{vmatrix}$ $= 3(-7) - 2(7)$ $= \boxed{-35}$

$x = \frac{D_x}{D} = \frac{21}{19}$ $y = \frac{D_y}{D} = \frac{-35}{19}$

Final Ans $\left(\frac{21}{19}, \frac{-35}{19}\right)$

✓ The sum of two numbers is 10.

3 times one reduce by twice the other one is 0.

use Cramer's rule to find both numbers.

$\begin{cases} x + y = 10 \\ 3x - 2y = 0 \end{cases}$

$D = \begin{vmatrix} 1 & 1 \\ 3 & -2 \end{vmatrix} = 1(-2) - 3(1) = \boxed{-5}$

$D_x = \begin{vmatrix} 10 & 1 \\ 0 & -2 \end{vmatrix} = 10(-2) - 0(1) = \boxed{-20}$

$D_y = \begin{vmatrix} 1 & 10 \\ 3 & 0 \end{vmatrix} = 1(0) - 3(10) = \boxed{-30}$

Cramer's Rule

$x = \frac{D_x}{D} = \frac{-20}{-5}$ $y = \frac{D_y}{D} = \frac{-30}{-5}$

$\boxed{x = 4}$ $\boxed{y = 6}$

The numbers are 4 & 6.

John has 8 Coins. ✓ $x \to$ Nickels

Nickels & Dimes only. $y \to$ Dimes

He has 50 ¢. ✓

How many of each?

Use Cramer's rule.

$$\begin{cases} x + y = 8 \\ 5x + 10y = 50 \end{cases} \div 5$$

$$\begin{cases} x + y = 8 \\ x + 2y = 10 \end{cases}$$

$$D = \begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix} = 1(2) - 1(1) = \boxed{1}$$

$$D_x = \begin{vmatrix} 8 & 1 \\ 10 & 2 \end{vmatrix} = 8(2) - 10(1) = \boxed{6}$$

$$D_y = \begin{vmatrix} 1 & 8 \\ 1 & 10 \end{vmatrix} = 1(10) - 1(8) = \boxed{2}$$

$$x = \frac{D_x}{D} = \frac{6}{1} = 6$$

$$y = \frac{D_y}{D} = \frac{2}{1} = 2$$

6 Nickels & 2 Dimes

Mary needs 50 lb of Candy @ \$1.60/lb.

She has unlimited supply of \$1.75/lb and \$1.50/lb.

How many pounds of each?

use Cramer's rule to Solve

$$\begin{cases} x + y = 50 \\ 1.75x + 1.50y = 1.60(50) \end{cases}$$

\$1.75	+	\$1.50	=	Mixture \$1.60
x		y		50 lb.

$$\begin{cases} x + y = 50 \\ 1.75x + 1.50y = 80 \end{cases} \times 100 \quad \Rightarrow \quad \begin{cases} x + y = 50 \\ 175x + 150y = 8000 \end{cases} \div 25$$

$$\begin{cases} x + y = 50 \\ 7x + 6y = 320 \end{cases}$$

$$D = \begin{vmatrix} 1 & 1 \\ 7 & 6 \end{vmatrix} = 1(6) - 7(1) = \boxed{-1}$$

$$D_x = \begin{vmatrix} 50 & 1 \\ 320 & 6 \end{vmatrix} = 50(6) - 320(1) = \boxed{-20}$$

$$D_y = \begin{vmatrix} 1 & 50 \\ 7 & 320 \end{vmatrix} = 1(320) - 7(50) = \boxed{-30}$$

$$x = \frac{D_x}{D} = \frac{-20}{-1} = \boxed{20}$$

$$y = \frac{D_y}{D} = \frac{-30}{-1} = \boxed{30}$$

20 lb @ \$1.75 & 30 lb @ \$1.50

Gary need 100 liters of 17% alcohol solution.
Gary has unlimited alcohol solution @ 8% and 20% alcohol.
How many liters of each should he combine to obtain what he needs? Use Cramer's rule.

8%	+	20%	=	17%
x		y		100

$$\begin{cases} x + y = 100 \\ 8\%x + 20\%y = 17\%(100) \end{cases} \xrightarrow{\cdot 100} \begin{cases} x + y = 100 \\ 8x + 20y = 17(100) \end{cases} \xrightarrow{\div 4}$$

$$\begin{cases} x + y = 100 \\ 2x + 5y = 17(25) \end{cases} \Rightarrow \begin{cases} x + y = 100 \\ 2x + 5y = 425 \end{cases}$$

$D = \begin{vmatrix} 1 & 1 \\ 2 & 5 \end{vmatrix} = 1(5) - 2(1) = \boxed{3}$

$D_x = \begin{vmatrix} 100 & 1 \\ 425 & 5 \end{vmatrix} = 100(5) - 425(1) = \boxed{75}$

$D_y = \begin{vmatrix} 1 & 100 \\ 2 & 425 \end{vmatrix} = 1(425) - 2(100) = \boxed{225}$

$x = \frac{D_x}{D} = \frac{75}{3} = \boxed{25}$ $\quad y = \frac{D_y}{D} = \frac{225}{3} = \boxed{75}$

25L @ 8% and 75L @ 20%.

3x3 Determinant

Always

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = a\begin{vmatrix} e & f \\ h & i \end{vmatrix} - b\begin{vmatrix} d & f \\ g & i \end{vmatrix} + c\begin{vmatrix} d & e \\ g & h \end{vmatrix}$$

Evaluate

Always

$$\begin{vmatrix} 2 & 3 & -1 \\ 4 & 2 & 1 \\ 0 & 5 & 2 \end{vmatrix} = 2\begin{vmatrix} 2 & 1 \\ 5 & 2 \end{vmatrix} - 3\begin{vmatrix} 4 & 1 \\ 0 & 2 \end{vmatrix} + (-1)\begin{vmatrix} 4 & 2 \\ 0 & 5 \end{vmatrix}$$

$$= 2(4-5) - 3(8-0) - 1(20-0)$$

$$= 2(-1) - 3(8) - 1(20) = -2 - 24 - 20$$

$$= \boxed{-46}$$

Evaluate $\begin{vmatrix} 3 & -2 & 0 \\ 0 & 1 & 8 \\ 2 & 0 & -4 \end{vmatrix}$

Always

$= 3\begin{vmatrix} 1 & 8 \\ 0 & -4 \end{vmatrix} - (-2)\begin{vmatrix} 0 & 8 \\ 2 & -4 \end{vmatrix} + 0\begin{vmatrix} 0 & 1 \\ 2 & 0 \end{vmatrix}$

$= 3(-4-0) + 2(0-16) + 0(\text{who cares!})$

$= 3(-4) + 2(-16) + 0$

$= -12 - 32 = \boxed{-44}$

Evaluate $\begin{vmatrix} 1 & -4 & 2 \\ 3 & 2 & 1 \\ 4 & -2 & 3 \end{vmatrix}$

Always

$= 1\begin{vmatrix} 2 & 1 \\ -2 & 3 \end{vmatrix} - (-4)\begin{vmatrix} 3 & 1 \\ 4 & 3 \end{vmatrix} + 2\begin{vmatrix} 3 & 2 \\ 4 & -2 \end{vmatrix}$

$= 1(6-(-2)) + 4(9-4) + 2(-6-8)$

$= 1(8) + 4(5) + 2(-14)$

$= 8 + 20 - 28 = \boxed{0}$

Locations & items

	coffee	tea	sugar	. . . .
East LA Walmart	50	30	80	. . . .
Long Beach				

Solve for z by Cramer's rule.

$$\begin{cases} x + y + z = 6 \\ 2x + y = 4 \\ y - z = -1 \end{cases}$$

$z = \dfrac{D_z}{D} = \dfrac{9}{3} = \boxed{3}$

Always

$D = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 1 & 0 \\ 0 & 1 & -1 \end{vmatrix} = 1\begin{vmatrix} 1 & 0 \\ 1 & -1 \end{vmatrix} - 1\begin{vmatrix} 2 & 0 \\ 0 & -1 \end{vmatrix} + 1\begin{vmatrix} 2 & 1 \\ 0 & 1 \end{vmatrix}$

$= 1(-1-0) - 1(-2-0) + 1(2-0)$

$= -1 + 2 + 2 = \boxed{3}$

Always

$D_z = \begin{vmatrix} 1 & 1 & 6 \\ 2 & 1 & 4 \\ 0 & 1 & -1 \end{vmatrix} = 1\begin{vmatrix} 1 & 4 \\ 1 & -1 \end{vmatrix} - 1\begin{vmatrix} 2 & 4 \\ 0 & -1 \end{vmatrix} + 6\begin{vmatrix} 2 & 1 \\ 0 & 1 \end{vmatrix}$

$= 1(-1-4) - 1(-2-0) + 6(2-0)$

$= 1(-5) - 1(-2) + 6(2) = -5 + 2 + 12 = \boxed{9}$

Solve for y by Cramer's rule:

$$\begin{cases} 2x - y + z = 4 \\ x + 2y = 5 \\ 4x - z = 0 \end{cases}$$

$y = \frac{D_y}{D} = \frac{-26}{-13}$

$\boxed{y = 2}$

$$D = \begin{vmatrix} 2 & -1 & 1 \\ 1 & 2 & 0 \\ 4 & 0 & -1 \end{vmatrix}$$

Always

$$= 2\begin{vmatrix} 2 & 0 \\ 0 & -1 \end{vmatrix} - (-1)\begin{vmatrix} 1 & 0 \\ 4 & -1 \end{vmatrix} + 1\begin{vmatrix} 1 & 2 \\ 4 & 0 \end{vmatrix}$$

$$= 2(-2-0) + 1(-1-0) + 1(0-8)$$

$$= -4 - 1 - 8 = \boxed{-13}$$

$$D_y = \begin{vmatrix} 2 & 4 & 1 \\ 1 & 5 & 0 \\ 4 & 0 & -1 \end{vmatrix}$$

Always

$$= 2\begin{vmatrix} 5 & 0 \\ 0 & -1 \end{vmatrix} - 4\begin{vmatrix} 1 & 0 \\ 4 & -1 \end{vmatrix} + 1\begin{vmatrix} 1 & 5 \\ 4 & 0 \end{vmatrix}$$

$$= 2(-5-0) - 4(-1-0) + 1(0-20)$$

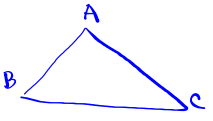
$$= -10 + 4 - 20$$

$$= \boxed{-26}$$

In triangle ABC,

Angle A is 5 times angle C.

Angle B is 3 times angle C.



Use Cramer's rule to find angle A.

$$\begin{cases} A + B + C = 180^\circ \\ A = 5C \\ B = 3C \end{cases} \quad \begin{cases} A + B + C = 180 \\ A - 5C = 0 \\ B - 3C = 0 \end{cases}$$

$$D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 0 & -5 \\ 0 & 1 & -3 \end{vmatrix}$$

Always

$$= 1\begin{vmatrix} 0 & -5 \\ 1 & -3 \end{vmatrix} - 1\begin{vmatrix} 1 & -5 \\ 0 & -3 \end{vmatrix} + 1\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$$

$$= 1(0+5) - 1(-3-0) + 1(1-0)$$

$$= 5 + 3 + 1 = \boxed{9}$$

$$D_A = \begin{vmatrix} 180 & 1 & 1 \\ 0 & 0 & -5 \\ 0 & 1 & -3 \end{vmatrix}$$

Always

$$= 180\begin{vmatrix} 0 & -5 \\ 1 & -3 \end{vmatrix} - 1\begin{vmatrix} 0 & -5 \\ 0 & -3 \end{vmatrix} + 1\begin{vmatrix} 0 & 0 \\ 0 & 1 \end{vmatrix}$$

$$= 180(0+5) - 1(0-0) + 1(0-0)$$

$$= \boxed{900}$$

$A = 100^\circ$

$A = \frac{D_A}{D} = \frac{900}{9} = \boxed{100}$

Graph of the equation $y = ax^2 + bx + c$ contains $(1,2)$, $(-1,6)$, and $(2,9)$. Find c using Cramer's rule.

$(1,2) \Rightarrow x=1, y=2 \Rightarrow a(1)^2 + b(1) + c = 2$

$(-1,6) \Rightarrow x=-1, y=6 \Rightarrow a(-1)^2 + b(-1) + c = 6$

$(2,9) \Rightarrow x=2, y=9 \Rightarrow a(2)^2 + b(2) + c = 9$

$$\begin{cases} a + b + c = 2 \\ a - b + c = 6 \\ 4a + 2b + c = 9 \end{cases}$$

$$D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 4 & 2 & 1 \end{vmatrix} = 6$$

$$D_c = \begin{vmatrix} 1 & 1 & 2 \\ 1 & -1 & 6 \\ 4 & 2 & 9 \end{vmatrix} = 6$$

$$c = \frac{D_c}{D} = \frac{6}{6} = \boxed{1}$$

Class QZ 9

1) Evaluate $\begin{vmatrix} 5 & -2 \\ 3 & 4 \end{vmatrix}$

2) Solve $x^2 - 6x - 16 = 0$