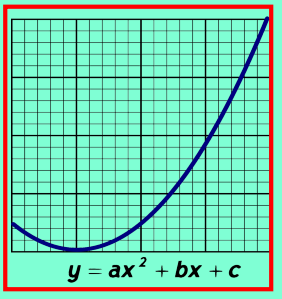
Math 125

Spring 2021

Lecture 11



Exam 1: Monday

New Materials from today will not be on Exam 1.

System of linear equations in 3 variables:

Equations have a form $ax + by + cz = d$

Ex.

$$\begin{cases} 2x + y - 3z = -2 \\ x - 4y + z = 24 \\ -3x - y + 4z = 0 \end{cases}$$

Solution is there is any is an ordered triple (x, y, z)

The solution must satisfy all equations.

Is $(3, -5, 1)$ a solution for above system? Yes

$2x + y - 3z = -2$ ✓

$2(3) - 5 - 3(1) = -2$

$6 - 5 - 3 = -2$

$1 - 3 = -2$

$-2 = -2$

$x - 4y + z = 24$ ✓

$3 - 4(-5) + 1 = 24$

$3 + 20 + 1 = 24$

$23 + 1 = 24$

$24 = 24$

$-3x - y + 4z = 0$ ✓

$-3(3) - (-5) + 4(1) = 0$

$-9 + 5 + 4 = 0$

$-4 + 4 = 0$

$0 = 0$

Is $(3,-1,5)$ a Solution for

$$\begin{cases} -x-y+z=3 \checkmark \\ 3x+4y-z=1 \\ 5x+7y-z=-1 \end{cases}$$

$-x-y+z=3\checkmark$

$-3-(-1)+5=3$

$-3+1+5=3$

$-2+5=3$

$3=3\checkmark$

$3x+4y-z=1$

$3(3)+4(-1)-5=1$

$9-4-5=1$

$5-5=1$

$0=1$ False

Equation 2 $\Rightarrow$ Fail to Satisfy

$\Rightarrow$ $(3,-1,5)$ is <u>not</u> a Solution.

Solve $\begin{cases} 3x+2y+5z=12 \\ 3y+8z=-8 \\ 10z=20 \end{cases}$

Back Subs.

$10z=20 \Rightarrow z=\frac{20}{10} \Rightarrow \boxed{z=2}$

One more back Subs.

$3y+8z=-8$

$3y+8(2)=-8$

$3y+16=-8$

$3y=-8-16$

$3y=-24$

$\boxed{y=-8}$

$3x+2y+5z=12$

$3x+2(-8)+5(2)=12$

$3x-16+10=12$

$3x-6=12$

$3x=18$ $\boxed{x=6}$

$\{(6,-8,2)\}$

Solve $\begin{cases} 2x+5z=2 \\ 3y-7z=9 \\ 5x-9y=-22 \end{cases} \Rightarrow \begin{cases} 2x \quad +5z=2 \\ \quad 3y \quad -7z=9 \\ 5x \quad -9y \quad =-22 \end{cases}$

$3\{ \quad 3y-7z=9 \quad \Rightarrow \quad 9y-21z=27$

$\{ \quad 5x-9y=-22 \quad\quad 5x-9y=-22$

Eliminate Y. $\qquad 5x-21z=5$

Solve $5\{ \quad 2x+5z=2 \quad \Rightarrow \quad 10x+25z=10$

$-2\{ \quad 5x-21z=5 \quad\quad -10x+42z=-10$

eliminate x. $\qquad 67z=0$

$z=\frac{0}{67}$

$\boxed{z=0}$

$3y-7z=9$

$3y-7(0)=9$

$3y-0=9$

$3y=9$

$\boxed{y=3}$

$2x+5z=2$

$2x+5(0)=2$

$2x+0=2$

$2x=2$

$\boxed{x=1}$

Ordered-Triple

$(x,y,z)=(1,3,0)$

Solve $\begin{cases} 4x=3y-2z-5 \\ 2(x+y)=y+z-6 \\ 6(x-y)+z=x-5y-8 \end{cases}$

Hint: Distribute, and Simplify. write all equations in the form of $ax+by+cz=d$

$\begin{cases} 4x-3y+2z=-5 \\ 2x+y-z=-6 \\ 5x-y+z=-8 \end{cases}$

$2\{ \quad 4x-3y+2z=-5 \quad \Rightarrow \quad 4x-3y+2z=-5$

$\{ \quad 2x+y-z=-6 \quad\quad 4x+2y-2z=-12$

eliminate z $\qquad \Rightarrow \boxed{8x-y=-17}$

$\{ \quad 2x+y-z=-6 \quad \Rightarrow \boxed{7x=-14} \quad \boxed{x=-2}$

$\{ \quad 5x-y+z=-8$

eliminate z

$5x-y+z=-8$

$5(-2)-1+z=-8$

$-10-1+z=-8$

$-11+z=-8$

$\boxed{z=3}$

$8(-2)-y=-17$

$-16-y=-17$

$-y=-1$

$\boxed{y=1}$

Ordered-Triple

$(x,y,z)=(-2,1,3)$

$\{(-2,1,3)\}$

Solve

$$\begin{cases} x + \frac{7}{2}y + \frac{1}{2}z = 4 & \text{LCD}=2 \\ \frac{3}{4}x + y + \frac{1}{2}z = -1 & \text{LCD}=4 \\ \frac{1}{10}x - \frac{2}{5}y - \frac{3}{10}z = 1 & \text{LCD}=10 \end{cases}$$

Hint: Use LCD to clear all fractions

$$\begin{cases} 2x + 7y + z = 8 \\ 3x + 4y + 2z = -4 \\ x - 4y - 3z = 10 \end{cases}$$

$$-2\begin{cases} 2x + 7y + z = 8 \\ 3x + 4y + 2z = -4 \end{cases}$$

$$-x - 10y = -20 \quad *$$

$$3\begin{cases} 2x + 7y + z = 8 \\ x - 4y - 3z = 10 \end{cases}$$

$$7x + 17y = 34 \quad **$$

$$7\begin{cases} -x - 10y = -20 \\ 7x + 17y = 34 \end{cases} \Rightarrow \begin{cases} -7x - 70y = -140 \\ 7x + 17y = 34 \end{cases}$$

$$-53y = -106$$

$$y = 2$$

$$7x + 17(2) = 34$$

$$7x + 34 = 34$$

$$7x = 0$$

$$x = 0$$

$$2x + 7y + z = 8$$

$$2(0) + 7(2) + z = 8$$

$$z = -6$$

Ordered-Triple $(0, 2, -6)$

Solution Set $\{(0, 2, -6)\}$

Solve

$$\begin{cases} x + 2y + 4z = 3 \\ y + 3z = 5 \\ x - 2z = -7 \end{cases}$$

$$-2\begin{cases} x + 2y + 4z = 3 \\ y + 3z = 5 \end{cases}$$

$$\begin{cases} x + 2y + 4z = 3 \\ -2y - 6z = -10 \end{cases}$$

$$x - 2z = -7 \quad ***$$

Pair with eqn 3

$$-1\begin{cases} x - 2z = -7 \\ x - 2z = -7 \end{cases}$$

$$0 = 0$$

True

$\Rightarrow$ infinitely many Solutions

System is consistent.

Solve

$$\begin{cases} -4x - 3y = 0 \\ 3y + z = -1 \\ 4x - z = 12 \end{cases}$$

$$\begin{cases} -4x - 3y = 0 \\ 3y + z = -1 \end{cases}$$

$-4x + z = -1$ *

Pair this with eqn 3 from the original Problem

$$\begin{cases} 4x - z = 12 \\ -4x + z = -1 \end{cases}$$

$0 = 11$

False $\Rightarrow$ No Solution

System is inconsistent.

Quadratic graph $y = ax^2 + bx + c$; $a \neq 0$

$a > 0$ $\quad$ $a < 0$

graph is called parabola.

Find an equation of a parabola that contains $(4,2)$, $(1,-1)$, and $(-1,7)$.

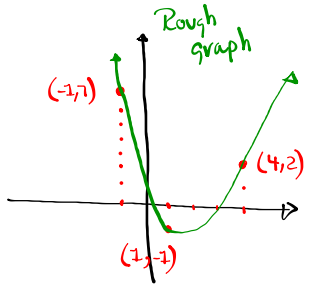
$y = ax^2 + bx + c$

At $(1,-1)$ $\quad a(1)^2 + b(1) + c = -1$

At $(-1,7)$ $\quad a(-1)^2 + b(-1) + c = 7$

At $(4,2)$ $\quad a(4)^2 + b(4) + c = 2$

Rough graph



$$\begin{cases} a + b + c = -1 \\ a - b + c = 7 \\ 16a + 4b + c = 2 \end{cases}$$

LeBron had 26 points in one game.

He scored four more 2-point baskets (y) than he did 3-point basket (z).

The number of free-throws was equal to the sum of 2-point and 3-point baskets.

How many of each did he make?

FT → 1 pt x → # FT

2-point shots → 2 pts y → # 2-pt shots

3-Point Shots → 3 pts. z → # 3-pt shots

$$\begin{cases} 1x + 2y + 3z = 26 \\ y = z + 4 \\ x = y + z \end{cases} \Rightarrow \begin{cases} x + 2y + 3z = 26 \\ y - z = 4 \\ x - y - z = 0 \end{cases}$$

$$\begin{cases} x + 2y + 3z = 26 \\ x - y - z = 0 \end{cases} \quad -1$$

$$3y + 4z = 26$$

$$\begin{cases} y - z = 4 \\ 3y + 4z = 26 \end{cases} \quad 4$$

$$7y = 42 \quad \boxed{y = 6}$$

$$6 - z = 4 \quad \boxed{z = 2}$$

$$x - y - z = 0$$

$$x - 6 - 2 = 0$$

$$\boxed{x = 8}$$

8 FT, 6 2-pt shots, and 2 3-pt shots

Jose invested \$8000 in 3 accounts.

Stock pays 6.2% in 1 yr.

Real estate (lost) 13.5% in 1 yr.

Bond pays 4.4% in 1 yr.

He earned a total of \$66 in return.

The amount invested in stock was twice the amount in real estate.

How much per account?

S → Stock

R → Real Estate

B → Bond

$$\begin{cases} S + R + B = 8000 \\ S = 2R \\ 6.2\%\,S - 13.5\%\,R + 4.4\%\,B = 66 \end{cases} \quad 100$$

$$\begin{cases} S + R + B = 8000 \\ S - 2R = 0 \\ 6.2S - 13.5R + 4.4B = 6600 \end{cases} \quad 10$$

$$\Rightarrow \begin{cases} S + R + B = 8000 \\ S - 2R = 0 \\ 62S - 135R + 44B = 66000 \end{cases}$$

$$\begin{cases} 2R + R + B = 8000 \\ 62(2R) - 135R + 44B = 66000 \end{cases}$$

$$124R - 135R$$

$$\Rightarrow \begin{cases} 3R + B = 8000 \\ -11R + 44B = 66000 \end{cases} \quad -4$$

$$\begin{cases} -132R - 44B = -352000 \\ -11R + 44B = 66000 \end{cases}$$

$$-143R = -286000$$

$$R = \frac{-286000}{-143}$$

$$\boxed{R = 2000}$$

$$3(2000) + B = 8000$$

$$\boxed{B = 2000}$$

$$\boxed{S = 4000}$$

He invested \$4000 in stock, \$2000 in bonds, and \$2000 in real estate.

$f(x) = 4x + 3 \qquad g(x) = 4x - 3$

Find

1) $(f+g)(x) = f(x) + g(x)$
$= 4x+3 + 4x-3$
$= \boxed{8x}$

2) $(f-g)(x) = f(x) - g(x)$
$= 4x+3 - (4x-3)$
$= 4x+3-4x+3 = \boxed{6}$

3) $(f \cdot g)(x) = f(x) \cdot g(x)$
$= (4x+3)(4x-3)$
$= 16x^2 - 12x + 12x - 9$
$= \boxed{16x^2 - 9}$

4) $(f \circ g)(x)$
$= f(g(x))$
$= 4\,g(x) + 3$
$= 4(4x-3) + 3$
$= 16x - 12 + 3$
$= \boxed{16x - 9}$

Simplify

$(3x+5)^2 - (3x-5)^2$

$= (3x+5)(3x+5) - (3x-5)(3x-5)$

$= 9x^2 + 15x + 15x + 25 - [9x^2 - 15x - 15x + 25]$

$= 9x^2 + 30x + 25 - [9x^2 - 30x + 25]$

$= 9x^2 + 30x + 25 - 9x^2 + 30x - 25 = \boxed{60x}$

$$\begin{cases} a + b + c = -1 \\ a - b + c = 7 \\ 16a + 4b + c = 2 \end{cases}$$

$$-1 \begin{cases} a + b + c = -1 \\ a - b + c = 7 \end{cases}$$

$$\begin{cases} a + b + c = -1 \\ -a + b - c = -7 \end{cases}$$

$2b = -8 \quad \boxed{b = -4}$

$$\begin{cases} a + b + c = -1 \\ 16a + 4b + c = 2 \end{cases} \Rightarrow \begin{cases} a - 4 + c = -1 \\ 16a + 4(-4) + c = 2 \end{cases} \Rightarrow -1 \begin{cases} a + c = 3 \\ 16a + c = 18 \end{cases}$$

$1 + c = 3 \quad \boxed{c = 2}$

$15a = 15 \quad \boxed{a = 1}$

$y = ax^2 + bx + c \qquad \boxed{y = x^2 - 4x + 2}$