Working with Quadratic Equation

\[ a \chi^2 + b \chi + c = 0 \text{, } a \neq 0 \]

\[ b^2 - 4ac \]

is called the discriminant.

Ex. \[ 3 \chi^2 - 5 \chi + 4 = 0 \]

Standard form

\[ a = 3 \]
\[ b = -5 \]
\[ c = 4 \]

\[ b^2 - 4ac = (-5)^2 - 4(3)(4) \]

\[ = 25 + 48 = 73 \]

Solving by \( \alpha \)-formula:

\[ \chi = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

\[ \chi = \frac{-(-5) \pm \sqrt{73}}{2(3)} = \frac{5 \pm \sqrt{73}}{6} \]

\[ \left\{ \frac{5 + \sqrt{73}}{6} \right\} \]
Given \( (2x - 7)(3x + 2) = -25 \)

1) Write in standard form
\[
6x^2 + 4x - 21x - 14 + 25 = 0 \\
6x^2 - 17x + 11 = 0
\]

2) Identify \( a, b, \) and \( c \) \( a = 6, \ b = -17, \ c = 11 \)

3) Find its discriminant \( b^2 - 4ac = (-17)^2 - 4(6)(11) \)
\[
= 25
\]

4) Solve by \( \pm \) formula.
\[
\chi = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-17) \pm \sqrt{25}}{2(6)} = \frac{17 \pm 5}{12}
\]
\[
\chi_1 = \frac{17 + 5}{12} = \frac{11}{6} \quad \chi_2 = \frac{17 - 5}{12} = \frac{1}{2}
\]
\[
\chi = \left\{ \frac{1}{2}, \frac{11}{6} \right\}
\]

**Properties of \( b^2 - 4ac \)**

- \( b^2 - 4ac > 0 \): Two different real solutions  
  - If it is a perfect-square: \( a \chi^2 + b \chi + c \) is factorable.
- \( b^2 - 4ac = 0 \): One repeated real solution
- \( b^2 - 4ac < 0 \): Two complex solutions  
  - Two imaginary solutions
Consider \(4x^2 - 3x - 7 = 0\)

1) Identify \(a, b,\) and \(c\).
   \[a = 4, \ b = -3, \ c = -7\]

2) Find its discriminant.
   \[b^2 - 4ac = (-3)^2 - 4(4)(-7) = 121\]

3) Discuss the type of solutions
   \[b^2 - 4ac > 0 \Rightarrow \text{Two different real solutions.}\]

4) Can the equation be solved by factoring?
   Yes, because \(b^2 - 4ac\) is a perfect square.

Consider \(9x^2 + 25 = -60x\)

1) Identify \(a, b,\) and \(c\).
   \[a = 9, \ b = 60, \ c = 25\]

2) Find its discriminant.
   \[b^2 - 4ac = 60^2 - 4(9)(25) = 3600 - 900 = 2700\]

3) Discuss the type of solutions
   \[b^2 - 4ac > 0 \Rightarrow \text{Two different real solutions}\]

4) Can the equation be solved by factoring?
   No, why not? \(b^2 - 4ac\) is not a perfect square.
Consider \(49\chi^2 + 4 = 28\chi\)

1) Identify \(a\), \(b\), and \(c\).
   \[49\chi^2 - 28\chi + 4 = 0\]
   \[a = 49, \quad b = -28, \quad c = 4\]

2) Find its discriminant.
   \[b^2 - 4ac = (-28)^2 - 4(49)(4) = 0\]

3) Discuss the type of solutions
   \[b^2 - 4ac = 0 \implies \text{one repeated real solution}\]

4) Can the equation be solved by factoring?
   Yes, \(b^2 - 4ac\) is a perfect square.

Consider \(36\chi^2 + 60\chi + 30 = 0\)

1) Identify \(a\), \(b\), and \(c\).
   \[a = 36, \quad b = 60, \quad c = 30\]

2) Find its discriminant.
   \[b^2 - 4ac = 60^2 - 4(36)(30) = -720\]

3) Discuss the type of solutions
   \[b^2 - 4ac < 0 \implies \text{two imaginary solutions}\]

4) Can the equation be solved by factoring?
   No, \(b^2 - 4ac\) is not a perfect square.
Discuss the type of solutions for 

\[(2x + 5)(3x + 2) = 10\]

\[6x^2 + 4x + 15x + 10 - 10 = 0\]
\[6x^2 + 19x = 0\]

\[a = 6 \quad b = 19 \quad c = 0\]

\[b^2 - 4ac = 19^2 - 4(6)(0) = 19^2 = 361\]

\[b^2 - 4ac > 0 \quad \text{Two different real solutions}\]

This problem can be solved by factoring since \[b^2 - 4ac\] is a perfect square.

Work on SG 16 & 17

Be ready to finish them tomorrow and turn in.